New Developments in the Design and Operation of the NEOS Server

Robert Fourer
Industrial Engineering & Management Sciences
Northwestern University, Evanston, IL, USA
4er@iems.northwestern.edu

Jorge Moré, Todd Munson, Jason Sarich
Mathematics and Computer Science Division
Argonne National Laboratory, Argonne, IL, USA
{more,tmunson,sarich}@mcs.anl.gov

APMOD 2004
Brunel University, London — Tuesday, June 22, 2004 — TB31

The NEOS Server

Server basics
- Use through a web browsers
- Use within a modeling environment

Frequently asked questions
- How is NEOS supported?
- Who uses it? How much?
- What solvers are there?
- How does a user know which solver to choose?

Recent and forthcoming developments
- XML-standard formats
- Problem analysis and solver choice
- Web services
- Benchmarking and verification
NEOS  www-neos.mcs.anl.gov/neos/

A general-purpose optimization server
- Several dozen solvers in all
- Commercial as well as experimental solvers
- Central scheduler with distributed solver sites

A research project
- Currently free of charge
- Supported through the Optimization Technology Center
  of Northwestern University & Argonne National Laboratory

. . . 4109 submissions last week
. . . as many as 6094 submissions per week

Using NEOS

Varied submission options
- E-mail
- Web forms
- TCP/IP socket-based submission tool: Java or tcl/tk
- Direct from optimization modeling environments

Numerous formats
- Low-level formats: MPS, SIF, SDPA
- Programming languages: C/ADOL-C, Fortran/ADIFOR
- High-level modeling languages: AMPL, GAMS

Example . . .
- Investigating solvers
- Using the Web interface
- Using AMPL via the Kestrel interface
Using NEOS

Learn About Your Problem

The NEOS Guide

- Optimization tree: Problem types
- Optimization software guide: Individual solvers
- Frequently asked questions: Varied listings & advice

Using NEOS

Investigate Solvers

NEOS Server home page
Investigate Solvers

NEOS Server solver type listing

Investigate Solvers

NEOS Server solver listing
Investigate Solvers

Individual solver listing

Try a Solver: Web Interface

Sample submission form

“Comments and Questions” button on every page
Using NEOS
Try a Solver: Web Interface

Submission form for your problem

Start of your run
Using NEOS

Try a Solver: Web Interface

Beginning of your solution listing

![Image of the NEOS Server web interface]

End of your solution listing

![Image of the NEOS Server solution listing]

Robert Fourer, Jorge Moré, Todd Munson & Jason Sarich, APMOD 2004, 22 June 2004, TB31
Try a Solver: Kestrel Interface

Kestrel client download page

Using NEOS

Try a Solver: Kestrel Interface

Applying a local solver to an AMPL model

AMPL Version 20040202 (MS VC++ 6.0)
ampl: model gs2000b.mod; data gs2000b.dat;
ampl: option solver minos;
ampl: solve;
Presolve eliminates 100 constraints.
Adjusted problem:
4290 variables: 4260 binary variables
30 linear variables
733 constraints, all linear; 36340 nonzeros
1 linear objective; 30 nonzeros.
MINOS 5.5:
Sorry, the student edition is limited to 300 variables and 300 constraints. You have 4290 variables and 733 constraints.
exit code 1
<BREAK>
Using NEOS

Try a Solver: Kestrel Interface

Applying a NEOS solver to an AMPL model . . .

```
ampl: option solver kestrel;
ampl: option kestrel_options 'solver=loqo';
ampl: option loqo_options 'minlocfil outlev=1';
ampl: solve;
```

Job has been submitted to Kestrel
Kestrel/NEOS Job number : 368607
Kestrel/NEOS Job password : OxBpVYMb
Check the following URL for progress report :
hhttp://www-neos.mcs.anl.gov/neos/
neos-cgi/check-status.cgi?job=368607&pass=OxBpVYMb

