Stochastic trees and the *StoTree* modeling software

*StoTree* is software tool for the formulation and solution of continuous-time Markov models of medical interventions. Markov chain models were introduced to the medical literature by Beck and Pauker [1] and provide a convenient means to account for medical treatment options and risks that occur not only in the present but also in the near and distant future. For a more recent introduction to Markov models in medicine, see [2] or [3]. *StoTree* represents Markov models using the graphic technique of *stochastic trees*, introduced by Hazen [4-6].

*StoTree* is implemented in the Excel spreadsheet environment. It is a Microsoft Excel addin for Office 2007, 2010 and 2013 that occupies its own tab on the Office ribbon. It is written in VBA for Office and therefore requires an Office installation on a PC – it will not function on a Macintosh computer. Although I have not checked, I do not believe it will function in Office Online or Office 365.

This document contains three major sections. The first section *Stochastic trees* gives an introduction to stochastic trees. This is the place to start if you are a beginner. Those familiar with stochastic trees may skip to the second section *Using StoTree to construct a stochastic tree*. The third major section *Factored stochastic trees* discusses factoring, a powerful tool that is one of the unique features of *StoTree*. In its final subsection *The benefits of factoring*, we describe the advantages of using factoring in model construction.

**Installing StoTree**

You can download a copy of *StoTree* from this website. It should appear as the file StoTree??X.xlam on your computer, where ? is a single letter indicating the version of StoTree7 (currently version f). Store it in any directory you like, but note where you have stored it. To activate it in Excel, follow the path File\Options\Addins, then click on Go… and Browse… to find the location at which you stored *StoTree*. (Alternately, you may follow the path Developer\Addins if you have made the Developer tab visible on the Excel ribbon.) Click on the appropriate buttons to complete the installation.

The default path for Office 2013 addins is c:\Users\[your identifier]\Appdata\Roaming\Microsoft\AddIns. Although it is not required, you may wish to store StoTree??X.xlam at this location, as it is the first place Excel looks for addins.

StoTree??X.xlam is an **unlicensed** copy of *StoTree*. An unlicensed copy allows you to formulate Markov models, but not solve them. You can obtain a license at my vendor [FastSpring](https://www.fastspring.com).

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**Stochastic trees**

For those familiar with stochastic or decision-analytic modeling, a stochastic tree can be characterized in several equivalent ways:

- as a continuous-time Markov chain transition diagram with chance and decision nodes added,
- as a decision tree with stochastic transitions added,
- as a continuous-time version of a Markov cycle tree.

A simple stochastic tree is shown in Figure 1, which depicts the risk of pulmonary embolism over time. It consists of nodes (*No Embolism, Embolism, Dead*) and arrows connecting the nodes. Nodes depict health states. The arrows are of two types, either *wavy* or *straight*. A node or health state can be either *persistant* (occupied continuously for some positive duration) or *instantaneous* (occupied for vanishly small duration), depending on the type of arrows emanating from it. Wavy arrows emanate from a persistant state (such as *No Embolism*) and indicate that the persistant state is occupied for a duration that is uncertain but dependent on the *rate* at which exit from that state occurs. In Figure 1, the exit rate from *No Embolism* is \( r_{PE} \), and it labels the wavy arrow emanating from that health state. The units of the rate \( r_{PE} \) is events per year (or per month or day or whatever the unit of time is). In *StoTree*, persistant states are known as *stochastic nodes*.

Straight arrows emanate from an instantaneous state (such as *Embolism*), and point to the possible new states immediately following. Each straight arrow is labeled with a *probability* representing the chance that exit from that state occurs along that arrow. Exit from that state occur immediately – the state is occupied for zero duration. (This is obviously an idealization – any health state must be occupied for a positive amount of time. However, zero duration can be a convenient modeling assumption for short-duration states.) Probabilities exiting instantaneous states must sum to 1. In *StoTree*, instantaneous states are known as *chance nodes*.

In Figure 1, exit from the instantaneous state *Embolism* leads to *Dead* with probability \( p_{DEmb} \), and back to *No Embolism* with probability \( 1 - p_{DEmb} \). The dashed border around the second *No Embolism* node means that it is a *copy* of the first node and that transition occurs back to the first node. Copy nodes are a stochastic tree convention meant to facilitate tree-like display of a Markov model. They are optional – one could equally well draw a (straight) arrow in this case from *Embolism* back to *No Embolism*.

Overall, this stochastic tree depicts a patient subject to repeated pulmonary embolisms with a chance of death after each embolism.

Stochastic trees are formulated in *continuous time*. Alternate graphical depictions for Markov models arise and are commonly used in the medical literature. In these, time is *discretized* into steps 0, \( \Delta s \), \( 2\Delta s \), \( 3\Delta s \), etc.

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1. All stochastic tree diagrams in this document are screen captures from *StoTree*. 

3Δs, … of length Δs. Here Δs is termed the cycle length. Rates r are converted into probabilities p via the formula \( p = 1 - \exp(-r \Delta s) \). (For more on this conversion, see the next section.)

For instance, Figure 2 portrays the Markov model of Figure 1 converted to discrete time. The fact that state No Embolism can persist over time is indicated by the self-loop with label \( 1 - p_{PE} \). The fact that the state Embolism is instantaneous is indicated by the lack of any such self-loop. In particular, this state must be departed for either No Embolism or Dead after only one cycle.

Figure 2. A discrete-time portrayal of the Markov model of Figure 1. The rate \( r_{PE} \) of Figure 1 has been converted into a transition probability \( p_{PE} = 1 - \exp(-r_{PE} \Delta s) \), where \( \Delta t \) is the cycle length. The loop with label \( 1 - p_{PE} \) on the No Embolism state means that the state can be occupied over multiple successive cycles. The lack of any such loop at the Embolism state means that once entered it must be departed immediately, that is, after only one cycle.

We provide Figure 2 in case the reader may be more familiar with graphical conventions traditional in the medical literature. However, the remainder of this documentation uses the stochastic tree conventions of Figure 1.

A second stochastic tree is depicted in Figure 3. Here the tree begins with a square node 0 depicting a decision that must be made initially. Choice must be made between Liver-Directed Therapy, Living Donor Transplant, and Resection (i.e. surgical removal of a tumor). Straight arrows emanate from node 0 with probabilities 1,0,0 summing to one like with chance nodes – but here each probability must be 1 or 0 to correspond to a decision chosen (or declined) with certainty. In Figure 3, the decision chosen is Liver-Directed Therapy. Decision nodes are also instantaneous, are therefore occupied for zero duration, and are exited immediately.
Figure 3. A stochastic tree with initial decision node.

