Modelling Stock Orders Using Hawkes's Self-Exciting Process

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Hawkes Process

- Univariate: intensity at time $t$ is given by

$$\lambda_t = \mu + \alpha \int_{u<t} e^{-\beta(t-u)} dN_u$$
Hawkes Process

- Bivariate: intensities of processes 1 and 2 are given by

\[ \lambda_t^1 = \mu_1 + \alpha_{11} \int_{u<t} e^{-\beta_{11}(t-u)} dN_u^1 + \alpha_{12} \int_{u<t} e^{-\beta_{12}(t-u)} dN_u^2 \]

\[ \lambda_t^2 = \mu_2 + \alpha_{22} \int_{u<t} e^{-\beta_{22}(t-u)} dN_u^2 + \alpha_{21} \int_{u<t} e^{-\beta_{21}(t-u)} dN_u^1 \]
Maximum Likelihood Estimation

- Hawkes log-likelihood function can be written explicitly (e.g. univariate case):

\[
\log L (t_1, \ldots, t_n | \mu, \alpha, \beta) = -\mu t_n + \sum_{i=1}^{n} \frac{\alpha}{\beta} \left[ e^{-\beta(t_n-t_i)} - 1 \right] + \sum_{i=1}^{n} \log \left( \mu + \alpha \sum_{t_j < t_i} e^{-\beta(t_i-t_j)} \right)
\]

- Gradients and Hessians are also available
Model Description

• Bivariate process: buy/sell orders for stocks arrive throughout the day
• Use Lee-Ready tick test to classify orders as buy/sell
• Ignore volume and price levels
Results

• Most companies (Google, Microsoft) exhibit negligible cross-correlation

• Most parameter estimates have very high decay, e.g. 3 seconds

A typical day’s parameter estimates of buy orders for Google stock:

\[ \mu = 0.1389; \; \alpha = 1.2131; \; \beta = 1.9805 \]
QQ plots

Fitted Poisson

Fitted Hawkes

True

True
Classification of Buy/Sell Orders

- Classical Lee-Ready ‘tick test’
- ~30000 data points
Data Thinning

- Dropped orders with $Price_{previous} = Price_{current}$
- ~10000 data points
Conditional intensity following a buy
“Most Recent” Conditional Intensity

• Suppose the most recent event occurred at time $t$. What is the intensity at time $t+s$?

• Want this to be

$$\lambda_{t+s} : \text{(most recent event at } t) = \mu + \alpha \int_{u<t} e^{-\beta(t-u)} dN_u$$

• In particular, it should decay to $\mu$!
“Most Recent” Conditional Intensity

Intensity

Time
Trading Strategy

- Performed across 3 counters (Dell, Yahoo, Oracle) from Nov 1\textsuperscript{st} – 7\textsuperscript{th}, 2006
- MLE parameters recalibrated 6 times a day, using 8000 second blocks
- Ignore Bid-Offer spread
- Ignore transaction costs
- No restrictions on short selling
Heuristic

• Begin with $100
  – If buy/sell intensity ratio > 5, purchase stock and exit position 10 seconds later
  – If sell/buy intensity ratio > 5, short sell stock and cover position 10 seconds later
Dell (DELL)

Price, $

PnL, $

$104.03$
Yahoo (YHOO)

$103.97
Challenges & Further Work

• Challenges
  – MLE algorithm computationally intensive

• Further Work
  – Incorporate volume into model
  – Cross counter trading strategies