In case of problems, e-mail neos-comments@mcs.anl.gov

Using NEOS

Try a Solver: Kestrel Interface

. . . and receiving a solution from NEOS . . .

```
LOQO 6.06: minlocfil
  1   0.000000e+00  2.1e-01 -4.266000e+05  3.1e+02
  2   2.840664e+03  9.8e-04 -4.206215e+05  1.5e+01
  3   2.796511e+03  4.8e-04 -3.078341e+05  6.2e-01
  4   1.769055e+03  2.8e-04 -2.948373e+04  0.0e+00           DF
  5   3.028480e+02  5.2e-05 -3.871922e+03  0.0e+00           DF
  6   3.705130e+01  5.9e-06 -2.158849e+02  0.0e+00           DF
  7   2.220340e+01  3.1e-06 -1.070050e+01  0.0e+00           DF
  8   1.685976e+01  1.4e-06  2.596295e+00  0.0e+00           DF
  9   1.534094e+01  6.7e-07  9.481761e+00  0.0e+00           PF DF
 10   1.445050e+01  2.1e-07  1.284805e+01  0.0e+00           PF DF
 11   1.405725e+01  8.3e-09  1.333832e+01  0.0e+00    1       PF DF
 12   1.400313e+01  4.9e-10  1.396657e+01  0.0e+00    3       PF DF
 13   1.400015e+01  2.5e-11  1.399833e+01  0.0e+00    4       PF DF
 14   1.400001e+01  1.2e-12  1.399992e+01  0.0e+00    5       PF DF
LOQO 6.06: optimal solution (14 QP iterations, 14 evaluations)
primal objective 14.00000783
dual objective 13.99991642
```
Using NEOS

Try a Solver: Kestrel Interface

... where it can be browsed interactively

```ampl
ampl: option display_eps .000001;
ampl: display MinType, MaxType;
: MinType  MaxType := Division Shipping          0        1
Division Logistics_and_Supply_Chain   1        2
Division Information_Technology       1        1
Division Production                   0        1
Division Production_Scheduling       1        2
Division Production_Scheduling_Research 1 2
Division Operations_Management      0        1
Division Finance                      3        4
Division Support                      0        1
Office Americas                      7        8
Office EMEA                          1        2
Office Far_East                      1        2
Gender F                              3        4
Gender M                              7        8
;
```

Using NEOS

Try a Solver: Kestrel Interface

Web form for checking your run's status
Using NEOS

Try a Solver: Kestrel Interface

Intermediate status listing

![Intermediate status listing](image1)

Using NEOS

Try a Solver: Kestrel Interface

Final result listing

![Final result listing](image2)
Frequently Asked Questions

**Who uses NEOS?**
- Where are its users from?
- How much is it used?

**What kinds of solvers does NEOS offer?**
- Who supplies them?
- Which are most heavily used?
- Where are they hosted?

**How is NEOS supported?**
- Who answers user questions?

---

**Who Uses NEOS? (a sample)**
- We are using NEOS services for duty-scheduling for ground handling activities in a regional airport environment.
- We used NEOS to solve nonlinear optimization problems associated with models of physical properties in chemistry.
- Our company is working with various projects concerning R&D of internal combustion engines for cars and brakes for heavy vehicles.
- We are working on bi-dimensional modeling of earth's conductivity distribution.
- I am dealing with ultimate limit-state analyses of large dams by means of a non-standard approach ("direct method"); this requires solving problems of linear and non-linear programming. The NEOS server is an extraordinary tool to perform parametric tests on small models, in order to choose the best suited solver.
- I have used NEOS with LOQO solver to optimize an interpolator. . . . My domain is digital receivers where the receiver clock is not changed to match the transmitter clock.
Who Uses NEOS? (more)

- I have been able to build and solve a prototype combinatorial auction MIP model using AMPL and NEOS in a fraction of the time it would have required me to do this had I needed to requisition a solver and install it locally.
- Our idea is trying to design antennas by using the computer. . . . We have tried various solvers on NEOS to see if this is possible at all.
- I am using the LOQO solver and code written in AMPL to perform numerical optimization of a spinor Bose-Einstein condensate.
- We are using the NEOS Server for solving linear and nonlinear complementarity problems in engineering mechanics and in robotics.
- I have been working on a system for protein structure prediction. . . . I had need to incorporate a nonlinear solver to handle packing of sidechain atoms in the protein.