Figure 3 includes non-death nodes without any emanating arrows – Cured, Not Cured, and Survive Rs. These are by convention stochastic nodes (persistent states), with the distinction that the state is occupied permanently and never departed.

Figure 4 shows a third stochastic tree, modeling HIV progression. Here all nodes are stochastic, and two stochastic transitions compete at both node A and node B.

Figure 4. A stochastic tree with competing stochastic transitions.

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Rates versus probabilities

The distinction between a rate and a probability may be made as follows. When an event (such as pulmonary embolism in Figure 1) can occur repeatedly over time, a rate satisfies

\[
\text{rate} = \frac{\text{event count per person per unit time}}{n \Delta t}.
\]

So if we observe \(k\) events among multiple subjects in a total of \(n \Delta t\) person-years, then the event rate \(r\) is estimated by

\[
r = \frac{k}{n \Delta t}.
\]

Rates have units of \(\text{time}^{-1}\). For example, we might observe \(k = 250\) events in \(n \Delta t = 200\) person-years. This would yield an estimated rate of \(250/200 = 1.25\) events per person per year. As you can see, a rate must be nonnegative, but can exceed one.

Rates are sometimes referred to as hazard rates, and under this name assume a key role in survival analysis and in standard statistical models, such as the proportional hazards model [7]. Stochastic trees have the conceptual advantage of speaking directly in the language of rates.

On the other hand, when time is not an issue, and we are only interested in whether or not a given event occurs (e.g., death given pulmonary embolism in Figure 1, or cure given liver-directed therapy in Figure 3), then the probability of that event satisfies

\[
\text{probability} = \frac{\text{event count per person}}{n}.
\]

So if we observe \(k\) events among \(n\) subjects, then the event probability \(p\) is estimated by

\[
p = \frac{k}{n}.
\]
The notions of rate and probability are related whenever we are counting events that occur over time but we are interested in a particular duration \( \Delta s \). If events occur at rate \( r \), and \( p \) is the probability of at least one event in duration \( \Delta s \), then

\[
p = 1 - \exp(-r\Delta s).
\] (1)

For small \( \Delta s \), probability is approximately rate times duration:

\[
p \approx r\Delta s.
\]

For example, if the estimated rate is 1.25 events per person-year, as above, and we are interested in a one-month duration, then \( \Delta s = 1/12 \text{ yr} \), and the probability of at least one event in a one-month duration is

\[
p = 1 - \exp(-(1.25)(1/12)) = 0.0989.
\]

Note that the approximation \( p \approx r\Delta s \) gives \( p = (1.25)(1/12) = 0.1042 \), reasonably close since \( \Delta s \) is small, but not exactly correct.

Most software for Markov modeling is inherently discrete-time, and allows only probability parameters as input. Users are then required to convert rates to probabilities via (1). **StoTree** models continuous-time Markov chains, and allows both rate and probability inputs. **StoTree** converts rates to probabilities automatically as needed for calculations using (1), relieving the user of this task.

The relation (1) assumes that the event rate \( r \) is constant over time. Some rates – for example, mortality rates – are time-dependent. In this case the relation between rates and particular-duration probabilities is more complex. See, for example [3, 7].

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**Cohort analysis: Quality, cost and discounting**

The traditional method of in the medical literature of solving Markov models is to perform a cohort analysis. This method assumes that a cohort of identical individuals begins in some initial health state \( x_0 \) at time \( t = 0 \), and progresses forward in time through the health states in the model, according to the assumed rates and probabilities in the model. The analyst selects a small time increment \( dt \), called the cycle length. The rates and probabilities in the model determine the transition probabilities

\[
\lambda_{xy}(dt,t) = \text{probability the state at time } t+dt \text{ is } y \text{ given that the state at time } t \text{ is } x.
\]

The analyst then sets the cohort size \( n_{x_0}(0) \) in state \( x_0 \) at time \( t = 0 \), and calculates the values of

\[
n_y(t) = \text{expected count of individuals in state } y \text{ at time } t
\]

for all time instants \( t = 0, dt, 2dt, 3dt, \ldots \) using the Kolmogorov forward equations

\[
n_y(t+dt) = n_y(t) + \sum_{x} n_x(t)\lambda_{xy}(dt,t) - n_y(t)\sum_{z} \lambda_{yz}(dt,t).
\]

The purpose of computing the expected counts \( n_y(t) \) is to use them to calculate expected quality adjusted life years (QALYs) and expected costs. The analyst uses the inputs

\[
Q_y = \text{expected quality accrued per unit time in state } y \text{ (the so-called utility of state } y)
\]

\[
C_y = \text{expected cost accrued per unit time in state } y
\]
to compute expected QALYs by forming the sum of products $\sum_y Q_n \eta_y (t)$, possibly weighting the result by a time-discount factor, and then summing the all resulting quantities over all time instants $t = 0, dt, 2dt, 3dt, \ldots$. The analogous operations are performed to calculate expected costs. It is also possible to include quality tolls $Q_{xy}$ and cost tolls $C_{xy}$ that are incurred whenever the cohort or a portion of it moves from state $x$ to state $y$.

Note that if the utilities $Q_x$ are all equal to 1 and there are no quality tolls, then expected $QALY$s is simply expected lifetime.
Using *StoTree* to construct a stochastic tree

Overview of *StoTree* capabilities

*StoTree* is a Microsoft Excel add-in possessing its own tab on Excel’s ribbon (see Figure 5). You can use *StoTree* to formulate a continuous-time Markov model of health interventions of interest. *StoTree* allows you to graphically characterize the consequences of interventions in one or more compact diagrams. You can next specify rates for events that occur over time, and probabilities for one-time events. You can also input quality and cost parameters. You can then instruct *StoTree* to perform a cohort analysis in order to compute comparative effectiveness and/or cost-effectiveness. Finally, if you are concerned about the impact of particular parameter inputs, *StoTree* can perform a sensitivity analysis on such parameters.

![Figure 5. The StoTree tab on the Microsoft Excel ribbon.](image)

*StoTree* does not arrange the layout of a tree for you. It places nodes you insert at the cell you have selected, and does not move or rearrange nodes – that is your responsibility.

Constructing a basic stochastic tree – in short

Once you install *StoTree* as an add-in, you can use the commands on the *StoTree* tab to construct, populate and solve a stochastic tree. In overview, the steps are:

1. Insert a new factor.
2. Add one or more nodes.
3. Connect the nodes with arrows as desired.
4. Populate arrows with rate or probability parameters.
5. Instruct *StoTree* to setup a cohort analysis worksheet, and have *StoTree* calculate that sheet to compute expected QALYs and costs.

We discuss these steps in detail in the following sections.

