IE310 Operations Research

solution

Assignment 2

1. **Problem 3.6**

   This represents a diseconomy of scale. A profit-maximizing linear program have an unintended option, which is to use overtime labor without fully utilizing the capacity of regular time labor. This unintended option will be ruled out by the optimization.

2. **Problem 3.11**

   a.

   Decision variables are number of cars need to repair without private contracting x_1, and number of cars need to refurnish without private contracting x_2. LP is as follows:

$$min \qquad 2000(100 - x_1) + 2500(50 - x_2)$$

$$\begin{aligned} x_1/150 + x_2/100 &\leq 1 \qquad (1)\\ x_1/60 + x_2/120 &\leq 1 \qquad (2)\\ x_2/40 &\leq 1 \qquad (3)\\ x_1/60 &\leq 1 \qquad (4)\\ x_1, x_2 &\geq 0 \qquad (5)\\ x_1 &\leq 100 \qquad (6)\\ x_2 &\leq 50 \qquad (7) \end{aligned}$$

   Solve it using solver, see excel file. optimal $x_1 = x_2 = 40$ with objective 145000.

   In the optimal solution, constraint (2), (3) are tight and the rest are slack. By the perturbation theorem, if the data are inexact, the tight constraints stay tight, the slack constraints stay slack.

   or you may say it based on the sensitivity report (see Excel file) generated by the solver. For example, line 1 of table 1 tells us the optimal value of x_1 won't change if the contracting cost are within the range [\$0, \$5000]. Line 1 of table 2 tells us if the rhs of constraint (1) is increased by 1 (but it should be within the range [0.667, $+\infty$]), the optimal objective value will increase by 0. The rest of the two tables can be explained in a similar way.

   b.

Note that the question asks only for the repairing part. Decision variables are number of cars need to repair without private contracting x_1, and number of cars need to refurnish without private contracting x_2.

$$max \qquad 2000x_1$$

$$\begin{aligned} x_1/150 + x_2/100 &\leq 1 && (8)\\ x_1/60 + x_2/120 &\leq 1 && (9)\\ x_2/40 &\leq 1 && (10)\\ x_1/60 &\leq 1 && (11)\\ x_1, x_2 &\geq 0 && (12)\\ x_1 &\leq 100 && (13)\\ x_2 &\leq 50 && (14)\end{aligned}$$

The solution is $x_1 = 60$, it changed! Because now we are only considering the repairing part without the refurnishing part, so the objective function changed.

Note that I give you full credits if you get the same optimal solution since you may consider the "repairing" here refers to both repairing and refurnishing. Then to minimize the monthly cost for private contracting is the same as to maximize the monthly saving.

3. **Problem 3.14**

a.

Let x_1 denotes the number of unit of A produced, x_2 denotes the number of unit of B produced. Then the LP reads:

$$max \qquad 1000x_1 + 2000x_2$$

such that

$$\begin{aligned} 2x_1 + x_2 &\leq 100 && (15)\\ 4x_1 + 3x_2 &\leq 240 && (16)\\ x_2 &\leq 60 && (17)\end{aligned}$$

$$x_1, x_2 \geq 0$$

In this LP, unit of measure for x_1, x_2 are units, for the objective function are dollars, for constraint (15) are dollars/hour, for constraint (16) are dollars/ounce, for constraint (17) are dollars/ounce.

b.

Note that constraint (16)(17) are tight, so if the data are inexact, they remain tight, constraint (15) remains slack.

Or you may state it based on the sensitivity report given, as we did in problem 3.11(a).

c.

optimal $x_1 = 15, x_2 = 60$ with optimal value equals 135000.

d.

Please see Figure 1.

e.

The value of relaxing the constraint on particulate emission and chemical emission by one ounce per week is \$250, \$1250, respectively. (equals shadow price)

f.

If the EPA allows to emit one additional ounce of particulate, the company gains \$250 more. Then the company is willing to reduce 250/1250=0.2 ounce of chemical emission. (This is to change rhs simultaneously)

g.

Now we drop the last two constraints and the new LP becomes:

$$max \qquad (1000 - 4P)x_1 + (2000 - 3P - C)x_2$$

such that

$$2x_1 + x_2 \leq 100$$

$$x_1, x_2 \geq 0$$

Dual problem:

$$min \qquad 100y$$

such that

$$2y \geq 1000 - 4P$$

$$y \geq 2000 - 3P - C$$

$$y \geq 0$$

In order to automatically satisfy the dropped two constraints, we need the iso-profit line to intersect the constraint line at a feasible point on Figure 1. Note that only vertex (50, 0) of the feasible region now is still feasible for the original feasible region of Figure 1. This indicates that we want the slope of the iso-profit line $-(1000-4P)/(2000-3P-C)$ change from -0.5 to some number that is no bigger than -2, which indicates

$$P + C \geq 1500$$

then for example we may let $P = C = 750$. Under this case, we solve the LP (see excel file attached), the optimal objective is equal to 0, which means the company won't prosper.

h.

Note that we only need $P+C \geq 1500$. If the value of P and C set all equal to 750, the company won't prosper, so it is not a good idea; however, if set $P = 0, C = 1500$, the company still can have profit (see excel file). Hence, using taxation to control is a good idea sometimes.

4. Problem 3.21

a. Refer to Figure 2.

b.