Who Uses NEOS? (academic)

- I am regularly suggesting my students to use NEOS as soon as their projects in AMPL cannot be solved with the student edition. So they debug their AMPL models locally . . . and then they run their real-life projects thanks to NEOS.
- I didn't even know what nonlinear programming was and after I discovered what it was, it became clear how enormous a task it would be to solve the problems assigned to me. . . . I had extremely complicated objective functions, both convex and nonconvex, an armload of variables, and an armload of convex, nonconvex, equality and inequality constraints, but when I sent off the information via the web submission form, within seconds I received extremely accurate and consistent results. The results were used for verifying a certain theory in my professor’s research and so accuracy was extremely important.
- NEOS has been a very valuable tool in the two graduate optimization courses that I teach. NEOS allows students to see a broader variety of solvers than we have available . . .
  ...more at www-neos.mcs.anl.gov/neos/stories.html
### NEOS Users

#### Where are They From?

**2004 through 24 April:**
*Identifiable domain and >= 20 submissions*

<table>
<thead>
<tr>
<th>Domain</th>
<th>Submissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(com)</td>
<td>3502</td>
</tr>
<tr>
<td>(edu)</td>
<td>13610</td>
</tr>
<tr>
<td>(gov)</td>
<td>2510</td>
</tr>
<tr>
<td>(net)</td>
<td>12310</td>
</tr>
<tr>
<td>(mil)</td>
<td>90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Country</th>
<th>Submissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium (be)</td>
<td>4918</td>
</tr>
<tr>
<td>Brazil (br)</td>
<td>2558</td>
</tr>
<tr>
<td>Canada (ca)</td>
<td>4514</td>
</tr>
<tr>
<td>Switzerland (ch)</td>
<td>1098</td>
</tr>
<tr>
<td>Chile (cl)</td>
<td>158</td>
</tr>
<tr>
<td>Colombia (co)</td>
<td>218</td>
</tr>
<tr>
<td>Cyprus (cy)</td>
<td>20</td>
</tr>
<tr>
<td>Czech Republic (cz)</td>
<td>554</td>
</tr>
<tr>
<td>Germany (de)</td>
<td>1834</td>
</tr>
<tr>
<td>Spain (es)</td>
<td>1548</td>
</tr>
<tr>
<td>Finland (fi)</td>
<td>270</td>
</tr>
<tr>
<td>Micronesia (fm)</td>
<td>36</td>
</tr>
<tr>
<td>France (fr)</td>
<td>2730</td>
</tr>
<tr>
<td>Greece (gr)</td>
<td>776</td>
</tr>
<tr>
<td>Hong Kong (hk)</td>
<td>...</td>
</tr>
</tbody>
</table>

Northwestern 1338  
Argonne 2194

#### What Countries are They From?

**2004 through 24 April**

<table>
<thead>
<tr>
<th>Country</th>
<th>Submissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium (be)</td>
<td>4918</td>
</tr>
<tr>
<td>Brazil (br)</td>
<td>2558</td>
</tr>
<tr>
<td>Canada (ca)</td>
<td>4514</td>
</tr>
<tr>
<td>Switzerland (ch)</td>
<td>1098</td>
</tr>
<tr>
<td>Chile (cl)</td>
<td>158</td>
</tr>
<tr>
<td>Colombia (co)</td>
<td>218</td>
</tr>
<tr>
<td>Cyprus (cy)</td>
<td>20</td>
</tr>
<tr>
<td>Czech Republic (cz)</td>
<td>554</td>
</tr>
<tr>
<td>Germany (de)</td>
<td>1834</td>
</tr>
<tr>
<td>Spain (es)</td>
<td>1548</td>
</tr>
<tr>
<td>Finland (fi)</td>
<td>270</td>
</tr>
<tr>
<td>Micronesia (fm)</td>
<td>36</td>
</tr>
<tr>
<td>France (fr)</td>
<td>2730</td>
</tr>
<tr>
<td>Greece (gr)</td>
<td>776</td>
</tr>
<tr>
<td>Hong Kong (hk)</td>
<td>466</td>
</tr>
<tr>
<td>Hungary (hu)</td>
<td>182</td>
</tr>
<tr>
<td>Israel (il)</td>
<td>1022</td>
</tr>
<tr>
<td>Italy (it)</td>
<td>1646</td>
</tr>
<tr>
<td>Japan (jp)</td>
<td>32</td>
</tr>
<tr>
<td>Luxembourg (lu)</td>
<td>40</td>
</tr>
<tr>
<td>Mexico (mx)</td>
<td>252</td>
</tr>
<tr>
<td>Malaysia (my)</td>
<td>912</td>
</tr>
<tr>
<td>Netherlands (nl)</td>
<td>4254</td>
</tr>
<tr>
<td>Norway (no)</td>
<td>82</td>
</tr>
<tr>
<td>New Zealand (nz)</td>
<td>20</td>
</tr>
<tr>
<td>Poland (pl)</td>
<td>302</td>
</tr>
<tr>
<td>Sweden (se)</td>
<td>3562</td>
</tr>
<tr>
<td>Singapore (sg)</td>
<td>236</td>
</tr>
<tr>
<td>USSR (former) (su)</td>
<td>834</td>
</tr>
<tr>
<td>Turkey (tr)</td>
<td>526</td>
</tr>
<tr>
<td>Taiwan (tw)</td>
<td>62</td>
</tr>
<tr>
<td>United Kingdom (uk)</td>
<td>4870</td>
</tr>
<tr>
<td>Venezuela (ve)</td>
<td>2042</td>
</tr>
</tbody>
</table>
NEOS Users

