IEMS 310 Final

6/5/2009

Solutions are due by 4pm on Saturday. Email me your solutions. Have the spreadsheet file name be `Lastname.xls`.

You may use your notes, Excel, your book, and material posted on the course website. You may not communicate with other people (except with me) about the exam. Best of luck.

1) Look at the `allocation.xls` spreadsheet used in the first lecture to describe the problem of investing the university's endowment. In particular look at the case involving all 6 asset classes that the university considers. Make sure to have a constraint that the allocations are nonnegative.

a) Solve it.

b) Add a bound that no more than 25% of the total may be invested in a single asset class. What is the new solution?

2) In the US there are flexible savings accounts (FSA) for medical expenses. These allow an employee to prepay for uninsured medical expenses in the coming calendar year with before-tax contributions from their pay-check. Flexible savings accounts have a "use it or lose it" feature in that all the money left at the end of the year goes to the company managing the accounts.

Consider a young professional, Sarah, whose marginal income tax rate is r and her uninsured medical expense X for the year is uncertain. Sarah's tax bracket is $r = 28\%$ and she estimates that X is normally distributed with a mean of \$2000 and a standard deviation of \$500. What amount, q, should Sarah contribute to the plan each year to maximize her after-tax income?

Here is a longer description of flexible savings accounts. Before the beginning of the calendar year, Sarah (like any employee at her firm) can declare an amount q of uninsured prepaid medical expenses. This reduces her salary for the year by q dollars and her firm will then reimburse her for the first q dollars of her medical expense for the forthcoming year. Sarah pays any excess, $\max(X - q, 0)$, out of after-tax income. Prepaying q dollars saves Sarah qr in after-tax income because she does not need to pay taxes on the q dollars she set aside in this account for medical expenses. The downside is that the law requires that Sarah forfeit any unspent money in the account, $\max(q - X, 0)$ (the money goes to the employer).

3) Kris Lee the owner and manager of the local hardware store is reassessing his inventory policy for hammers because reordering hammers at the end of each month is taking a great deal of his time. He sells an average of 60 hammers per month. So at the end of each month, he has been placing an order to purchase 60 hammers from a wholesaler at a cost of \$10 per hammer. He estimates that the value of his time placing each order for hammers is \$50. What would the unit holding cost for hammers need to be for Kris' current inventory policy to be optimal for the basic EOQ model?

4) Consider a clinic with 6 staff and an average client arrival rate of 15/hr. Each visit consists of a work-up (taking 5 minutes on average) followed by a consultation (taking 10 minutes on average). Assume all queues are Markov. What will the average waiting time be if the steps are combined — if each client has both steps performed by the same staff member?

What will the average total waiting time be if the clinic separates the two steps with 2 people responsible for the work-ups and the remaining 4 responsible for the consultation? Give a reason why one might want to go with the case with a longer waiting time.

5) Extra Credit. Solve the following MDP relating to setting ticket prices for a concert. The idea is that you may want to increase the ticket price if there are few seats left or few days remaining until the concert. The state $i = (\#seats\ left, \#days\ left)$ is the pair of numbers giving the number of seats left (0, 1, ...) at the beginning of the day and the number of days left (1, 2, ...) until the concert (1 being the day of the concert). The action k is the ticket price to charge on a particular day. The reward $R(i,k)$ is the expected revenue from the tickets you will sell in that state.

Suppose you have initially 3 seats to sell and 5 days until the concert. Hence you have 20 states of the form (s,t) where $s = 0,\ldots,3$ and $t = 1,\ldots,5$. You have two ticket prices \$50 and \$100. The demand for tickets is Poisson(λ) each day with $\lambda = 1$ when the price $k = \$50$ and $\lambda = 1/2$ when the price $k = \$100$. What is the optimal action in each state?

Here I setup the MDP in more detail. Clearly the value function $f(i) = 0$ for states $i = (s,t)$ without any seats left, where $s = 0$. Also, $P(j|i,k) = 0$ on the day of the concert, when $i = (s,t)$ and $t = 1$. Otherwise, if $i = (s,t)$ with $t > 1$ and $s > 0$, and $j = (s-n, t-1)$ then

$$P(j|i,k) = \begin{cases} P(Poisson(\lambda) = n) = \mathtt{POISSON(n}, \lambda, \mathtt{FALSE)} & \text{if } s > n \\ P(Poisson(\lambda) \geq s) = \mathtt{1} - \mathtt{POISSON(s} - \mathtt{1}, \lambda, \mathtt{TRUE)} & \text{if } s = n \end{cases} \tag{1}$$

and the reward is the price times the expectation of the number of seats sold:

$$R(i,k) = k \sum_{n=0}^{s} nP((s-n, t-1)|i,k). \tag{2}$$