1 Interest Rates etc.

Definitions With a *discount rate* of r, a cash flow of size X, t periods (e.g., years) in the future, is worth $X(1+r)^{-t}$ today (that is the *present value*). Similarly, an amount Y today is worth $Y(1+r)^t$ in t periods (i.e., the *future value*).

If the *APR* (the Annual Percentage Rate) is r and interest is *compounded* (i.e., calculated) n times a year (e.g., n equals 12 for monthly and 1 for yearly), then \$1 will grow to be $(1 + r/n)^n$ in a year. The *APY* (the Annual Percentage Yield) is the equivalent annual interest rate and is $(1 + r/n)^n - 1$ in this case.

The amount of a loan (e.g., a *mortgage* on a house) is called the *principal*. The size of the loan outstanding (i.e., the amount not yet paid) decreases as payments are made.

Excel There are some Excel functions for a stream of payments beginning one period from now that might be useful. PV calculates its present value; FV calculates its value after the last payment is made; PMT calculates the size of the payment; NPER calculates the number of payments; and RATE calculates the interest rate. NPV calculates the present value for a stream of payments of unequal sizes (again the first payment is one period from now). Their parameters are described in the Excel function wizard. Be careful to check the sign of the result because it may not be what you expect.

2 Probability

Let X and Y be random variables. Then $\mathrm{E}[X]$ is the expectation of X; $\mathrm{Cov}(X,Y) = \mathrm{E}[XY] - \mathrm{E}[X]\,\mathrm{E}[Y]$ is the covariance of X and Y; $\mathrm{Var}[X] = \mathrm{Cov}(X,X)$ is the variance of X; $\sqrt{\mathrm{Var}[X]}$ is the standard deviation of X; and $\mathrm{corr}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}[X]\,\mathrm{Var}[Y]}}$ is the correlation of X and Y. If X and Y are independent then $\mathrm{Cov}(X,Y) = 0$.

Let X_1, X_2, X_3, ... be a sequence of random variables. Then

$$\mathrm{E}\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} \mathrm{E}[X_i] \quad \text{and} \quad \mathrm{Var}\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{Cov}(X_i, X_j). \tag{1}$$

Bernoulli If a random variable X is distributed according to a Bernoulli distribution, $X \sim \mathrm{Bernoulli}(p)$, then $X = 1$ with probability p and $X = 0$ with probability $1 - p$.

Normal We write $X \sim \mathcal{N}(\mu, \sigma^2)$ to describe a normally distributed random variable with mean or expectation μ and variance σ^2. *Standard normal* random variables, $\mathcal{N}(0, 1)$, are those with mean 0 and variance 1. The probability $\Pr[\mathcal{N}(\mu, \sigma^2) < x]$ (i.e., the value of the cumulative distribution function at x) can be calculated with the Excel function NORMDIST. If $X \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $Y \sim \mathcal{N}(\mu_2, \sigma_2^2)$ are two independent normally distributed random variables, then $X + Y$ is also normally distributed with mean $\mu_1 + \mu_2$ and variance $\sigma_1^2 + \sigma_2^2$.

An iid (that is *independent, identically distributed*) sequence of random variables X_i is one where all have the same distribution and are independent of each other. So for any $i \neq j$, X_i and X_j have the same distribution and $\mathrm{Cov}(X_i, X_j) = 0$. Suppose $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}[X_i] = \sigma^2$. If $Z_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ is the average of the first n random variables in the sequence, then the *central limit theorem* states that (under reasonable conditions) as $n \to \infty$: $\sqrt{n}\frac{Z_n - \mu}{\sigma}$ converges in distribution to a standard normal distribution, $\mathcal{N}(0, 1)$.