How Much Do They Use It?

Submissions by month, 1/1999 through 3/2004

... 15 / hour over past year
... 20 / hour in peak months

What Solvers Does NEOS Offer?

For familiar problem types
- Linear programming
- Linear network optimization
- Linear integer programming
- Nonlinear programming
- Stochastic linear programming
- Complementarity problems

For emerging problem types
- Nondifferentiable optimization
- Semi-infinite optimization
- Global optimization
- Nonlinear integer programming
- Semidefinite & 2nd-order cone programming

... virtually every published semidefinite programming code
NEOS Solvers

Who Supplies Them?

Some commercial solver vendors
- Xpress, FortMP (mixed integer)
- CONOPT, KNITRO, MOSEK (nonlinear)

Universities and their researchers
- BonsaiG (mixed integer)
- DONLP2, FILTER, LANCELOT, LOQO, MINOS, SNOPT (nonlinear)

Open-Source Enthusiasts
- GLPK (mixed integer)

with thanks to . . .
- Modeling language vendors (AMPL, GAMS)
- Hans Mittelmann, University of Arizona

NEOS Solvers

Which are Most Heavily Used?

Totals for 2002 . . .
NEOS Solvers

Which are Most Heavily Used?

... totals since January 1st, 2003

NEOS Solvers

Where are They Hosted?

Varied workstations at

- Aachen University of Technology
- Argonne National Laboratory
- Arizona State University
- Lehigh University
- National Taiwan University
- Northwestern University (with support from Sun Microsystems)
- University of Wisconsin at Madison

... new hosts are readily added anywhere on the Internet
How is NEOS Supported?

Grants
- National Science Foundation, Operations Research Program, grant DMI-0322580
- National Science Foundation, Information Technology Research Program, grant CCR-0082807
- National Science Foundation, Challenges in Computational Science Program, grant CDA-9726385

Donations
- Processor cycles
- Many people’s time

... no user charges as yet
Recent & Forthcoming Developments

**Problem analysis and solver choice**

**Web services**

**Benchmarking and verification**

**XML-standard formats**

- see next talk

---

Problem Analysis & Solver Choice

**Motivation**

- Confirm problem type
- Choose appropriate solver

**Problem analysis**

- Information included with problem instance
- Characteristics determined by analyzer
- Convexity

**Solver choice**

- Relational database
- Database queries
Example 1

**Transportation with nonlinear costs**

```plaintext
set ORIG;   # origins
set DEST;   # destinations

param supply {ORIG} >= 0;   # amounts available at origins
param demand {DEST} >= 0;   # amounts required at destinations
param rate {ORIG,DEST} >= 0;   # base shipment costs per unit
param limit {ORIG,DEST} > 0;   # limit on units shipped

var Trans {i in ORIG, j in DEST} >= 1e-10, <= .9999 * limit[i,j], := limit[i,j]/2;

minimize Total_Cost:
  sum {i in ORIG, j in DEST}
    rate[i,j] * Trans[i,j]^0.8 / (1 - Trans[i,j]/limit[i,j]);

subject to Supply {i in ORIG}:
  sum {j in DEST} Trans[i,j] = supply[i];

subject to Demand {j in DEST}:
  sum {i in ORIG} Trans[i,j] = demand[j];
```

