A model of disease spread and containment

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March 14, 2006 / Stanford MS&E 292 guest lecture
Goal

• What is the cost-minimizing mix of screening and contact tracing
  – in order to find \( n \) disease cases?
  – in order to keep long-term disease prevalence below \( P \)?
  – in order to reduce disease prevalence below \( P \) by time \( T \)?
Intervention 1: random screening

routine/recommended screening used in high risk groups

• TB screening
  – school employees
  – prisons
  – immigrants

• syphilis screening for pregnant women

• STD screening for high risk populations
Intervention 2: contact tracing

health care provider’s perspective:
• infected person found (*index case*)
• treated
• asked for list of *contacts*
• contacts found and tested
• if contact infected go to step 1.

• standard practice for Tuberculosis (TB)
• common for HIV and other STDs
  – called *partner notification*
Outline

1. Model the dynamics
2. Optimal policies
3. Discussion
4. An extended model
S→I→S model

cured (produces costs)

birth

S

infection

I

dead

dead

cured (produces costs)
\[\text{S} \rightarrow \text{I} \rightarrow \text{S} \text{ model}\]

\[N=S+I, \quad N'=0\]

\[\gamma[\beta I(S/N) + \eta N]C_t\]

cured (produces costs)

\[\gamma[\beta I(S/N) + \eta N]\]

\[\mu N\]

birth

\[\mu S\]

death

\[\beta I(S/N) + \eta N\]

infection

\[\mu I\]

death

\[N=S+I, \quad N'=0\]
$S \rightarrow I \rightarrow S$ model

$S' = -[\beta I(S/N) + \eta N] - \mu S + \gamma[\beta I(S/N) + \eta N] + \mu N$

$I' = +[\beta I(S/N) + \eta N] - \mu I - \gamma[\beta I(S/N) + \eta N]$

$N = S + I$

$\hat{C} = \gamma[\beta I(S/N) + \eta N]C_t$
\[ S' = -\left[ \beta I(S/N) + \eta N \right] - \mu S + \gamma \left[ \beta I(S/N) + \eta N \right] + \mu N \]

\[ I' = +\left[ \beta I(S/N) + \eta N \right] - \mu I - \gamma \left[ \beta I(S/N) + \eta N \right] \]

\[ N = S + I \]

\[ \hat{C} = \gamma \left[ \beta I(S/N) + \eta N \right] C_t \]
Reduced model

\[ S' = - [\beta I(S/N) + \eta N] - \mu S + \gamma [\beta I(S/N) + \eta N] + \mu N \]

\[ I' = + [\beta I(S/N) + \eta N] - \mu I - \gamma [\beta I(S/N) + \eta N] \]

\[ \hat{C} = \gamma [\beta I(S/N) + \eta N] C_t \]

\[ p = I/N \quad \quad C = \hat{C}/N \]

\[ p' = [\beta p(1 - p) + \eta] - \mu p - \omega(p) \]

\[ C = \omega(p) C_t \]
Intervention 1: random screening

random screening at rate $\lambda$

• reduces prevalence at rate $\lambda p$

• cost per capita is $\lambda(C_S + C_t p)$

$$p' = [\beta p(1 - p) + \eta] - \mu p - \omega(p) - \lambda p$$

$$C = \omega(p)C_t + \lambda(C_S + C_t p)$$
Intervention 2: contact tracing

health care provider’s perspective:

• infected person found (index case)
• treated
• asked for list of contacts
• contacts found and tested
• if contact infected go to step 1.

• standard practice for Tuberculosis (TB)
• common for HIV and other STDs
  – called partner notification
Intervention 2: contact tracing

node 2 infected
Intervention 2: contact tracing

node 2 infects nodes 1, 4, 5
Intervention 2: contact tracing

node 4 gets tested (maybe has symptoms)
Intervention 2: contact tracing

node 4
- tests positive, gets treated
- becomes a contact tracing *index case*
- names nodes 1,2,3,6,7 as contacts

nodes 1,2,3,6,7 scheduled to be tested
Intervention 2: contact tracing

node 3 tests negative
Intervention 2: contact tracing

node 6 tests negative
Intervention 2: contact tracing

- Node 1:
  - Tests positive, gets treated
  - Becomes a contact tracing index case
  - Names nodes 2, 4 as contacts

- Node 4 already tested
  - Testing node 2 gets higher priority as named by both nodes 1, 4
Intervention 2: contact tracing

- Node 2 tests positive, gets treated
- Becomes a contact tracing index case
- Names nodes 1, 4, 5 as contacts

