Simulation

Decision Trees
1) Calculate the NPV of your outcomes
2) Combine to get E[NPV] for decision.
3) choose action with highest E[NPV]

ex. Stock Option

Dec. 5, AAPC $94
- 12/5 (cool) $104
- $94
- $84 (lame)

Question: To buy or not to?

$E[\text{buy}] = 6\% (104-94) + 40\% (84-94) = 2$

$E[\text{not buy}] = 0$

So you should buy.

Option with strike price of $90

$\frac{1}{6}$ profit

$6\%$ $104$ 104-90

$40\%$ $84$ 0
Value of the option is:
\[
E_{\text{value}} = \begin{cases} 
S - 90 & \text{if } S > 90 \\
0 & \text{if } S \leq 90
\end{cases}
\]
S is stock price.

\[E[\text{value}] = 0.6 \times 14 = 8.4\]

Now let us assume the stock price is normally distributed, i.e.
\[S \sim N(94, 10^2)\]

Then we have infinite scenarios on Jan 6th.

We can simulate S from its distribution.

for example, we get

<table>
<thead>
<tr>
<th>S</th>
<th>option value</th>
<th>The average of all option values.</th>
<th>The standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>89</td>
<td>0</td>
<td>(\approx E\left[\max(S - 90, 0)\right])</td>
<td>(\approx 0) [option value]</td>
</tr>
<tr>
<td>92</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>97</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>95</td>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Generate random values from Normal distribution in Excel.

\[ \text{NORMINV (RAND(), 94, 10)} \]

For every value you get from this function, it is a realized stock price in the future (1/6).

Assume you have got \( S_1, \ldots, S_n \) from \( \mathcal{N}(\mu, \sigma) \).

That means you have \( n \) scenarios in the future.

For \( v = 1, \ldots, n \), calculate the option payoff \( S_i - K \), where \( K \) is the strike price.

Then you can calculate the sample average of option payoff and the sample standard deviation of option payoff based on \( (S_i - K, \ldots, S_n - K) \).

Another way to generate random numbers in Excel.

DATA \( \rightarrow \) DATA ANALYSIS \( \rightarrow \) random number generation.

Simulate stock price via binomial tree

\[ S = S_0 + X_1 + X_2 + \ldots + X_n \]

\( (X_i, i = 1, \ldots, n \) are random variables \)

\[
X_i = \begin{cases} 
S + 1 & \text{with probability } p \\
0 & \text{with probability } \frac{1}{2} \\
-1 & \text{with probability } 1 - p - \frac{1}{2}
\end{cases}
\]

EX. \( S = 94 + X_1 + X_2 + \ldots + X_{30} \)

\[
X_i = \begin{cases} 
S + 1 & \text{35\%} \\
0 & \text{35\%} \\
-1 & \text{30\%}
\end{cases}
\]
Generate Normal and Bernoulli random numbers:

\[ N(\mu, \sigma^2) : \text{NORMINV} \left( \text{RAND()}, \mu, \sigma \right) \]

\[ \text{Bernoulli}(p) : \text{IF} \left( \text{RAND()} < p, 1, 0 \right) \]