---

Problem analysis

Example 1 (cont’d)

**Transportation data**

```plaintext
param: ORIG:  supply :=
  GARY  1400
  CLEV  2600
  PITT  2900 ;

param: DEST:  demand :=
  FRA     900    STL    1700
  DET    1200    FRE    1100
  LAN     600    LAF    1000
  WIN     400 ;

param rate :  FRA  DET  LAN  WIN  STL  FRE  LAF :=
  GARY  39   14   11   14   16   82   8
  CLEV  27    9   12    9   26   95   17
  PITT  24   14   17   13   28   99   20 ;

param limit :  FRA  DET  LAN  WIN  STL  FRE  LAF :=
  GARY  500 1000 1000 1000 800 500 1000
  CLEV  500  800  800  800  500  500  1000
  PITT  800  600  600  600  500  500  900 ;
```
Example 1 (cont’d)

AMPL’s .nl file: Summary information

| 0 1 | # nonlinear constraints, objectives |
| 0 0 | # network constraints: nonlinear, linear |
| 0 21 0 | # nonlinear vars in constraints, objectives, both |
| 0 0 0 1 | # linear network vars; functions; arith, flags |
| 0 0 0 1 | # discrete vars: binary, integer, nonlinear (b,c,o) |
| 42 21 | # nonzeros in Jacobian, gradients |
| 0 0 | # max name lengths: constraints, variables |
| 0 0 0 0 | # common exprs: b,c,o,c1,o1 |

AMPL does all the work here

Example 1 (cont’d)

AMPL’s .nl file: Nonlinear expressions

| 00 0 | # \text{Total Cost} |
| 054 | # \text{sumlist} |
| 21 | |
| 03 | #/ |
| 02 | #* |
| n39 | |
| 05 | ^ |
| v0 | #\text{Trans[‘GARY’, ‘FRA’]} |
| n0.8 | |
| 01 | # - |
| n1 | |
| 03 | #/ |
| v0 | #\text{Trans[‘GARY’, ‘FRA’]} |
| n500 | |
| 03 | #/ |
| 02 | #* |
| n14 | |
| 05 | ^ |
| ........ |

Problem analysis
Problem analysis

Example 2

Torsion model (parameters and variables)

```
param nx > 0, integer;    # grid points in 1st direction
param ny > 0, integer;    # grid points in 2nd direction
param c;                  # constant
param hx := 1/(nx+1);     # grid spacing
param hy := 1/(ny+1);     # grid spacing
param area := 0.5*hx*hy;  # area of triangle
param D {i in 0..nx+1,j in 0..ny+1} =
   min( min(i,nx-i+1)*hx, min(j,ny-j+1)*hy);  # distance to the boundary
var v {i in 0..nx+1,j in 0..ny+1};
```

# definition of the
# finite element approximation

Problem analysis

Example 2 (cont’d)

Torsion model (objective and constraints)

```
var linLower = sum {i in 0..nx, j in 0..ny}
   (v[i+1,j] + v[i,j] + v[i,j+1]);
var linUpper = sum {i in 1..nx+1, j in 1..ny+1}
   (v[i,j] + v[i-1,j] + v[i,j-1]);
var quadLower = sum {i in 0..nx,j in 0..ny} (
   ((v[i+1,j] - v[i,j])/hx)**2 + ((v[i,j+1] - v[i,j])/hy)**2);
var quadUpper = sum {i in 1..nx+1, j in 1..ny+1} (
   ((v[i,j] - v[i-1,j])/hx)**2 + ((v[i,j] - v[i,j-1])/hy)**2);
minimize Stress:
   area * ((quadLower+quadUpper)/2 - c*(linLower+linUpper)/3);
schedule
   subject to distanceBound {i in 0..nx+1, j in 0..ny+1}:
      -D[i,j] <= v[i,j] <= D[i,j];
```

Robert Fourer, Jorge Moré, Todd Munson & Jason Sarich, APMOD 2004, 22 June 2004, TB31 60
Example 2 (cont’d)

Output from AMPL’s presolver

Presolve eliminates 2704 constraints and 204 variables.
Substitution eliminates 4 variables.

Adjusted problem:
2500 variables, all nonlinear
0 constraints
1 nonlinear objective; 2500 nonzeros.