Constructing a stochastic tree – philosophy

*StoTree* uses a node-focused philosophy in constructing stochastic trees. By this we mean that most of the important graphical construction techniques you will use begin with creating, selecting or deleting a node. You should keep the following points in mind.

- Node selection is sticky in *StoTree* – if you click on two or more nodes consecutively, they will all be selected (in contrast to Excel’s normal behavior, where clicking on a second graphic object will deselect the first one).
- If you want to change the layout of a tree you have constructed, simply drag one or more nodes to new locations, and then click on Refresh on the *StoTree* toolbar – the arrows will follow. *StoTree* does not arrange the layout of a tree for you. It places nodes you insert at the cell you have selected, and does not move or rearrange nodes – that is your responsibility.
- If you want to delete a node, simply select it and press Delete on your keyboard.
To construct a basic stochastic tree using **StoTree**, you should open an empty Excel workbook, and proceed more or less from left to right on the **StoTree** tab. You should execute steps in roughly the following order:

1. **Insert a new factor.**
   
   Click on **Insert Factor** to insert a blank worksheet into your workbook, and set up your workbook to host stochastic tree construction. (You will see a dialog asking whether you wish to insert a stochastic tree into this workbook, and you should click **Yes**.) The resulting worksheet will have cells but no cell borders, and there will be an Active/Inactive toggle button in its upper left corner.

2. **Add one or more nodes.**
   
   a. Select a cell in the newly created worksheet where you would like **StoTree** to place a new node, and click on **Add Node**. A userform will appear that allows you to name the node and choose its type. **StoTree** will insert the name you provide as a caption in the node it draws. Click on **Insert** and a node will appear at your selected cell.
   
   b. Resize the node if it is too large or too small by clicking on it and dragging the shape handles that appear.
   
   c. Add other nodes to other locations in the worksheet by selecting other cells and clicking on **Add Node** again.
   
   d. You can move a node around by dragging it with your mouse. If needed, you can type in a new caption as well.

3. **Connect the nodes with arrows as desired.**
   
   a. To draw an arrow from one node to another:
      
      i. Click first on an empty cell to deselect any node you may have currently selected.
      
      ii. Click on the node where you want your new arrow to start.
      
      iii. Click on the node where you want your new arrow to end. (Note: You do not have to press the Ctrl key – node selection in **StoTree** is **sticky**, and both nodes will remain selected when you click on the second one.)
      
      iv. Click on one of the arrow button on the **StoTree** tab.
      
      **StoTree** will draw an arrow of the type you chose from the first node to the second.
   
   b. You can drag to move nodes around your worksheet, but doing this will leave the arrows behind. *Do not attempt to drag the arrows.* Instead, click on the **Refresh** command on the **StoTree** tab, and arrows will re-attach to all nodes you have selected (or will reattach to all nodes if none are selected when you click **Refresh**).
4. Revise your construction as needed.
   a. You may copy/paste entire subtrees as follows. (However, using factoring is potentially more simple and elegant – see the section below.)
      i. Select the root node of any subtree you wish to copy.
      ii. Click the Copy Subtree button on the StoTree tab. StoTree will copy the subtree to a new blank worksheet.
      iii. Click on a cell in your original worksheet where you would like the copied subtree to be pasted.
      iv. Click on the Paste Subtree node. StoTree will paste the copied subtree into your worksheet with its root node at the cell you selected.

      (Please note that you cannot use Excel’s Copy and Paste commands to duplicate existing nodes or arrows – or rather you can, but StoTree will not recognize the duplicates as part of your stochastic tree.)
   b. If needed, you can reroute arrows you have mistakenly drawn. Select any part of the arrow, and click on Reroute on the StoTree tab.

5. Populate arrows with rate or probability parameters.
   You will notice that the arrows that StoTree draws are initially labeled “Needs populating”. Click on any such arrow and then click the Parameters button on the StoTree tab. StoTree will open a user form where you may enter a desired rate (for a wavy arrow) or probability (for a straight arrow). You can ignore the Quality Toll and Cost Toll entries for now. You can enter a numeric value, or any numeric expression that will successfully evaluate in your Excel spreadsheet. You can include spreadsheet cell references in your expression if you provide defined names for the relevant cells (see below).

6. Instruct StoTree to setup a cohort analysis worksheet.
   a. Click on Setup Cohort Sheet. StoTree will open a new worksheet called the cohort sheet, and will insert formulas that will perform a cohort analysis when the sheet is repeatedly calculated.

   b. You can manually perform a cohort analysis by clicking on the button Step ahead one cycle. Or you can simply click on the Calculate Sheet button on the cohort sheet or the Calc Cohort button on the StoTree tab to complete the cohort analysis in one step.

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Calculation settings
StoTree provides default calculation settings for the cohort sheet it creates. However, you may modify these settings. Click on the Calc Settings button on the StoTree tab to enter your desired settings. The settings you can choose are

- **Initial Cohort.** The number of individuals in the cohort. StoTree calculates fractional values of the expected counts \( n_i(t) \) as needed, so it is adequate to set the initial cohort size to 1.
- **Initial Time.** The clock time \( t \) at which the cohort begins. Usually this would be \( t = 0 \).
• **Cycle Time.** The increment $dt$ by which time is incremented in each cycle of the cohort analysis. Ideally this should be a small value, but small $dt$ increases calculation time. Larger values of $dt$ will calculate quickly but results will be less accurate. Use large values of $dt$ when debugging your model, but switch to small values after debugging is complete and you want accurate results.

• **Time Horizon.** The clock time at which you wish cohort analysis to stop.

• **Discount Rate.** If you choose discount rate to have value $r$, then QALYs and costs incurred at time $t$ will be weighted by the discount factor $e^{-rt}$. Set the discount rate to zero if you want no discounting.

• **Calculate Expected QALYs (Yes/No).**

• **Calculate Expected Costs (Yes/No).**

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**The cohort sheet**

We illustrate here the cohort sheet output that *StoTree* produces for the HIV-AIDS model of Figure 4. The user has supplied values for the parameters $r_{AB}$, $r_{BC}$, $r_D$, $r_{D_AIDS}$ (these are defined names in Excel – see below), and in addition has entered cost rates in each state. When the user clicks on the Setup cohort sheet button on the *StoTree* tab, *StoTree* produces the cohort sheet in Figure 6. In that figure, *StoTree* has performed one cycle of cohort analysis: The cycle time is 0.1, and in the Time column, only the time-instants $t = 0$, $t = 0.1$ appear. The initial cohort size is 1000 in state A, and during this cycle, the state counts have moved from $(1000,0,0,0)$ to $(948.749, 34.3946, 0, 16.8563)$ in states A,B,C and Terminal Node (death). Accrued QALYs for the cohort are 99.157 yr., and accrued costs are $225,880. The user can click the plus (+) signs on the left to inspect details in the calculations for counts, QALYs and costs that may be useful for debugging.

