Exciting Times for Trade Arrivals

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Finance Project Course MS&E 444 06/04
Road Map

- use of a bivariate Hawkes process to model buy and sell trade arrivals

- a more general Hawkes model
  - much, much faster to fit
  - more accurate function for excitation decay
  - higher likelihood
  - more flexible to incorporate additional data sources (e.g. price, volume, market index, option data)

- simple trading strategy showing promising results
Bivariate Hawkes Model

- processes $N_1(t), N_2(t)$ that correspond to buy, sell orders

- intensities at time $t$, for $i = 1, 2$:

$$\lambda_i(t) = \mu_i + \int_{u<t} h_{i1}(t-u) dN_1(u) + \int_{u<t} h_{i2}(t-u) dN_2(u)$$

- log-likelihood function

$$\mathcal{L} = \int_{u<t} - (\lambda_1(u) + \lambda_2(u)) \, du + \log(\lambda_1(u)) dN_1(u) + \log(\lambda_2(u)) dN_2(u)$$
Traditional Hawkes Model Fit

- basic assumption: exponential decay

- fitting done by maximizing log-likelihood $\mathcal{L}$:

$$\begin{align*}
\text{maximize} \quad & \mathcal{L} \\
\text{subject to} \quad & h_{ij}(t) = \alpha_{ij} e^{-\beta_{ij} t} \\
& \text{positivity, stationarity}
\end{align*}$$

- this optimization program involves typically 10 variables

- but, it is not convex...

  $\Rightarrow$ no efficient way of solving it
  $\Rightarrow$ no optimality certificate
A New Way To Do It...

- idea: consider a time-limited, piecewise linear form of $h_{ij}(t)$
  - if the length of each piece $\delta t$ is $\leq$ to the time resolution of the data, no information is lost
  - exponentials are also 'time-limited', since they die off after 20-30 secs

- intensities can be written as:

$$\lambda_i(t) = \mu_i + \sum_{k=t-n}^{t-1} w_{i1}(t - k) N_1(k\delta t) + \sum_{k=t-n}^{t-1} w_{i2}(t - k) N_2(k\delta t)$$

where $h_{ij}(t)$ consists of $n$ pieces, with the $t$-th having value $w_{ij}(t)$
Problem Formulation

the fitting problem can now be cast as a convex optimization program:

\[
\begin{align*}
\text{maximize} & \quad \mathcal{L} \\
\text{subject to} & \quad \mu_i \geq \epsilon \\
& \quad w_{ij} \geq 0 \\
& \quad z_{ii} \geq \epsilon \\
& \quad \frac{z_{12}}{z_{11}} \leq z_{22} \\
& \quad z_{ii} = 1 - 1^T w_{ii} \\
& \quad z_{ij} = 1^T w_{ij} \\
& \quad z_{12} = z_{21} + s \\
& \quad s \geq 0
\end{align*}
\]

- constraints correspond to positivity and stationarity
• the piecewise linear form of $h_{ij}(t)$ results in many more variables, $w_{ij}(t)$, typically 200

• still, it can now be readily solved (using a primal-dual interior point solver for instance)
  – much, much faster
  – with an optimality certificate
Regression Interpretation

- this model corresponds to a linear Poisson regression with constraints — with features being past data

- other information can easily be used as features: price, volume, market index, option data
Comparison with Hewlett (2006)

- fitting time drops from 1 hour to 30 sec!!!
- mle objective value is also 33% higher!
QQ plot

(data from YHOO 07/03/06)
Strategy Simulation

- data from NYSE TAQ

- market assumptions
  - discrete time (1-second resolution for TAQ)
  - we execute a trade at worst possible price over next five seconds
  - trades do not affect the market
  - no short selling constraints
Use of Hawkes Model

- types:
  - exponential
  - piecewise linear

- parameter estimation using data from the previous day

- parameters applied to calculate buy and sell intensities, $\lambda_{\text{buy}}, \lambda_{\text{sell}}$ throughout trading days
Scalp Reverse Strategy

- one share position limit
- target intensity ratio $\rho$ that defines enter strategy:
  - buy when $\lambda_{buy}/\lambda_{sell} > \rho$
  - sell when $\lambda_{sell}/\lambda_{buy} > \rho$
- no explicit exit strategy
- can only reverse positions intraday
- liquidate position at end of day
Performance for MSFT, 01/03/07-01/11/07

![Graph showing the performance of MSFT from 01/03/07 to 01/11/07. The x-axis represents the intensity ratio ρ, while the y-axis represents the average profit per trade ($). The graph compares piecewise linear and exponential models.]
Performance for TIE, 09/01/06-01/11/07

\[ \rho = 30, \text{ average profit per trade } \$0.087, \text{ 86 trades (43 roundturns)} \]
### Performance for Other Stocks

<table>
<thead>
<tr>
<th>stock</th>
<th>avg. profit/stock/day</th>
<th>no. of trades</th>
<th>$\rho$</th>
<th>period</th>
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<td>YHOO</td>
<td>$.00565</td>
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<td>17</td>
<td>07/03/06 – 01/31/07</td>
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<tr>
<td>SNDK</td>
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<td>15</td>
<td>08/01/06 – 12/29/06</td>
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<td>15</td>
<td>03/08/06 – 03/24/06</td>
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<td>24</td>
<td>7.5</td>
<td>01/03/07 – 01/11/07</td>
</tr>
</tbody>
</table>
Good Model, Bad Strategy?

- technical indication of fundamental change?

- improvements:
  - trade size increases with intensity ratio threshold
  - exit strategy depending on price, time, and/or intensity

- incorporate price, volume or market index information in features