Formula Sheet

1 Time Value of Money

1.1 Future Value
The future value of $x$ after $n$ periods of growth at (annual) interest rate $a$ compounded $m$ times per year is

$$x(1 + r)^n$$

where $r = a/m$ is the per-period interest rate.

The effective annual interest rate is

$$i = (1 + a/m)^m - 1.$$

The future value of $x$ after $t$ years of growth at annual growth rate $d$ is

$$x(1 + d)^t.$$

1.2 Present Value
In the following, $r$ is the per-period discount rate, $d$ is the annual discount rate, and there are $m$ periods per year.

The present value of $y$ to be received $n$ periods later is

$$y(1 + r)^{-n} = \frac{y}{(1 + r)^n}.$$

The present value of $y$ to be received $t$ years later is

$$y(1 + d)^{-t} = \frac{y}{(1 + d)^t}.$$

The relationship between $r$ and $d$ is

$$d = (1 + r)^m - 1 \quad \text{and} \quad r = (1 + d)^{1/m} - 1.$$

1.3 Present Value: Perpetuities and Annuities
When the discount rate is $r$ per period, an annuity making $n$ payments of $C$, each one period apart, starting in one period:

$$\frac{C}{r}(1 - (1 + r)^{-n}).$$

Present value of a perpetuity of $C$ per period, starting in one period:

$$\frac{C}{r}.$$
2 Bonds

A coupon payment of a bond with face value \( F \), coupon rate \( c \) and \( m \) coupon payments per year is

\[
Fc/m.
\]

If the yield (quoted annually) is \( y \) for a bond making \( m \) coupon payments per year, the corresponding per-period discount rate is (because of the yield quotation convention)

\[
r = y/m.
\]

The price of a bond with face value \( F \), coupon rate \( c \), \( m \) coupon payments per year, next coupon payment in 1 period, \( n \) coupon payments remaining, and yield \( y \) is

\[
F(1 + r)^{-n} + \frac{Fc}{y}(1 - (1 + r)^{-n}).
\]

3 Inflation

The real cost, as measured in base-\( b \) dollars, of an actual cost \( A \) at time \( t \), is

\[
R = A(1 + f)^{(b-t)},
\]

where \( f \) is the annual rate of inflation.

If the actual cost of something at time \( t \) is \( A_t \), and its actual cost changes at an annual rate \( g \), then its actual cost at time \( u \) is

\[
A_u = A_t(1 + g)^{(u-t)}.
\]

The relationship between the inflation rate \( f \), the actual discount rate \( d_A \), and the real discount rate \( d_R \) is

\[
(1 + f)(1 + d_R) = (1 + d_A).
\]