Nodes 1, 4 already tested
Node 5 scheduled to be tested
Intervention 2: contact tracing

node 5
• tests positive, gets treated
• becomes a contact tracing *index case*
• names node 2 as a contact

node 2 already tested
Intervention 2: contact tracing

node 7 tests negative
Intervention 2: contact tracing

• $\delta=1$ if program exists, 0 if it does not
• $K_T$ number of infected contacts per index case
  - $\delta K_T (\lambda p + \omega(p))$ total
• CT cost per index case
  - $\delta C_T (\lambda p + \omega(p))$

\[ p' = [\beta p(1 - p) + \eta] - \mu p - (1 + \delta K_T) (\lambda p + \omega(p)) \]
\[ C(\lambda, \delta; p) = \omega(p) C_t + \lambda (C_S + C_d p) + \delta C_T (\lambda p + \omega(p)) \]
Optimal intervention

tradeoff $C$ and $p$
by choosing $\lambda$ and $\delta$

1. small changes (unchanged prevalence)
2. long term costs ($p' = 0$)
3. transition costs
Unchanged prevalence

\[ \text{Cost} = \min_{\lambda, \delta} C(\lambda, \delta; p) \]

s.t. \[ N(1+\delta K_T) (\lambda p + \omega(p)) = n \]

\[ \lambda \geq 0, \quad \delta = 0, 1 \]
Unchanged prevalence

\[ \text{Cost} = \min_{\lambda, \delta} C(\lambda, \delta; p) \]

s.t. \[ N(1+\delta K_T) (\lambda p + \omega(p)) = n \]

\[ \lambda \geq 0, \; \delta = 0, 1 \]

this is the number of people we find assuming \( p \) doesn’t change
contact tracing not optimal

contact tracing optimal

\[ \bar{\lambda}_U(n; p) \]

\[ \delta^*_U = 1 \]

\[ \delta^*_U = 0 \]

0

P'

p

1
Long term costs

\[ \text{Cost} = \min_{\lambda(t), \delta(t)} C(\lambda, \delta; p) \]

\[ \text{s.t.} \quad p(t) = P \text{ for all } t \]

\[ \lambda(t) \geq 0, \quad \delta(t) = 0, 1 \]

\[ \min_{\lambda, \delta} C(\lambda, \delta; P) \]

\[ \text{s.t.} \quad 0 = p' = [\beta P(1 - P) + \eta] - \mu P - (1 + \delta K_T) (\lambda P + \omega(P)) \]

\[ \lambda \geq 0, \quad \delta = 0, 1 \]
Transition costs

\[ C^*_T(p_0, T, P_1) := \min_{\delta(t), \lambda(t)} \int_0^T e^{-rt} C(\lambda(t), \delta(t); p(t)) \, dt \]

s.t. \[ \dot{p}(t) = f(p(t), \lambda(t), \delta(t)) \ \forall t \]

\[ p(0) = p_0, \quad p(T) \leq P_1, \quad \dot{p}(t) \leq 0 \ \forall t \]

\[ \lambda(t) \geq 0, \quad \delta(t) \in \{0, 1\} \ \forall t. \]
Results

\( \delta = 1 \) optimal if and only if \( p < \frac{C_S}{(C_T/K_T - C_t)} \)

equivalently \( \frac{C_S}{p} < \frac{C_T}{K_T - C_t} \)

- Unchanged prevalence
  - \( \lambda \) uniquely determined by feasibility
- Long term costs
  - \( \lambda \) uniquely determined by feasibility
- Transition Costs, \( p(t) \) flat or makes jumps
Insights

- contact tracing cost-effective only when $p$ below some threshold
- screening rate $\lambda$ decreasing in prevalence $p$
- as $p$ increases above the threshold $\lambda$ jumps up
- model robust to different cost formulations
Model criticisms

• population in steady state: births correlated exactly balance deaths
• infections from abroad don’t depend on p (or number susceptible)
• # found by contact tracing doesn’t depend on p (or number infected)
Model criticisms

- homogenous mixing
- no delay terms for infection or contact tracing
- no model of effort or contact tracing capacity
- deterministic
- lacking realistic parameters
Current work

• modeling how an STD spreads in a high school

“One in 12 Philly teenage girls has chlamydia.”

Stochastic $S \rightarrow I \rightarrow R$ model

- New students enter the $S$ (susceptible) state.
- Infection leads to transitioning from $S$ to $I$ (infected).
- Cure with relapsing (to risky behavior) can lead back to $R$ (recovered).
- Drop-out or graduate can lead to $R$.

States:
- $S$: Susceptible
- $I$: Infected
- $R$: Recovered
Stochastic $S \rightarrow I \rightarrow R$ model

relapsing (to risky behavior) $R/\tau_r$

\begin{align*}
N &= S + I + R \\
ir &= +S/\tau_{oi} + I(S/N)/\tau_{ii} \\
cr &= +I/\tau_h + (I/N)/\tau_s + \delta \ R \ d \ (I/N)/\tau_c
\end{align*}
A network model
size of largest connected component

average number of contacts
Questions?

Optimal mix of screening and contact tracing for endemic diseases
www.stanford.edu/~barmbrus/policystatics.pdf