MSE 444 Final Presentation

Implied Volatility Modeling

M. Günhan Ertosun, Sarves Verma, Wei Wang
Advisors: Prof. Kay Giesecke, Benjamin Ambruster
Outline

Brief Literature Review
Data and Methodology
Results
Interpretations
Conclusions
Four Different Ways to model:

- Using a Deterministic Volatility Function (DVF) used by Derman\(^1\), Dupire\(^2\)
- Using Stochastic Volatility Model such as in Hull-White\(^3\)
- Using factor based models constructed using time dependent parameters such as Rama Cont. et. al\(^4\) which used O-U process
- Using empirical statistical techniques to fit data and then use PCA (principal component analysis) to understand the dynamics (as in Roux et.al\(^5\))

\(^2\) B. Dupire, “Pricing with a smile”, RISK, 1994
Data & Methodology

- Use S&P 500 index options (daily data) from June 2000-June 2001
- Sort Data:
  - All options with less than 15 days of maturity were ignored as they result in high volatility.
  - Data values with call prices less than 10 cents were also ignored.
  - Average value of ask & bid price was taken to represent the call price.
  - All call prices which were less than the theoretical value (calculated using Black-Scholes) were ignored for arbitrage reasons.
- Divide the data into:
  - Moneyness Buckets (New!)
  - Maturity Buckets (Skiadoupoulos et.al) (New!)
- Model Implied Volatility by incorporating both maturity & moneyness (New!)
- Ultimately, answer the following question:
- Which Principal component is important for different regimes of moneyness & maturity

<table>
<thead>
<tr>
<th></th>
<th>Out of Money</th>
<th>At the money</th>
<th>In the Money</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short Term Maturity (8-30 days)</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Medium Term Maturity (60-90 days)</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Long term Maturity (150-250 days)</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

Implied Volatility Models

Model 1: \( I(m, \tau) = \beta_0 + \varepsilon \)

Model 2: \( I(m, \tau) = \beta_0 + \beta_1 m + \beta_2 m^2 + \varepsilon \)

Model 3: \( I(m, \tau) = \beta_0 + \beta_1 m + \beta_2 m^2 + \beta_3 \tau + \beta_4 \tau m + \varepsilon \)

Model 4: \( I(m, \tau) = \beta_0 + \beta_1 m + \beta_2 m^2 + \beta_3 \tau + \beta_4 \tau m + \beta_5 \tau^2 + \varepsilon \)

\[ m = \frac{\log(S_t e^{r \tau_t} / K)}{\sqrt{\tau_t}} \]

- S&P 500 index options (daily data) from June 2001-June 2002 (i.e. Next years’) is used to verify our models via out of sample prediction
Model I

- Black-Scholes like model assuming constant volatility

Model 1: \[ I(m, \tau) = \beta_0 + \varepsilon \]
Model II

- Model accounting for slope & curvature of moneyness

\[ I(m, \tau) = \beta_0 + \beta_1 m + \beta_2 m^2 + \epsilon \]
Model III

In Sample Fit

Out of Sample Prediction

- This model takes into account the slope contribution of maturity as well as the mixed contribution from maturity & moneyness.

Model 3:  
\[ I(m, \tau) = \beta_0 + \beta_1 m + \beta_2 m^2 + \beta_3 \tau + \beta_4 \tau m + \varepsilon \]
This model takes into account the slope contribution of maturity as well as mixed contribution from maturity & moneyness.

Model 4: \[ I(m, \tau) = \beta_0 + \beta_1 m + \beta_2 m^2 + \beta_3 \tau + \beta_4 \tau m + \beta_5 \tau^2 + \varepsilon \]
Comparison of In Sample Vs Out of Sample Prediction

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_4$</th>
<th>$\beta_5$</th>
<th>RMSE (In Sample) (Fitting)</th>
<th>RMSE (Out of Sample) Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model I</td>
<td>-1.4876</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.3033</td>
<td>0.3362</td>
</tr>
<tr>
<td>Model II</td>
<td>-1.6352</td>
<td>0.2702</td>
<td>0.8836</td>
<td></td>
<td></td>
<td></td>
<td>0.1805</td>
<td>0.2001</td>
</tr>
<tr>
<td>Model III</td>
<td>-1.6244</td>
<td>0.2504</td>
<td>0.8779</td>
<td>-0.1208</td>
<td>0.2565</td>
<td></td>
<td>0.1802</td>
<td>0.1999</td>
</tr>
<tr>
<td>Model IV</td>
<td>-1.6108</td>
<td>0.2538</td>
<td>0.8783</td>
<td>-0.5613</td>
<td>0.2202</td>
<td>2.5269</td>
<td>0.1801</td>
<td>0.1998</td>
</tr>
</tbody>
</table>
### PCA (Principal Component Analysis)

