LP Formulations for Radiation Treatment Planning (IMRT)

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joint work with Russell Hamilton, Martin Lachaine, and J. Cole Smith
Radiation Therapy

1. meet doctor
2. CT scan
3. M.D. identifies tumor, organs, writes prescription
4. dosimetry/physics creates plan
5. treatment: M-F, 5-8 wks
6. followups

Goals: tumor control and organ functionality
Leaves allow custom apertures
Typical Data and Problem Complexity

- leaves 0.5-1 cm thick
- max aperture 40 cm
- ~5-9 beam angles
- ~5-30-100 aperture shapes per beam angle

- CT 512x512 pixels (0.5-1mm), 1-5mm slices
- dose calculation 2-5mm mesh
The Model

dose at location \( j \) = \( \sum_i \) weight given to beamlet \( i \) \times \text{normalized dose to location } j \text{ from beamlet } i

or \( D_j = \sum_i w_i \times d_{ij} \)
Cumulative dose-volume histogram

PTV:  
-Requested,  
-Optimizer result

Post-rectum:  
-Requested,  
-Optimizer result
Plan 1

Constraints:
- PTV > 1
- PTV < x

Goal: \( \text{min } x \)  

Result: \( x = 1.01 \)

Dose contour levels:
- 1.15
- 1.00
- 0.80
- 0.50
- 0.30

*Dose normalized to prescription dose

PTV = tumor
Plan 2

constraints:
• PTV > 1
• PTV < 1.15
• rectum outside PTV < x

goal: min x
result: x = 0.59
Plan 3

Constraints:
• PTV > 1
• PTV < 1.15
• Avg. dose in rectum outside PTV < x

Goal: Min x
Result: x = 0.15
Plan 4

cconstraints:
• PTV > 1
• PTV < 1.15
• femurs < 0.25
• avg. dose in rectum outside PTV < x

goal: min x

result: x=0.30

dose contour levels

<table>
<thead>
<tr>
<th>level</th>
<th>color</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.15</td>
<td>red</td>
</tr>
<tr>
<td>1.00</td>
<td>red</td>
</tr>
<tr>
<td>0.80</td>
<td>yellow</td>
</tr>
<tr>
<td>0.50</td>
<td>blue</td>
</tr>
<tr>
<td>0.30</td>
<td>green</td>
</tr>
</tbody>
</table>
Plan 5

constraints:
• PTV > 1
• PTV < 1.15
• femurs < 0.25
• avg. dose in rectum outside PTV < 0.33
• avg. dose in bladder < x

goal: min x
result: x=0.54
Plan 6

constraints:
• PTV > 1
• PTV < 1.15
• femurs < 0.375
• avg. dose in rectum outside PTV < 0.50
• avg. dose in bladder < 0.71
• avg. external dose < x

goal: min x
result: x=0.17
Plan 7

Constraints:
- **PTV > 1**
- **PTV < 1.15**
- **femurs < 0.375**
- **avg. dose in rectum outside PTV < 0.50**
- **avg. dose in bladder < 0.71**
- **external < 1**
- **avg. external dose < x**

Goal: min x

Result: x = 0.17
Plan 8

constraints:
• PTV > 1
• PTV < 1.15
• femurs < 0.375
• avg. dose in rectum outside PTV < 0.50
• avg. dose in bladder < x
• external < 1
• avg. external dose < 0.26

goal: min x
result: x=0.37
Plan 9

Constraints:
- PTV > 1
- PTV < 1.15
- Femurs < 0.375
- Avg. dose in rectum outside PTV < 0.50
- Avg. dose in bladder < x
- External < 0.80
- Avg. external dose < 0.26

Goal: Min x

Result: x = 0.39

Dose distribution

<table>
<thead>
<tr>
<th>dose contour levels</th>
<th>1.15</th>
<th>1.00</th>
<th>0.80</th>
<th>0.50</th>
<th>0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>GTV</td>
<td>Red</td>
<td>Red</td>
<td>Red</td>
<td>Red</td>
<td>Red</td>
</tr>
<tr>
<td>Bladder</td>
<td>Yellow</td>
<td>Yellow</td>
<td>Yellow</td>
<td>Yellow</td>
<td>Yellow</td>
</tr>
<tr>
<td>Left femur</td>
<td>Pink</td>
<td>Pink</td>
<td>Pink</td>
<td>Pink</td>
<td>Pink</td>
</tr>
<tr>
<td>Right femur</td>
<td>Cyan</td>
<td>Cyan</td>
<td>Cyan</td>
<td>Cyan</td>
<td>Cyan</td>
</tr>
<tr>
<td>Rectum</td>
<td>Blue</td>
<td>Blue</td>
<td>Blue</td>
<td>Blue</td>
<td>Blue</td>
</tr>
<tr>
<td>PTV</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
</tbody>
</table>
Hard constraints fit tumor control

- probability cell survives: $s_j = \exp[-D_j \alpha]$
  - $s_j << 1$

- probability all clonagenic tumor cells die:
  
  $$p = \prod (1-s_j) \approx 1-\sum s_j \approx 1-\max s_j = 1-\exp[-\alpha \min D_j]$$

  hence $p = f(\min \text{dose})$
Hard constraints fit serial organs?

- serial organs: need all cells to functions
  - example: spinal cord
- probability all cells survive, $p$
  
  recall $s_j = \exp[-D_j \alpha]$
  
  $p = \prod s_j = \exp[-\alpha \sum D_j]$ 
  hence $p = f(\text{mean dose})$

- caveat: $s_j = \exp[-D_j \alpha - D_j^2 \beta]$ and data ambiguous on importance of $\beta$
Cumulative dose-volume histogram

- $D_j \leq 40$ for all $j$ in Rectum
- $D_j \leq 30 + \text{BigNum} \times x_j$
  - $x_j$ binary
  - $\sum x_j \leq 0.5 \times \text{num mesh points in Rectum}$
penalty functions

penalty function for organ

Dose (Gy)
Tail averages

• $\text{avg}_{\text{over } X\% \text{ of volume with highest dose}} D_j \leq U$
• $\text{avg}_{\text{over } X\% \text{ of volume with lowest dose}} D_j \geq L$
• each such constraint needs an artificial variable for every mesh point
• introduced by Romeijn et al.

• formulation for upper tail average:
  – $w_j \geq D_j - z, \quad w_j \geq 0$
  – $z + 1/(X\% \text{ vol}) \sum w_j \leq U$
  – we want $w_j = \max(0, D_j - z)$
  – we want $z = \min_{\text{over } X\% \text{ of volume with highest dose}} D_j$
gradient objective

\[
\min \sum_{\nu} \mathbf{u}_\nu \cdot \nabla \text{dose}_\nu
\]
Questions