Change to $A + B$ then the LP has infinitely many optimal solutions. For example, (0, 12), (0.1, 11.9), (0.2, 11.8).

c.

We may drop $A+B \leq 12$. Some feasible solutions that make the objective goes to infinity can be $(2, +\infty), (3, +\infty), ...etc.$

5. Problem Additional

Note that a and b are decision variables. The trick used here is not easy to come up with so if you can solve this by excel, you are good.

a.

$$min \qquad \sum_{i=1}^{14}(ax_i + b - y_i)$$

to solve, use the solver (see excel file attached). We get the value of a, b equal to 0.517967316 and 6.442515655, respectively.

$$a = 0.518, b = 6.443$$

b.

$$min \qquad \sum_{i=1}^{14}|ax_i + b - y_i|$$

, to transform it into a standard LP, we do the following:

$$min \qquad \sum_{i=1}^{14}|s_i|$$

such that

$$ax_i + b - y_i = s_i$$

. Now note that for any number s_i, we have

$$s_i = s_i^+ - s_i^- \quad and \quad s_i^+ \geq 0 \quad and \quad s_i^- \geq 0$$

, then we have the standard LP:

$$min \qquad \sum_{i=1}^{14}(s_i^+ + s_i^-)$$

such that

$$ax_i + b - y_i = s_i^+ - s_i^-$$
$$s_i^+ \geq 0$$

$$s_i^- \geq 0$$

Note that $|s_i| = s_i^+ + s_i^-$ since we are minimizing $\sum_{i=1}^{14} |s_i|$, then at least one of s_i^+ or s_i^- must equal to 0.

$$a = 0.525, b = 6.601$$

c.

The LP reads

$$min \qquad \sum_{i=1}^{14}(10s_i^+ + s_i^-)$$

such that

$$ax_i + b - y_i = s_i^+ - s_i^-$$

$$s_i^+ \geq 0$$

$$s_i^- \geq 0$$

Note that if we overestimate, $s_i^+ > 0, s_i^- = 0$; if we underestimate, $s_i^+ = 0, s_i^- > 0$.

$$a = 0.473, b = 5.649$$

To plot (a)-(c), use solver to directly solve the optimization problems, and draw a straight line for the fitted data (see excel file attached).

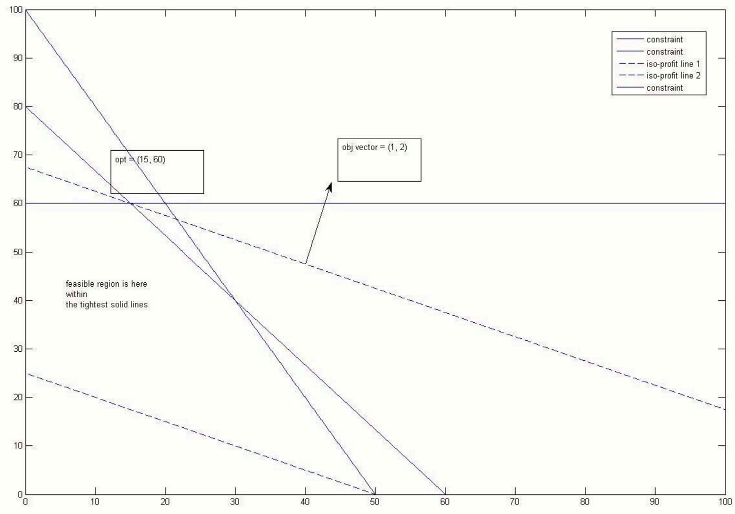

Fig. 1: Graphical Demonstration, magnify the pdf file to see the caption more clearly

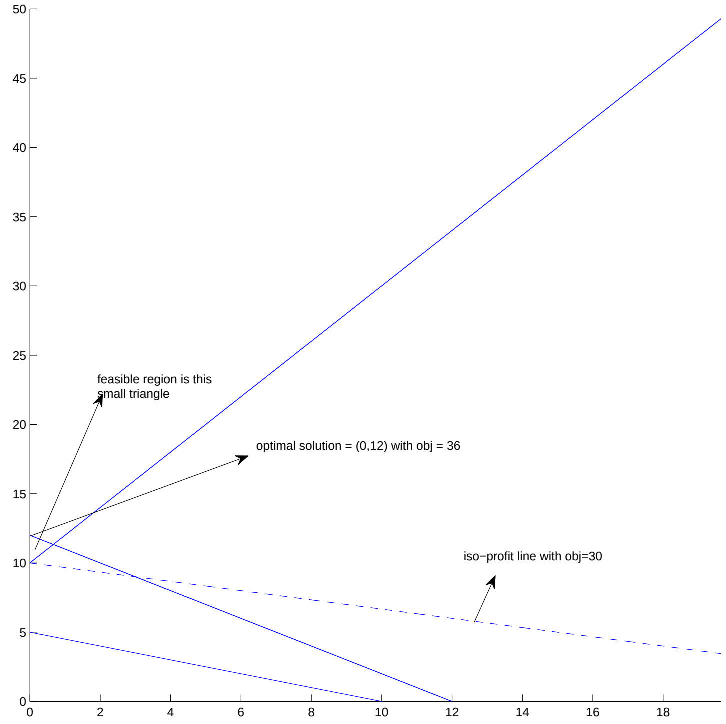

Fig. 2: Graphical Demonstration, magnify the pdf file to see the caption more clearly