1 Time Value of Money

1.1 Future Value
The future value of $x$ after $n$ periods of growth at (annual) interest rate $a$ compounded $m$ times per year is

$$x(1 + r)^n$$

where $r = a/m$ is the per-period interest rate.

The effective annual interest rate is

$$i = (1 + a/m)^m - 1.$$ 

The future value of $x$ after $t$ years of growth at annual growth rate $d$ is

$$x(1 + d)^t.$$ 

1.2 Present Value
In the following, $r$ is the per-period discount rate, $d$ is the annual discount rate, and there are $m$ periods per year.

The present value of $y$ to be received $n$ periods later is

$$y(1 + r)^{-n} = \frac{y}{(1 + r)^n}.$$ 

The present value of $y$ to be received $t$ years later is

$$y(1 + d)^{-t} = \frac{y}{(1 + d)^t}.$$ 

The relationship between $r$ and $d$ is

$$d = (1 + r)^m - 1 \quad \text{and} \quad r = (1 + d)^{1/m} - 1.$$ 

1.3 Present Value: Perpetuities and Annuities
When the discount rate is $r$ per period, an annuity making $n$ payments of $C$, each one period apart, starting in one period:

$$\frac{C}{r}(1 - (1 + r)^{-n}).$$

Present value of a perpetuity of $C$ per period, starting in one period:

$$\frac{C}{r}.$$
2 Bonds

A coupon payment of a bond with face value $F$, coupon rate $c$ and $m$ coupon payments per year is

$$Fc/m.$$  

If the yield (quoted annually) is $y$ for a bond making $m$ coupon payments per year, the corresponding per-period discount rate is (because of the yield quotation convention)

$$r = y/m.$$

The price of a bond with face value $F$, coupon rate $c$, $m$ coupon payments per year, next coupon payment in 1 period, $n$ coupon payments remaining, and yield $y$ is

$$F(1 + r)^{-n} + \frac{Fc}{y}(1 - (1 + r)^{-n}).$$

3 Inflation

The real cost, as measured in base-$b$ dollars, of an actual cost $A$ at time $t$, is

$$R = A(1 + f)^{(b-t)},$$

where $f$ is the annual rate of inflation.

If the actual cost of something at time $t$ is $A_t$, and its actual cost changes at an annual rate $g$, then its actual cost at time $u$ is

$$A_u = A_t(1 + g)^{(u-t)}.$$  

The relationship between the inflation rate $f$, the actual discount rate $d_A$, and the real discount rate $d_R$ is

$$(1 + f)(1 + d_R) = (1 + d_A).$$