Problem analysis

Example 2 (cont’d)

Output from Dr. AMPL prototype (analysis)

Problem type
-------------------
-Problem has bounded variables
-Problem has no constraints

Analyzing problem using only objective
----------------------------------------
-This objective is quadratic
-Problem is a QP with bounds
-0.833013 <= objective <= 0.8359

Problem convexity
-------------------
Nonlinear objective looks convex on its domain.
Detected 0/0 nonlinear convex constraints,
0/0 nonlinear concave constraints.
### Solver choice

#### Example 2 (cont’d)

**Output from Dr. AMPL (solver recommendations)**

#### Specialized solvers, based on all properties ###

- MOSEK
- OOQP

#### Specialized solvers, excluding "hard" properties ###

- BLMVM
- FortMP
- L-BFGS-B
- MINLP
- MOSEK
- OOQP
- PathNLP
- SBB
- TRON

#### General-purpose solvers ###

- KNITRO
- LANCELOT
- LOQO

---

### Solver choice

#### Example 2 (cont’d)

**Output from MOSEK solver run**

```
ampl: model torsion.mod;
ampl: data torsion.dat;
ampl: option solver kestrel;
ampl: option kestrel_options 'solver=mosek';
ampl: solve;
Job has been submitted to Kestrel
Kestrel/NEOS Job number = 280313
Kestrel/NEOS Job password = ExPXrRcP
MOSEK finished.
t (interior-point iterations - 11, simplex iterations - 0)
Problem status : PRIMAL AND DUAL_FEASIBLE
Solution status : OPTIMAL
Primal objective : -0.4180876313
Dual objective : -0.4180876333
```
Problem Instance Characteristics

Read from .nl file header
- Size
- Differentiability
- Linearity
- Sparsity

Deduce from expression trees
- Quadraticity
- Smoothness
- Convexity . . .

Convexity

Significance
- For an optimization problem of the form

\[
\begin{align*}
\text{Minimize} & \quad f(x_1, \ldots, x_n) \\
\text{Subject to} & \quad g_i(x_1, \ldots, x_n) \geq 0, \quad i = 1, \ldots, r \\
& \quad h_j(x_1, \ldots, x_n) = 0, \quad j = 1, \ldots, s
\end{align*}
\]

a local minimum is global provided
* \( f \) is convex
* each \( g_i \) is convex
* each \( h_i \) is linear

- Many physical problems are naturally convex if formulated properly

Analyses . . .
- Disproof of convexity
- Proof of convexity
Disproof of Convexity

Find any counterexample
- Sample in feasible region
- Test any characterization of convex functions

Sampling along lines
- Look for \( f(\lambda x_1 + (1-\lambda)x_2) > \lambda f(x_1) + (1-\lambda)f(x_2) \)
- See implementation in mProbe
  (John Chinneck, www.sce.carleton.ca/faculty/chinneck.html)

Sampling at points
- Look for \( \nabla^2 f(x) \) not positive semi-definite
- Implemented in Dr. AMPL . . .

Problem analysis

Disproof of Convexity (cont'd)

Sampling
- Choose points \( x_0 \) such that \( x_{01}, \ldots, x_{0n} \) are within inferred bounds

Testing
- Apply GLTR (galahad.rl.ac.uk/galahad-www/doc/gltr.pdf) to
  \[
  \min_d \ nabla f(x_0)d + \frac{1}{2} d\nabla^2 f(x_0) d \\
  \text{s.t. } \|d\|_2 \leq \max\{10,\|\nabla f(x_0)\|/10\}
  \]
- Declare nonconvex if GLTR's Lanczos method finds a direction of negative curvature
- Declare inconclusive if GLTR reaches the trust region boundary without finding a direction of negative curvature
Proof of Convexity