#### PCA on Moneyness Bucket

<table>
<thead>
<tr>
<th>Moneyness of Call Option (in %)</th>
<th>1\textsuperscript{st} PC (in %)</th>
<th>2\textsuperscript{nd} PC (in %)</th>
<th>3\textsuperscript{rd} PC (in %)</th>
<th>Total explained Variance by 1\textsuperscript{st} three PCs (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>m&lt;-1</td>
<td>51.561</td>
<td>39.379</td>
<td>9.0596</td>
<td>100</td>
</tr>
<tr>
<td>-1&lt;m&lt;-0.5</td>
<td>50.548</td>
<td>26.729</td>
<td>11.646</td>
<td>88.923</td>
</tr>
<tr>
<td>-0.5&lt;m&lt;0</td>
<td>45.248</td>
<td>23.932</td>
<td>18.656</td>
<td>87.836</td>
</tr>
<tr>
<td>0&lt;m&lt;0.5</td>
<td>50.017</td>
<td>19.536</td>
<td>16.346</td>
<td>85.899</td>
</tr>
<tr>
<td>0.5&lt;m&lt;1</td>
<td>37.732</td>
<td>24.999</td>
<td>22.1</td>
<td>84.831</td>
</tr>
<tr>
<td>m&gt;1</td>
<td>62.871</td>
<td>23.417</td>
<td>10.996</td>
<td>97.284</td>
</tr>
</tbody>
</table>

- Moneyness\(m = \ln\left(\frac{S_te^{r\tau}}{K}\right)/\sqrt{\tau}\)

#### PCA on Maturity Bucket

<table>
<thead>
<tr>
<th>Maturity of Call Option</th>
<th>1\textsuperscript{st} PC (in %)</th>
<th>2\textsuperspcert{nd} PC (in %)</th>
<th>3\textsuperspcert{rd} PC (in %)</th>
<th>Total explained Variance by 1\textsuperspcert{st} three PCs (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-30</td>
<td>56.929</td>
<td>21.359</td>
<td>12.072</td>
<td>90.41</td>
</tr>
<tr>
<td>30-60</td>
<td>69.426</td>
<td>15.266</td>
<td>10.496</td>
<td>95.188</td>
</tr>
<tr>
<td>60-90</td>
<td>88.71</td>
<td>5.41</td>
<td>2.79</td>
<td>96.92</td>
</tr>
<tr>
<td>90-150</td>
<td>81.419</td>
<td>10.712</td>
<td>7.2489</td>
<td>98.83</td>
</tr>
<tr>
<td>150-250</td>
<td>77.38</td>
<td>15.55</td>
<td>4.58</td>
<td>97.5</td>
</tr>
</tbody>
</table>

- For short term maturities: All three PCs important.
- For long term maturities: Only the first PC most important
PCA Analysis on Moneyness Bucket

![Graphs showing percentage contribution by different principal components towards total variance.]

Novel Way of Option Hedging
Observations:
- At the money regime most sensitive; hence 1st three principal components not sufficient
- ‘In the money’ Regime, 1st PC most important
- ‘Out of Money’ Regime, All three PCs Important
- Note both out of money & In the money options are illiquid

Model Constructed

\[ \log(I) = \eta_0 + \varepsilon \]
\[ \log(I) = \eta_0 + \eta_1 m + \eta_2 m^2 + \varepsilon \]
\[ \log(I) = \eta_0 + \eta_1 m + \eta_2 m^2 + \eta_3 \tau + \eta_4 \theta + \varepsilon \]
- Incorporates both Maturity & Moneyness
- R² & RMS taken to check for accuracy
- The model fitting is sensitive to data sampling

Summary of PCA & Model
Conclusions

- Developed an Implied Volatility model on S&P 500 Index options (from June 2000-June 2001)
- The model incorporated slope and curvature of moneyness and maturity
  - Incorporating maturity (slope and curvature) does not improve the model appreciably
- Out of sample prediction shows good matching with our model
  - The coefficients change with time, however, for a shorter to medium horizon they are pretty constants
- PCA analysis was done on moneyness & maturity (see Clewlow 1999) buckets
  - We observed that the three components (corresponding to moneyness buckets) are significant enough & have shapes confirming our intuitive understanding
- The shapes of different principal components are important to develop hedging strategy