If the user presses the Calculate Sheet button, or the Calc Cohort button on the *StoTree* tab, *StoTree* will perform all additional cycles of cohort analysis, using a cycle length of 0.1 yr., until the time horizon $t = 20$ is reached. The results are presented in Figure 7. Discounted cumulative QALYs for this cohort of 1000 is 3588.62, and discounted cumulative cost is $10^7$. 
Figure 6. The cohort sheet StoTree produces for the stochastic tree model of Figure 4 when the user clicks the Setup cohort sheet button on the StoTree tab.
As suggested by the other two buttons on the cohort worksheet, the user also has the option of resetting calculations to the start time, which will produce once again the worksheet displayed in Figure 6; and then stepping the calculations ahead one cycle at a time. This procedure could be useful for debugging.

Once you have instructed *StoTree* to create a cohort sheet of this type, you should save your workbook as a macro-enabled workbook, that is, with a “.xlsm” suffix. The workbook now contains Visual Basic macros in a hidden code sheet, and must be saved with this suffix in order for the macros to function.

Although you need a licensed copy of *StoTree* to create a cohort sheet, the resulting workbook may be viewed and manipulated by anyone who has even an unlicensed version of *StoTree* installed. You may therefore share your workbook (see below) with colleagues possessing only unlicensed copies of the *StoTree* addin. Your colleagues may wish to perform what-if analyses on your model, and may do so without acquiring a license. See the section Defined names for documenting and sensitivity below for more information on how to do what-if analyses.

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**Tips and shortcuts for constructing a basic tree**

There are a number of shortcuts you can use to construct basic trees more quickly and elegantly.

1. You can instruct *StoTree* to draw multiple arrows in one click by first selecting node you want to serve as origin (call it node A), and next selecting all destination nodes (call them nodes B,C and D). Then a single click of an arrow button on the *StoTree* tab will draw arrows from A to B, A to C and A to D.
2. As already mentioned, node selection in StoTree is sticky – you can click several nodes in succession to select them all, without using the Ctrl key in Excel. In fact, if you wish to take advantage of the shortcut in point 1, you must select nodes without using the Ctrl key – otherwise StoTree does not recognize the multiple selection. You should also be careful, if you want to select only one node, that other nodes are not already selected. To achieve this, simply click on any cell in the worksheet and all nodes will be de-selected.

3. If you determine that an arrow or a node is no longer needed, you can simply click on the node or any part of the arrow and press the Delete key on your keyboard. StoTree will notice the deletion the next time you click one of the buttons on its tab. At this point, if you have deleted a node, then StoTree will also delete any arrows connected to that node. If you delete just part of an arrow, then StoTree will delete the entire arrow when it notices the partial deletion.

4. If you mistakenly draw an arrow of the wrong type or orientation, you can correct your mistake without deleting the arrow and redrawing it. Instead select the arrow and click the desired arrow button on the StoTree tab. StoTree will replace the mistaken arrow with the corrected arrow.

5. You can move labels on arrows from one side of the arrow to the other, or slide them along the arrow. Clicking the Refresh button will leave the label on the side of the arrow on which StoTree finds it.

6. You may wish to move or delete a group of nodes and arrows together. You can do so by activating Excel’s Select Objects pointer (Home/Editing/Find & Select). You can use this pointer to select multiple graphics at once by dragging it around the objects. Once a group of nodes and arrows are selected, you can drag them to any desired location or delete them. If you wish to access Excel’s Select Objects pointer more quickly, you can add it to the Quick Access Toolbar (File/Options/Quick Access Toolbar).

7. Do not attempt to use Excel’s Copy/Paste tools to create new nodes or arrows. This will only result in new worksheet graphics that StoTree will not recognize as being part of your model. The reason is that StoTree maintains hidden databases of nodes and arcs, and these databases will not be updated by Excel’s Copy/Paste tools. You must use the StoTree tab on the Office ribbon to create new nodes or arrows. For similar reasons, do not attempt to create a new factor (see below for a discussion of factors) by having Excel copy a worksheet.

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QALYs and costs
You can enter quality accrual rates (quality coefficients or so-called utilities) and cost accrual rates for any stochastic node in your tree. (Note chance and decision nodes are instantaneous, so quality and cost accrual does not apply to them.) Simply select the node and then click on the Parameters button on the StoTree tab. A userform appears into which you may enter these values for the selected node. You may enter quality tolls and cost tolls for any arrow in your model – again simply select part of the arrow and then click on the Parameters button.

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Sharing your constructed workbook
Once you have used the StoTree addin to construct a Markov model in your workbook, and have saved your workbook as a macro-enabled workbook (.xlsm), you may wish to share your workbook with colleagues. Because it is an Excel workbook, anyone with Microsoft Office can view it. However the
graphic objects and cohort-sheet formulas in your workbook contain references to macros in your copy of the StoTree addin. When a user on another computer recalculates the cohort sheet, or clicks on a graphic, error messages will result – because your workbook is on her computer, the macro references there will not be able to find your copy of the StoTree addin.

However, if the user has a licensed or unlicensed copy of the StoTree addin installed, she can convert her copy of your workbook to access her copy of StoTree. She does this by opening her copy of your workbook and clicking on the Convert button in the About section of the StoTree tab. A successful conversion will result in a confirmation message similar to the one illustrated in Figure 8.

After conversion, a user with a licensed copy of StoTree has complete access to the structural details of your workbook, and may add to or modify its contents in whatever way she desires, for instance by adding or deleting nodes or arcs, setting up a revised cohort sheet, and performing sensitivity analyses. A user with an unlicensed version of StoTree can also convert in this way, but does not have access to the calculation features on the StoTree menu system. An unlicensed user does, however, have access to the calculation buttons on your workbook’s cohort sheet, and may use these to perform what-if analysis on your model – again, see the section Defined names for documenting and sensitivity below for more information on how to do what-if analyses.

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Defined names for documenting and sensitivity
You can enter any numeric value or expression (e.g., 22.3, 1/10, 12+3) into the Parameters userform. However, to properly document your model and make it more transparent and flexible, you are strongly advised to avoid numeric expressions, and instead use Excel’s defined names feature (on the Formulas tab) to provide names for cells containing your input data. Then use these names, or expressions containing these names, in StoTree’s Parameter userform when you populate parameters for your model. StoTree displays rate and probability expressions next to the appropriate arrows, so using defined names

Figure 8. To convert another user’s workbook to a form compatible with your copy of Microsoft Office, you should clicking on the Convert button in the About section of the StoTree tab. This figure illustrates the confirmation message displayed when conversion is complete.
makes it easy to see to what arrows these inputs apply. See for instance, Figure 9. This is a method you can use to document your model.