**Proof of Convexity**

**Recurisely assess each expression tree node for**
- Bounds
- Monotonicity
- Convexity / Concavity

**Apply properties of functions**
- \(|x|_p\) is convex, \(\geq 0\) everywhere
- \(x^\alpha\) is convex for \(\alpha \leq 0, \alpha \geq 1\); \(-x^\alpha\) is convex for \(0 \leq \alpha \leq 1\)
- \(x^p\) for even \(p > 0\) is convex everywhere, decreasing on \(x \leq 0\), increasing on \(x \geq 0\), etc.
- \(-\log x\) and \(x \log x\) are convex and increasing on \(x > 0\)
- \(\sin x\) is concave on \(0 \leq x \leq \pi\), convex on \(\pi \leq x \leq 2\pi\), increasing on \(0 \leq x \leq \pi/2\) and \(3\pi/2 \leq x \leq 2\pi\), decreasing . . . \(\geq -1\) and \(\leq 1\) everywhere
- \(x^TMx\) is convex if \(M\) is positive semidefinite
- \(e^{\alpha x}\) is convex, increasing everywhere for \(\alpha > 0\), etc.
- \(-\Pi_i x_i^{1/n}\) is convex where all \(x_i > 0\) . . . etc., etc.

**Problem analysis**

**Proof of Convexity (cont’d)**

**Apply properties of convexity**
- Certain expressions are convex:
  * \(-f(x)\) for any concave \(f\)
  * \(\alpha f(x)\) for any convex \(f\) and \(\alpha > 0\)
  * \(f(x) + g(x)\) for any convex \(f\) and \(g\)
  * \(f(Ax + b)\) for any convex \(f\)
  * \(f(g(x))\) for any convex nondecreasing \(f\) and convex \(g\)
  * \(f(g(x))\) for any convex nonincreasing \(f\) and concave \(g\)
- Use these recursively to show that
  nodes in the expression tree are convex

**Apply properties of monotonicity & bounds**
- Work recursively as for convexity
- Determine all properties in one recursive pass
**Problem analysis**

**Testing Convexity Analyzers**

**Principles**
- Disprovers can establish nonconvexity, suggest convexity
- Provers can establish convexity, suggest nonconvexity

**Test problems**
- Established test sets:
  - COPS (17), CUTE (734), Hock & Schittkowski (119),
  - Netlib (40), Schittkowski (195), Vanderbei (29 groups)
- Submissions to NEOS Server

**Design of experiments**
- Run a prover and a disprover on each test problem
- Check results for consistency
- Collect and characterize problems found to be convex
- Inspect functions not proved or disproved convex, to suggest possible enhancements to analyzers

---

**Solver choice**

**Problem / Solver Data**

**Relational database**
- Table of identifiable problem characteristics
- Table of solvers and general information about them
- Table of all valid problem-solver pairs

**Database queries**
- Most specialized solvers
- Moderately specialized solvers:
  - "hard" criteria such as convexity not used
- General-purpose solvers

**Room for enhancement**
- Add data from NEOS Server runs
- Automatically apply "best" solver (or solvers)
Web Services

**NEOS currently uses**
- E-mail
- Web forms
- TCP/IP sockets (for submission tool)
- CORBA (for Kestrel calls from modeling environments)
- *Ad hoc central server design*

**NEOS of the future could use**
- More general, flexible design
  * SOAP
  * UDDI
  * WSDL
- Less centralized architecture

---

Web services

**New Web Service Standards**

**SOAP**
- Simple Object Access Protocol
- Access to remote objects’ methods and data
- Via standard web servers & HTTP protocols
- Using XML description of the interface

... more general & flexible than CORBA, DCOM
... less client-server coordination required
Web services

New Web Service Standards (cont’d)

UDDI

- Universal Description, Discovery, and Integration
- Specification for online web services registry
- Providers list their services
- Users search listings in a standard way

... decentralized listing of solvers?

WSDL

- Define XML tags used in accessing a remote service
- Make available through UDDI listings

... incorporate optimization XML standard
... add XML tags for solver-specific directives

Web services

Issues for NEOS Redesign

Special nature of optimization services

- Potentially huge resource requirements
- Hard-to-predict relations between size and requirements
- Dependence on mathematical form of problem

Client vs. server, local vs. remote

- Scheduling requests
- Analysis of problems
- Databases of problem and solver characteristics
- Choice of solver

... prefer to decentralize as much as possible
Web services

Issues for NEOS Scheduler

Many jobs, one processor
- Allow 1 large + several small jobs to share a processor?
- How does the scheduler tell what’s “large”?

One job, many processors
- Generalize metaNEOS experiments?

