

IEMS 326 Midterm 2

3/3/2009

Solutions are due by noon on Wednesday in my email inbox. State your final answers in the email body and include as an attachment a document (such as a spreadsheet) showing your work. This document should show your work in a clear and organized fashion, and its file name should be your last name.

You may use your notes, Excel, your book, and material posted on the course website. You may not communicate with other people (except with me) about the exam.

1) Why do longer maturity bonds have higher interest rates? Give a one sentence answer.

A: They have a greater chance of defaulting.

2) Calculate the duration of a zero coupon bond with yield y (assume it is compounded annually) and maturity m ?

A: Let the principal be P . Then $NPV(y) = P(1 + y)^{-m}$ and thus $NPV'(y) = P(-m) * (1 + y)^{-m-1}$. Therefore, duration equals $-NPV'(y)/NPV(y) = m/(1 + y)$. (Note that if the bond were compounded n times per year then the duration would be $m/(1 + y/n)$.)

3) The APY on the Vanguard Prime Money Market fund is 1.16% while that of the Vanguard Tax-Exempt Money Market fund is 0.79%. Money market accounts are like savings accounts and have very low risks. As the names indicate, income from the latter fund is tax-exempt while income from the former is not. Assuming that taxes are paid annually, at which tax rates is the tax-exempt fund preferred to the first fund?

A: For fund 1, the income equals $principal * APY1 * (1 - taxrate)$ while for fund 2 the income equals $principal * APY2$. Fund 2 is preferred to fund 1 when its income is larger: $APY1 * (1 - taxrate) < APY2$ or equivalently when $taxrate > 1 - APY2/APY1 = 32\%$.

4) Suppose Facebook is worth \$2b today and has a an annual net income of \$100m. What rate of income growth must they sustain to justify their valuation at a 10% discount rate?

A: An annuity throwing out \$100m a year is worth \$2b at a $100m/2b = 5\%$ discount rate. This is equivalent to an annuity growing at rate f discounted at 10% if $5\% = \frac{10\% - f}{1 + f} \approx 10\% - f$. Thus the growth rate $f = \frac{10\% - 5\%}{1 + 5\%} \approx 10\% - 5\% = 5\%$.

5. (*This problem is from the fall 2008 final.*) You work for an oil company which has discovered a large new oil field. Your job is to decide whether to develop the field, and if so, which of two alternative oil rigs to use to pump the oil. The main source of uncertainty is the amount of oil in the field. With probability 20%, this is 6 million barrels, with probability 50%, it is 10 million, and with probability 30%, it is 20 million. You must pay for the rig now (Year 0) and production will begin in Year 1 and last until the field is exhausted. Rig A costs \$50 million while Rig B costs \$150 million, but it allows you to pump oil faster. The following table shows what your annual cashflow (in millions of dollars) would be in each scenario and with each rig, and the duration i.e. how many years that annual cashflow would last.

	Rig A			Rig B		
	Scenario			Scenario		
Probability	20%	50%	30%	20%	50%	30%
Cashflow	34	34	34	71	71	71
Duration	6	10	20	3	5	10

The discount rate is 10%. (Hint: ignore taxes and anything else not mentioned.)

a) Suppose you have no way of resolving your uncertainty about the size of the field before making a decision. Should you install Rig A, Rig B, or neither?

A: Install Rig A since it has a higher expected net present value.

b) Now suppose that you can do further testing in order to reveal the exact size of the oil field, i.e. find out what scenario is true before installing a rig. This test would not delay installation of the rig or the beginning of production, but it would cost money. In each scenario of oil field size, once you have done the test and know the size, would you develop the field, and which rig would you install? What is the value to you of doing the test?

A: Install Rig A if field size 6 million or 10 million barrels (scenario 1 or 2) since $NPV(A) > NPV(B)$, install Rig B if the field size is 20 million barrels (scenario 3).

$$E [NPV] = \$98.08 * 20\% + \$158.92 * 50\% + \$286.26 * 30\%$$

$$E [NPV] = \$184.95$$

The value of the test is $\$184.95 - \$170.91 = \$14.04$ million.