A second advantage of using Excel’s defined names in the Parameter userform is that you may change the values in the named cells, and then Calculate the cohort sheet to obtain new expected QALYs and costs, without having to request StoTree to setup the cohort sheet again. As mentioned above, someone inspecting your workbook can do this even if his or her computer possesses only an unlicensed copy of the StoTree addin (e.g., perhaps you have sent others a copy of the workbook you have constructed). You or others can thereby quickly perform a manual sensitivity analysis by repeatedly changing values in named cells and clicking on Calculate Sheet. Note, however, that StoTree has automated this process via the One-Way Sensitivity button on the StoTree tab.

**Figure 9. The use of Excel’s defined names in StoTree. The user has given cell C5 the name rPE, and cell C6 the name pDEmb.**

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**One-way sensitivity analysis**

By clicking on the One-Way SA button on the StoTree tab, you can obtain instructions in how to set up a one-way sensitivity analysis on any input parameter that you have stored in a cell of your workbook. You proceed by setting up one of Excel’s column data tables, then selecting its corner cell, and finally pressing Ctrl+Shift+s. Details are provided by clicking on the One-Way SA button on the Excel ribbon’s StoTree tab.

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**Probabilistic sensitivity analysis**

Probabilistic sensitivity analysis is a type of sensitivity analysis in which the uncertainty in parameters is represented by probability distributions. The primary advantage of probabilistic sensitivity analysis is that it allows analysts to perform a sensitivity analysis on all parameters jointly, instead of only one or two at a time.

If you have set up your workbook for probabilistic sensitivity analysis, then you can obtain sensitivity estimates via Monte Carlo simulation, by having StoTree repeatedly recalculate your workbook, once for each Monte Carlo iteration. These recalculations are done in a structured way that is not equivalent to using the F9 key on your keyboard.

For information on how to set up your workbook for probabilistic sensitivity analysis, click on the Probabilistic SA button on the Excel ribbon’s StoTree tab.
Time-dependent parameters
As mentioned above, you may enter any valid Excel formula or expression containing defined names into StoTree’s Parameter userform. There is an additional defined name _t that StoTree automatically creates when you instruct it to setup a cohort sheet. The name _t represents the elapsed time in the cohort sheet, and you may use this name in any of the formulas or expressions you enter into the Parameter userform. You would most commonly use _t in a rate formula when you desire the rate to depend on elapsed time in your cohort analysis. Figure 10 shows an example. However, _t may be used in any parameter expression, for example for quality or cost coefficients. Figure 11 illustrates how you might want to use _t in a quality coefficient formula.

Figure 10. You can use the defined name _t to represent elapsed time in parameter expressions. Here, in a model of the outcome of hip replacement, the rate out of Success Primary is the Weibull hazard rate \( h(t) = \lambda \gamma t^{\gamma-1} \).

\[
\begin{align*}
\text{Lbda} &= 0.02 \\
\text{Gmma} &= 1.45 \\
Lbda^*Gmma^* t^{(Gmma-1)}
\end{align*}
\]

Figure 11. You can use the elapsed-time defined name _t in a quality coefficient. Here the node No Embolism has zero quality after 5 years, and quality 1 before that. The resulting cohort analysis (with initial cohort set to 1.0) will calculate the 5-year survival probability.

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Human mortality rates
StoTree includes a function GmpHR that returns time-dependent human mortality rates for any age, gender and race. GmpHR stands for “Gompertz hazard rate” and is based on a Gompertz approximation to human mortality rates. You can employ this function in conjunction with the current time variable _t.
to supply the correct human time-dependent mortality at any clock time in a cohort analysis – see for example Figure 12. The Gompertz approximation to human mortality is valid for ages 30 or greater. See the appendix *Gompertz hazard rate for human survival* for documentation on this function.

Figure 12. Use of StoTree’s Gompertz hazard rate to model human mortality.

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**Constant intervals of clock time**

Occasionally you may wish a state transition to occur repeatedly at time intervals of length \( t_{\text{inc}} \). In essence this means that the associated transition rate is infinite every \( t_{\text{inc}} \) units of clock time and zero for other clock times. The *StoTree* function `Spike_()` produces this effect (the name indicating that the transition rate “spikes” to infinity every \( t_{\text{inc}} \) units of time). In cohort analysis, *StoTree* steps off time using the Excel defined name \( _t \) in units of the Excel defined name \( _dT \), which *StoTree* sets equal to the cycle time you specify in the calculation settings. The function call `Spike_\( (t_{\text{inc}}) \)` returns a rate \( r_{\text{Inf}}/_dT \) (for some large constant \( r_{\text{Inf}} \) – effectively an infinite rate) when \( _t \) is within \( _dT/2 \) of \( k \cdot t_{\text{inc}} \) for any integer \( k \).

Figure 13 illustrates the use of `Spike_()` in a model of testing at multiples of time increment \( t_{\text{inc}} \). Because `Spike()` produces zero or infinite rates, it can only be used on stochastic (wavy) arrows.

Figure 13. The use of the *StoTree* function `Spike_()` in a model of testing at multiples of time increment \( t_{\text{inc}} \). Here the transition rate from Wait to Test is zero unless \( _t \) is close to some multiple of \( t_{\text{inc}} \), in which case the rate is “infinite” (equal to the inverse cycle time \( 1/_dT \)). That is, `Spike_\( (t_{\text{inc}}) \)` produces a transition every \( t_{\text{inc}} \) units of time.

`Spike_()` also has an optional parameter \( t_0 \) that specifies the start time for spikes. The function call `Spike_\( (t_{\text{inc}}, t_0) \)` produces the first transition spike at time \( t_0 \), and successive spikes at \( t_0+k \cdot t_{\text{inc}} \) for every positive integer \( k \). If \( t_0 \) is omitted (as above), then its value is assumed to be zero.

It is possible you may wish transitions to occur at predetermined times \( t_1, t_2, \ldots \) that are not equally spaced. For example, you might want testing to occur more frequently initially than later. The multi-spike
function SpikeM_() accomplishes this. If you enter the times \( t_1, t_2, \ldots \) into a spreadsheet region and assign that region an Excel defined name, then using the region name in a call to SpikeM_() will result in transitions that spike at those times and no others. For example, if the name of the region containing times \( t_1, t_2, \ldots \) is \( t\text{Times} \), then the call should be SpikeM_\((t\text{Times})\).

