

## Midterm 1

1. (12 points) What is the effective annual interest rate in each situation?

a. A savings account with 4% annual interest rate compounded daily (assume a year consists of 365 days)?

ANSWER:  $(1+0.04/365)^{365} - 1 = 0.0408 = \mathbf{4.08\%}$

b. A savings account with 4% annual interest rate compounded monthly?

ANSWER:  $(1+0.04/12)^{12} - 1 = 0.0407 = \mathbf{4.07\%}$

2. Northwestern's endowment bought a year ago a bond with face value \$10,000, paying semiannual coupons at an annual coupon rate of 10%.

2.1 (5 points) What is the dollar amount of each coupon payment?

- (a) \$1200
- (b) \$1000
- (c) \$600
- (d) \$500
- (e) zero
- (f) Something else.
- (g) More information is required to answer the question.

ANSWER:  $10000 * 10\% / 2 = \mathbf{\$500}$ , (d)

2.2 (10 points) The bond's yield fell by 1.5% (150 basis points) over the last year. This implies that:

- a) The value of this bond increased.
- b) The value of this bond stayed the same.
- c) The value of this bond fell.

ANSWER: (a) Yield goes down, so value goes up.

3. (10 points) You need to invest money for one year and decide to buy a 30-year Treasury bond issued this month with a 5% yield. What risk results from this mismatch of when you need the money and when the bond matures?

- a) inflation risk
- b) interest-rate risk
- c) reinvestment risk
- d) credit risk
- e) funding liquidity risk

ANSWER: (b) You will need to sell the bond after 1 year (i.e., before it matures), so its value depends on the current interest rates.

4. (10 points) Which type of risk is most relevant for the bond issuer?
- a) inflation risk
  - b) interest-rate risk
  - c) reinvestment risk
  - d) credit risk
  - e) funding liquidity risk

ANSWER: (e) This is the only of these risks that apply to issuers of bonds (the other risks apply to bond investors).

5. (11 points) Consider the following cashflow stream and a bank account paying 3% annual interest. What is the present value? Is the account value ever negative?

Year	Cashflow
0	8
1	2
2	4
3	-15
4	16

ANSWER: Present value equals  $8 + 2 \cdot 1.03^{-1} + 4 \cdot 1.03^{-2} - 15 \cdot 1.03^{-3} + 16 \cdot 1.03^{-4} = 14.20$ . If the account value is ever negative, then it will be at the end of year 3. The present value up cashflows through year 3 is  $8 + 2 \cdot 1.03^{-1} + 4 \cdot 1.03^{-2} - 15 \cdot 1.03^{-3} = -0.01$ . Since this is negative, **the account will be negative at the end of year 3.**

6. (24 points) Which of the following cashflows do you most prefer using a discount rate of 10%? Using a discount rate of 1%? Show and explain all supporting calculations!

Cashflow A: receive \$10 every year, forever, with the first payment next year

Cashflow B: receive \$19 every other year, forever, with the first payment being next year

Cashflow C: pay \$5 every year for 20 years, with the first payment being today, and then subsequently receive \$30 every year for 20 years.

Cashflow D: receive \$70 today and then receive \$50 in five years.

ANSWER: The present value of cashflow A is  $10/r$ , or 100 when  $r = 10\%$  and 1000 when  $r = 1\%$ . The two period interest rate is  $s = (1+r)^2 - 1$ , or 21% when  $r = 10\%$  and 2.01% when  $r = 1\%$ . The present value of cashflow B is  $(1+r) \cdot 19/s$  where the  $1+r$  factor accounts for the fact that the first payment is in one year (half of a two year period). Thus the present value is 99.52 when  $r = 10\%$  and 955 when  $r = 1\%$ . The present value of cashflow C is  $-5/r \cdot (1 - (1+r)^{-19}) + (1+r)^{-19} \cdot (30/r \cdot (1 - (1+r)^{-20}))$ , or -5.06 when  $r = 10\%$  and 357 when  $r = 1\%$ . The present value of cashflow D is  $70 + 50 \cdot (1+r)^{-5}$ , or 101 when  $r = 10\%$  and 118 when  $r = 1\%$ . **Thus when  $r = 10\%$  then cashflow D is preferred and when  $r = 1\%$  then cashflow A is preferred.**

7. (18 points) Irene Engels recently graduated with an MBA. In August 2007, she borrowed \$50,000, and she borrowed another \$50,000 in August 2008. Her student loan

has an annual interest rate of 2% compounded monthly. Irene doesn't make any payments on her student debt until she starts a lucrative Wall St. job. Then starting in September 2009 she makes a payment of \$1000 every month. Now bonus time is coming near. For January 2010 she plans to make another \$1000 payment (her 5<sup>th</sup>) and also apply her bonus to the debt. How big must her bonus be so that she will have completely paid-off the debt at the end of this January?

ANSWER: Let  $r=0.02/12$  be the monthly interest rate. The future value of the debt at the end of August 2009 is  $50000*(1+r)^{24}+50000*(1+r)^{12} = 103,048$ . The present value at the end of August 2009 of the future payments is  $1000/r*(1-(1+r)^{-5}) = 4975$ . Thus the value of the debt at the end of August 2009 is  $103,048-4975=98,073$ . Thus the future value of the debt at the end of January 2010 is  $98,073*(1+r)^5=\mathbf{\$98,893}$ . A bonus this big would allow her to pay off the debt.

8. (10 points extra credit) You are analyzing the value of the company Twitter using a 15% discount rate. You expect its cashflows over the next 4 years to be as shown below and you estimate its NPV as \$1B. Explain.

Year	Cashflow
0	-20M
1	-10M
2	0
3	12M
4	40M

ANSWER: Clearly the present value of the cashflows over the next 4 years is less than \$1B. So to have a present value of \$1B the cashflows after year 4 must be pretty big. Another way of saying the same thing is that the value of Twitter, X, at the end of year 4 must be quite high. We can actually calculate X. The future value X at year 4 is  $X=(1B-20M)*1.15^4 - 10M*1.15^3+12M*1.15^1+40M = 1.753B$ .