Many jobs, many processors
- Use CONDOR to run on workstations when they are idle

Benchmarking

Fundamentals
- Collection of test problems
- Collection of solvers & solver parameter settings

Performance measures
- Number of iterations
- Number of function evaluations
- Number of conjugate gradient iterations
- Computing time

Issues
- Running the solvers
- Verifying the results
- Comparing performance
**Running Solvers**

**NEOS benchmarking tools**
- User submits one problem to NEOS benchmark "solver"
- User selects solvers to be compared
- NEOS tries all solvers, using the same computer
- NEOS verifies reported solutions
- NEOS returns listing of results

**Other current benchmarking resources**
- Hans Mittelmann’s benchmark pages, [plato.la.asu.edu/bench.html](http://plato.la.asu.edu/bench.html)

...access to numerous solvers is essential

**NEOS Tools**

**Benchmarking web page (instructions)**
Benchmarking

NEOS Tools (cont’d)

Benchmarking web page (solver choice)

Verifying Results

Comparable running environments

- Same computer and operating system
- User’s choice of solver parameters
- User’s choice of tolerances for feasibility, optimality, complementarity

Independent assessment of solutions

- Based only on solution returned

E.D. Dolan, J.J. Moré and T.S. Munson,
“Optimality Measures for Performance Profiles”
(available from www.optimization-online.org)
Benchmarking

NEOS Tools (cont’d)

Benchmark verification results

<table>
<thead>
<tr>
<th>Solver</th>
<th>feasi. error</th>
<th>comp. error</th>
<th>opti. error</th>
<th>scaled opti. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>lbfgsb.</td>
<td>0.000000e+00</td>
<td>0.000000e+00</td>
<td>1.923416e-07</td>
<td>3.827304e-06</td>
</tr>
<tr>
<td>loqo.</td>
<td>0.000000e+00</td>
<td>7.554012e-05</td>
<td>6.588373e-06</td>
<td>1.311233e-04</td>
</tr>
</tbody>
</table>

Solver solution optimality and complementarity found acceptable.

Solver solution not acceptable by this analysis because the scaled optimality error is greater than your limit of 1.0e-05 and the complementarity error is greater than your limit of 1.0e-05.

Benchmarking

Comparing Performance

Average or cumulative totals of metric

- Sensitive to results on a small number of problems

Medians or quartiles of a metric

- Information between quartiles is lost

Number of k-th place entries

- No information on the size of improvement

Number of wins by a fixed amount or percentage

- Dependent on the subjective choice of a parameter
Performance Profiles

Quantities to compute

- For each solver $s$ on each test problem $p$:
  - ratio $r_{ps}$ of that solver’s metric to best solver’s metric
- For each solver $s$:
  - fraction $\rho_s(\tau)$ of test problems that have $\log_2 r_{ps} \leq \tau$

Values to display

- Plot $\rho_s(\tau)$ vs. $\tau$ for each solver $s$
- $\rho_s : \mathbb{R} \to [0,1]$ is a non-decreasing, piecewise constant function
- $\rho_s(0)$ is the fraction of problems on which solver $s$ was best
- $\rho_s(\infty)$ is the fraction of problems on which solver $s$ did not fail
- Emphasis goes from performance to reliability as you go from left to right in the plot

E.D. Dolan and J.J. Moré,
"Benchmarking Optimization Software with Performance Profiles."

Benchmarking

Performance Profiles (cont’d)

COPS optimal control problems
Performance Profiles (cont’d)

Mittelman test problems

Advantages
- Not sensitive to the data on a small number of problems
- Not sensitive to small changes in the data
- Information on the size of improvement is provided
- Does not depend on the subjective choice of a parameter
- Can be used to compare more than two solvers

Further research interests
- Multi-problem NEOS benchmarking tool
- Automated benchmark runs
- Automated generation of result tables & performance profiles
For more information . . .

List of published papers on NEOS
   ➢ www-neos.mcs.anl.gov/neos/faq.html

Recent writings on NEOS
   ➢ Optimization on the NEOS Server,
   ➢ The NEOS Server: Version 4 and Beyond,
   ➢ Kestrel: An Interface
      from Modeling Systems to the NEOS Server,
      www-neos.mcs.anl.gov/neos/ftp/kestrel2.pdf

Server websites
   ➢ NEOS, www-neos.mcs.anl.gov
   ➢ OSP, www.osp-craft.com
   ➢ WEOPT, www.webopt.org — see TB33, this session