Asset Management Strategies:

Fat Tails and Risk Control

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Layout

- Hedge Fund Returns: Distributional Characteristics
- How to Calculate Robust VaR Numbers
- Portfolio Construction in the presence of fat tails
Ideally: A trading strategy transforms underlying asset return distribution favorably.
Tsallis distribution: Fits well to daily returns also with $q = 1.4$.

Used for non-Gaussian option pricing
Lipper TASS Database: 2883 Funds, 1300 Funds of Funds, Monthly Returns

Hedge Fund Returns

Mean = 0.43

q = 1.38
• Hedge Fund managers do shift the mean from 0.16 to 0.43

• Tails are much fatter, monthly returns well-fit by q=1.4

• The “ideal” distribution (small left tail) is not achieved, but also no significant negative skew
• $q = 1.3 - 1.5$ fits well to hedge fund monthly returns

• How can we use this for
  
  - Risk Control
  - Portfolio Construction
VaR

• 5% VaR: You have a 5% chance of getting returns less than VaR (per $)

• Common calculation methods:
  - Assume a distribution (eg. Gaussian)
  - Use the past $N$ days historical price changes
  - Use MC simulations of future returns
VaR

- 5% VaR: You have a 5% chance of getting returns less than VaR (per $)

- Common calculation methods:
  - Assume a distribution (eg. Gaussian)  
    Can’t be good! Fat tails!
  - Use the past N days historical price changes  
    Simple. Can we do better?
  - Use MC simulations of future returns  
    Very compute-intensive!
Robust Calculation of VaR

An experiment:

• Simulate 500 returns drawn from q = 1.4 Tsallis distribution. Repeat 250 times.

• For each sample:

  Method 1: Estimate 5%-ile from 500 day generated data \( \rightarrow 250 \) values of VaR.

  Method 2: Fit Tsallis distribution of index q to 500 day generated data. Then calculate 5%-ile of that fitted distribution \( \rightarrow 250 \) values of VaR.
Robust Calculation of VaR (Example 5%)

\[ \langle \text{VaR} \rangle = -2.12 \pm 0.11 \]
\[ \langle \text{VaR} \rangle = -2.10 \pm 0.17 \]

VaR from Method 1

VaR from Method 2

True VaR = -2.13

VaR from 250 runs each of length 500
• Fitting Tsallis distribution to data and then calculating VaR → More robust estimate

• Using q=1.4 is a better prior than the Gaussian distribution

• Better than unconditional VaR using historical data (recent history might be anomalous)
Portfolio construction in the presence of fat-tails

- Single strategy case:

  How to calculate optimal holdings?
One strategy is:

Maximize expected long-run profit based on log-utility function (Kelly criterion)

\[
\left\langle \log(1 + hx) \right\rangle_P(x|\mu,\sigma)
\]

h = holding (position size)
\(\mu\) = expected return
\(\sigma\) = standard deviation (volatility)
q-Kelly criterion

\[ \int \log(1 + hx) N \left( 1 + (q - 1) \frac{(x - \mu)^2}{\sigma^2} \right)^{\frac{1}{1-q}} \, dx \]

Gaussian, q=1  Not good – any slightly positive expected return implies an extremely large position because there is no tail risk

Tsallis, q = 1.5  Good – large position sizes are penalized by the tail risk
Example: Daily expected return $\mu = 25$ bp and $\sigma = 1$ %

Calculating Optimal Holdings with $q = 1$ (Gaussian)

Optimal position is where ??

![Graph showing Optimal position is where ??]
Example: Daily expected return $\mu = 25$ bp and $\sigma = 1$ %

Calculating Optimal Holdings with $q = 1.5$
These portfolios might be optimal, but some investors might not like the high leverage

i) Might not be log-utility maximizers
ii) Might be irrational

• One more ingredient:

- Prospect Theory
  (Tversky and Kahneman, Nobel Prize 2002)

\[
\langle \log(1 + hx) \rangle_{\mu, \sigma}^a \leq 1
\]

Gives even more weight to the tails – incorporates subjective investor fear, not just actual probability of losses
Results using q-Kelly & Prospect Theory  \( q=1.5, a = 0.8 \):

A real trading strategy: returns with and without scaling

Same predictive signal but better risk control \( \rightarrow \) superior returns
Remember our cartoon!

Ideally: A trading strategy transforms underlying asset return distribution favorably
Results using q-Kelly & Prospect Theory  q=1.5, a = 0.8:

Another real trading strategy: returns with and without scaling

![Graph showing risk-adjusted returns with and without scaling. The graph compares q-Kelly & Prospect strategy with an unscaled strategy.](image)

- **q-Kelly & Prospect**
  - Mean = 0.093
  - St. Dev = 1
  - Skewness = 6.5

- **Unscaled**
  - Mean = 0.017
  - St. Dev = 1
  - Skewness = -0.18
• Multi-strategy case:
Results using q-Kelly & Prospect Theory  $q=1.5$, $a = 0.8$:

Applied to a multi-strategy portfolio of real trading strategies

- **q-Kelly & Prospect**
  - Mean  = 0.086
  - St. Dev  = 1
  - Skewness  = 3.5

- **Unscaled**
  - Mean  = 0.058
  - St. Dev  = 1
  - Skewness  = -0.15
• Multi-strategy case:
  
  - Combined strategies in a naive approximation
  
  - Used q-Kelly & Prospect Theory to get leverage rule for whole portfolio

Work still to be done:

- Use q-Kelly & Prospect Theory directly on the multivariate distribution

- Incorporate asymmetry between profit seeking and loss aversion.
Conclusions

• Hedge fund monthly returns distributed according to Tsallis distribution with $q = 1.4$

• This is quite stable across strategy types

• Using $q=1.4$, more robust VaR numbers can be calculated

• By taking tail risk into account, optimal position sizes can be found that – at least for the strategies studied here – produce more desirable return distributions