See the appendix for detailed documentation on Spike_() and SpikeM_().

Clock time versus sojourn time

You may at times wish to have a rate out of some state depend not on the clock time \( t \) but on the so-called sojourn time in a state – the time elapsed since the state was entered. Unfortunately, Markov models as usually formulated do not explicitly include a sojourn-time variable, except in the special case in which sojourn time and clock time coincide, which happens only for states occupied initially at time 0.

The distinction can be seen in Figure 10, where the failure rate of the initially implanted artificial hip depends on its age, which is equal to the sojourn time in the Success Primary state. In that state, age is equal to clock time \( t \) because a new hip is implanted at time 0. Therefore the age-dependent failure rate can be made a Weibull function of \( t \). On the other hand, in the state Success Revision, age is never equal to clock time because the replacement hip is never implanted (if it is at all) at time 0. The modeler would like to make the failure rate of the replacement hip depend on its age, but cannot because there is no sojourn time variable available. As an approximation, the modeler has in this case entered a constant failure rate \( rrr \) for the replacement hip.

This approximation may be adequate in some cases, but if it is not, then the modeler must resort to using what are known as tunnel states, a sequence of states that mimics elapsed sojourn time. StoTree can handle tunnel states, but we do not discuss the method here.

Decision nodes

In a stochastic tree, a decision node is meant to indicate that an individual has a choice among the branches leading from that node. However, in StoTree, decision nodes are treated the same as chance nodes. You the user would assign probability one to the branch corresponding to whatever decision is to be analyzed, and probability zero to other branches. StoTree does not perform any kind of optimization among decision branches – it merely evaluates QALYs and/or costs associated with the desired decision branch. To evaluate a different decision, you would have to change its branch probability to one. This can be automated using Excel’s defined names, as is illustrated in Figure 14.
Figure 14. You can use a defined name (here, Decn) to specify which decision branch is to be selected. Change the value of the Decn cell and press Calculate Sheet to evaluate a different decision. You can also use StoTree’s One-Way Sensitivity command to perform a sensitivity analysis on Decn, and thereby evaluate all decision alternatives in one step.

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**Terminal nodes**

*Death* states are common in medical decision analyses. They can be represented in *StoTree* as stochastic nodes with zero quality and no emanating arrows. However, it is computationally more efficient to use the *StoTree* node type *terminal* to represent death states. When you create a terminal node, it appears shaded in your spreadsheet (see several of the preceding figures). You can change an existing stochastic node to terminal in the *Parameters* userform if the node has no successors.

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**Root nodes**

*StoTree* begins its cohort analysis by assuming all cohort members occupy the *root node* in the stochastic tree. You can set which node in your tree is the root node using a checkbox in the *Parameters* userform for that node. If you neglect to do so, *StoTree* will make an educated guess about which node is the root node in your tree. However, for some types of trees, there may be more than one plausible root node, and you should make sure *StoTree* has selected the one you desire.

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Factored stochastic trees

Often in constructing models of medical prognosis, one encounters processes that evolve in parallel. Consider for example, the pulmonary embolism model depicted in Figure 1. Tsevat et al.[8] consider the use of Warfarin to reduce the incidence of pulmonary embolism. However, Warfarin can increase the incidence of systemic hemorrhage, a process depicted in Figure 15. The pulmonary embolism and systemic hemorrhage processes are proceeding in parallel – both processes evolve simultaneously. It is possible, but cumbersome, to formulate a single Markov model or stochastic tree that depicts both processes. Fortunately, this is not necessary in StoTree. One of the most important features of StoTree is that it allows the modeler to formulate parallel processes as inputs. It also allows the modeler to specify links between the processes, so that what happens in one process can influence what happens in others, and can in turn be influenced by others. Parallel processes are known as factors, and the overall combination of processes is called a factored stochastic tree.

![Diagram of factored stochastic tree]

Figure 15. A model of systemic hemorrhage. This process evolves in parallel with the pulmonary embolism process depicted in Figure 1.

Using StoTree to construct a factored stochastic tree

How do you construct a factored stochastic tree? The procedure in StoTree is quite simple: Click on the Insert Factor button on the StoTree tab for each factor you wish to create. StoTree will insert new worksheets into your workbook in which you can construct new stochastic factors in the same way as described in the sections above. Once you have created multiple worksheets, each containing its own factor, StoTree allows you to link what happens in one factor to what happens in others using dependencies and triggers. We discuss each of these in turn.

Dependencies

In StoTree, a dependency indicates how parameters in one factor depend on the state or states of other factors. For example, in the StoTree model with factors depicted in Figure 1 and Figure 15, both the rate of embolism and the rate of hemorrhage should depend on whether Warfarin is being administered. The decision on whether to do so is shown in Figure 16, which depicts a third factor with an initial decision node Choose selecting between Warfarin and No Warfarin.
Once the user has created the Warfarin factor, s/he can make the hemorrhage rate in the Systemic Hemorrhage factor depend on whether Warfarin is being administered. The user activates the Systemic Hemorrhage factor, clicks on the arrow out of No Hemorrhage, and proceeds as in Figure 17, Figure 18, Figure 19, and Figure 20 below.

Figure 16. The Warfarin factor, depicting the decision of whether to administer Warfarin.

Figure 17. Creating a dependency in the Systemic Hemorrhage factor. The user has selected the arrow from No Hemorrhage and clicked on the Parameters button to bring up the userform pictured here. Next the user has clicked on the dropdown arrow in the Rate combobox to reveal the option (more).

Figure 18. The user has clicked on the (more) entry in the Rate combobox dropdown. As a result, the ===> button next to the combo box has been activated. The user will next click on this button to bring up the Dependency userform.
Figure 19. The Dependency userform. The user has selected Warfarin from the If Factor dropdown, and indicated that the chosen rate will equal \( r_{	ext{HemW}} \) if the Warfarin factor is in state Warfarin, and will equal \( r_{	ext{HemNo}} \) if the Warfarin factor is in state No Warfarin. The user will next click on Back to return to the Parameters userform.

Figure 20. The process is complete. The arrow out of No Hemorrhage is labeled with a partial description of the dependency just entered.

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**Triggers**

Sometimes it is desirable to have a transition in one factor produce a state change in another factor. The user can arrange this by clicking on the Trigger button on the StoTree tab. Consider once more the Tsevat model of Warfarin treatment (Embolism in Figure 1, Hemorrhage in Figure 15, and Warfarin in Figure 16). If a hemorrhage occurs while the patient is on Warfarin, it would seem natural to discontinue Warfarin for some period of time. In other words, the occurrence of a hemorrhage should trigger transition from Warfarin to No Warfarin in the Warfarin factor. To make this a temporary change, the user can add a Temporarily Discontinue node to the Warfarin factor, as shown in Figure 21. The user has also added an arrow from Temporarily Discontinue back to Warfarin, with an associated rate of 12/yr (resume Warfarin in an average of one month). Finally, the user needs to add a trigger that sends the Warfarin factor from Warfarin to Temporarily Discontinue. This is shown in Figure 22.
Note that the dependencies that the user has added to the Pulmonary Embolism and Systemic Hemorrhage factors – see Figure 19 – do not account for the new state \textit{Temporarily Discontinue}. The user must revisit these dependencies and add this new state to them.

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{figure21.png}
\caption{The Warfarin factor with a new node Temporarily Discontinue added.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{figure22.png}
\caption{Inserting a trigger. The user has selected the arrow out of No Hemorrhage and clicked on the Trigger button on the StoTree tab. The user has clicked on Add to add the chosen trigger to the list above. The user will click on OK to complete the process.}
\end{figure}

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\textbf{Priorities among arrows*}

In a multifactor tree, there can be many combinations of health states. Consider a particular health-state combination. There may be multiple ways to exit this combination health state, corresponding to multiple exit arrows across different factors.

Recall that \textit{StoTree} treats decision nodes the same as chance nodes. You the user would assign probability one to the branch corresponding to whatever decision is to be analyzed, and probability zero to other branches. For the purpose of assigning priorities, there are therefore only two types of nodes that can appear in a combination health state – decision/ chance nodes, and stochastic nodes.

The following precedence rules are enforced:

1. If a combination state contains only one decision/ chance node, \textit{StoTree only} executes arrows out of this node, thereby reaching a new combination state. No other arrows are executed.

For instance, consider the combination state (Choose, No Embolism, No Hemorrhage) in the model composed of the Warfarin factor (Figure 16), the Embolism factor (Figure 1), and the
Hemorrhage factor (Figure 15). The combination state contains only one decision/ chance node Choose (a decision node). It takes priority, and the only possible transitions out of this combination state are those in the Warfarin factor, namely the transitions to Warfarin and to No Warfarin. StoTree ignores the transitions from No Embolism in the Embolism factor, and from No Hemorrhage in the Hemorrhage factor. (Note, however, that these transitions will be considered subsequently because they will be present at resulting combination states.)

As another example, consider the combination state (Warfarin, Embolism, No Hemorrhage) in this Warfarin model. Warfarin and No Hemorrhage are stochastic nodes, and there is one decision/ chance node Embolism (a chance node). Therefore StoTree considers only transitions out of Embolism and ignores transitions out of No Hemorrhage (and would ignore transitions out of Warfarin if there were any). Again, these transitions will be subsequently be executed because they will be present at the resulting combination states.

2. If the combination state contains decision/ chance nodes in two or more factors, StoTree executes only the decision/ chance arrows from the alphanumerically first factor. No other arrows are executed. (But again, decision/ chance arrows in other factors would subsequently be executed because they would be present at the new combination states.)

3. If only stochastic nodes are present, they are all executed across all factors, that is transitions compete across factors.

For instance, consider the combination state (Warfarin, No Embolism, No Hemorrhage) in the Warfarin model discussed in point 1. This state consists of three stochastic nodes. StoTree considers all transitions from these three nodes, which in this case consist of a transition to Embolism at rate rPE and a transition to Hemorrhage at rate rSysHem. These two transitions compete, and StoTree accounts for the possibility that either one may occur first in time.

Priorities among triggers*
A user may enter several triggers on an arrow. StoTree executes these in the order entered, which is the order listed in the trigger userform.

Triggers may also be attached to nodes. StoTree executes such a trigger just after the node is entered.

The cohort sheet for a factored tree
It is important to remember that in a factored stochastic tree, the factors depict processes that are proceeding in parallel. So any combination of states, one from each factor, may in principle occur. StoTree accounts for this in constructing the cohort sheet, and lists many such state combinations, as long as they can be reached from the combination root node. Figure 23 illustrates this for the example tree of this section.
You may note that the three factor trees in this model contain respectively 3, 5, and 5 states. Yet there are nowhere near $3 \times 5 \times 5 = 75$ combination states in the cohort sheet of Figure 23. *StoTree* exploits known properties of stochastic processes to bypass the portrayal of terminal (death) nodes, chance nodes, and decision nodes (unless they are root nodes). Only stochastic nodes and root chance/decision nodes need be portrayed on the cohort sheet. This is the primary reason you should use a terminal node rather than a stochastic node to model the death state—a stochastic-node death state will add to the number of combinations that need display, whereas a terminal node will not. The three factors in this model contain respectively 1, 2, and 4 stochastic or root nodes, producing $1 \times 2 \times 4 = 8$ combinations. The cohort sheet contains only 7 combination states because one combination state cannot be reached from the combination root node.

**Active and Inactive factors**
*StoTree* will form the cohort sheet using only combinations of states from *active* factors. You can use the active/inactive toggle button in each factor worksheet to change a factor from active to inactive and vice versa. This toggle button lies in the upper left corner of each factor worksheet (for example, see Figure 9).

You might, for example, wish to deactivate a factor rather than deleting it if you think you might use it later and do not wish to have to reconstruct it.

**Quality and cost for factored trees**
In order to perform cohort analysis for a factored stochastic tree, *StoTree* must assign quality coefficients and cost rates to *combinations* of stochastic nodes. The rule *StoTree* uses to accomplish this is that the cost rate for a combination is the sum of the cost rates in each factor, and the quality coefficient for a combination is the product of the quality coefficients in each factor.

**Tips and tricks for constructing factored stochastic trees**
Use an additional factor instead of duplicate subtrees
As mentioned above, *StoTree* has *Copy Subtree/Paste subtree* features for easily duplicating subtrees in any particular factor. A disadvantage of doing so, however, is that copy/paste can produce large bushy
trees. For example, in Figure 24, the user has duplicated the Test subtree, because periodic screening applies to both the healthy and diseased populations. A more elegant approach is to create a separate factor depicting the outcomes of testing, and use dependencies to link its parameters back to health and disease. This approach is illustrated in Figure 25.

![Stochastic Tree Diagram](image)

**Figure 24. A generic stochastic tree for periodic screening. Note the Test subtree is repeated.**

The advantage is a simpler and more elegant model formulation. However, the factored approach does require the use of dependencies at the Test branches and the Positive branches (see Figure 25). The resulting model details are thereby obscured. Whether to factor in this way is largely a matter of taste – computationally there are no advantages. Had the parameters on the Test and Positive branches been independent of health or disease, dependencies would not be necessary and the factored approach would be superior. The general rule is to consider factoring when you find yourself duplicating subtrees.
Dependencies may only reference stochastic nodes
You may notice that the dependency userform only allows you to access stochastic nodes in each factor. Recall that chance nodes, being instantaneous, have zero duration. They therefore cannot influence rates, probabilities, QALYs or costs in other factors.

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The benefits of factoring
The ability to handle multiple factors is one of the primary strengths of StoTree. Formulating factored stochastic trees has several advantages:

- It simplifies the model formulation process. The analyst can focus attention on one component (factor) of the model at a time, refining it as needed, and only later worry about other components and interactions between components.
- It allows the easy formulation of large Markov models. The models shown in this document are small examples. In our experience, models containing tens or hundreds of states may easily arise. These are readily formulated in factored form because most factors contain on the order of 2–5 states. Formulating large models such as these is impractical without factoring.
- Greater transparency in the presentation of the model to others. Even those not versed in the theory of continuous-time Markov decision processes can gain an intuitive graphical understanding of simple 2- to 5-state factors of most models.
- It allows the easy swapping or adding of model components. The StoTree software allows the analyst to add, remove, or substitute model components (factors) as needed. This opens up the possibility that model components formulated and populated by other analysts may be imported, or that repositories of model components may be created for the community of modelers.
Appendix

Built-in functions

Gompertz hazard rate for human survival

It is known that for ages 30 or more, human mortality rates can be closely approximated by the Gompertz mortality rate function

$$\mu(t) = b_1 \cdot \exp(b_0 + b_1 t)$$

with values of $b_0, b_1$ depending on age, gender and race. *StoTree* provides a VBA function GmpHR that returns these mortality rates for different ages ($\geq 30$), genders (Male or Female), and races (White or Black). We provide documentation of this function below.

'See my Excel workbook 'Deaths 2005 by Age,Race,Sex'.xls.
'
Function GmpHR(Age As Double, Gender As String, Optional Race As String = "White") As Double
Dim b0 As Double, b1 As Double
If Race = "White" Then
    If Gender = "Male" Then
        b0 = -6.707225204
        b1 = 0.079711236
    ElseIf Gender = "Female" Then
        b0 = -7.709089201
        b1 = 0.086873918
    End If
ElseIf Race = "Black" Then
    If Gender = "Male" Then
        b0 = -5.631988662
        b1 = 0.071532881
    ElseIf Gender = "Female" Then
        b0 = -6.586692612
        b1 = 0.077624643
    End If
End If
If Age < 0 Then
    GmpHR = 0
Else
    GmpHR = b1 * Exp(b0 + b1 * Age)
    If GmpHR > 1E+100 Then GmpHR = 1E+100  'To prevent overflow issues in Excel
End If
End Function

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Spike functions

'Returns rInf/dT if _t-t0 is closest to some multiple of tInc, and zero otherwise. By 
'closest is meant within _dT/2. In other words, returns the "infinite" rate rInf/_dT when 
' _t is closest to t0+k*tInc for any integer k (positive, negative or zero); and zero otherwise. 
'Extracts t = _t and dT = _dT from names on the cohort sheet. t0 is an optional parameter that 
'takes value zero if it is omitted.

Function Spike_(tInc As Double, Optional t0 As Double = 0) As Double
    Dim M As Double, T As Double, dT As Double, rInf As Double
    'An "infinite" rate
    rInf = 1E+20
    'Extract values of _t and _dT from cohort sheet
    T = Evaluate(ActiveWorkbook.Names(nmt).Value)
    dT = Evaluate(ActiveWorkbook.Worksheets(CohortSheetName).Names(nmdT).Value)
    'Calculate elapsed time modulo tInc, and spike if it is close to zero.
    M = ModC(T - t0, tInc)
    If M >= -dT / 2 And M < dT / 2 Then
        Spike_ = rInf / dT
    End If
End Function

'Spike at multiple times. Returns rInf/dT if _t is close to some numerical entry in 
'the range Times, and zero otherwise. By close is meant within dT/2. In other words, 
'returns the "infinite" rate rInf/_dT when _t is one of the numerical entries in Times, 
'and zero otherwise.

Function SpikeM_(Times As Range) As Double
    Dim M As Double, T As Double, dT As Double, rInf As Double, Etr As Range, Diff As Double
    'An "infinite" rate
    rInf = 1E+20
    'Extract values of _t and _dT from cohort sheet
    T = Evaluate(ActiveWorkbook.Names(nmt).Value)
    dT = Evaluate(ActiveWorkbook.Worksheets(CohortSheetName).Names(nmdT).Value)
    'Spike if _t is one of the numerical entries in Times.
    For Each Etr In Times
        If TypeName(Etr.Value) = "Double" Then
            Diff = T - Etr.Value
            If Diff >= -dT / 2 And Diff < dT / 2 Then SpikeM_ = rInf / dT
        End If
    Next Etr
End Function
Modulo function

'Due to the binary representation of floating point numbers in Excel, Excel's Mod function can return erroneous results. For example, Mod(.03,.01) returns .01 in Excel, not 0. This function corrects the result to 0 if mod/Modulus is within dM of 1.

Function ModC(Argument As Double, Modulus As Double) As Double
    Dim dM As Double
    dM = 10 ^ (-12)
    ModC = Argument - Modulus * Int(Argument / Modulus)
    If ModC / Modulus > 1 - dM Then ModC = 0
End Function

Weibull hazard rate function

The Weibull is a commonly used distribution for modeling survival. With parameters $\lambda > 0$ and $\gamma > 0$, it’s hazard rate function is

$$h(t) = \lambda \gamma t^{\gamma-1} \quad t > 0.$$ 

The corresponding survival function is

$$S(t) = P(T > t) = \exp(-\lambda t^\gamma) \quad t > 0.$$ 

'Returns the Weibull hazard rate at time T.

Function WeiHR(t As Double, Lambda As Double, Gamma As Double) As Variant
    If Gamma > 0 And Lambda >= 0 Then
        If t > 0 Then
            WeiHR = Lambda * Gamma * t ^ (Gamma - 1)
        ElseIf t = 0 Then
            If Gamma >= 1 Then
                WeiHR = 0
            Else
                WeiHR = "Error"
            End If
        Else
            WeiHR = "Error"
        End If
    Else
        WeiHR = "Error"
    End If
End Function
Function WeiPct(Alpha As Double, Lbda As Double, Gamma As Double) As Double
    If Gamma > 0 Then
        WeiPct = (-1 / Lbda * Log(1 - Alpha)) ^ (1 / Gamma)
    Else
        WeiPct = -1
    End If
End Function


