Department of Industrial Engineering and Management Sciences

Northwestern University, Evanston, Illinois, 60208-3119, U.S.A.

Working Paper No. 14-03

Network Repair Crew Scheduling and Routing for Emergency Relief Distribution Problem

Pablo A. Maya Duque

Universidad de Antioquia, Faculty of Engineering

e-mail: pmayaduque@gmail.com

Irina S. Dolinskaya

Department of Industrial Engineering and Management Sciences, Northwestern University

e-mail: dolira@northwestern.edu

Kenneth Sorensen

University of Antwerp, Faculty of Applied Economics, ANT/OR

e-mail: kenneth.sorensen@ua.ac.be

November 6, 2014

Network Repair Crew Scheduling and Routing for Emergency Relief Distribution Problem

Pablo A. Maya Duque[a,*], Irina S. Dolinskaya[b], Kenneth Sörensen[c]

[a] *Universidad de Antioquia, Faculty of Engineering*

[b] *Northwestern University, Department of Industrial Engineering and Management Sciences*

[c] *University of Antwerp, Faculty of Applied Economics, ANT/OR*

Abstract

Every year, hundreds of thousands of people are affected by natural disasters. The number of casualties is usually increased by lack of clean water, food, shelter, and adequate medical care during the aftermath. One of the main problems influencing relief distribution is the state of the post-disaster road network. In this paper, we consider the problem of scheduling the emergency repair of a rural road network that has been damaged by the occurrence of a natural disaster. This problem, which we call the Network Repair Crew Scheduling and Routing Problem addresses the scheduling and routing of a repair crew optimizing accessibility to the towns and villages that demand humanitarian relief by repairing roads. We develop both an exact dynamic programming (DP) algorithm and a GRASP metaheuristic to solve the problem and compare the performance of both approaches on small- to medium-scale instances. Our numerical analysis of the solution structure validates the optimization model and provides managerial insights into the problem and its solutions.

Keywords: Network repair, repair crew scheduling, repair crew routing, dynamic programming, greedy randomized adaptive search procedure (GRASP).

1. Introduction

Every year, hundreds of thousands of people are affected by natural disasters such as floods and earthquakes, especially in less developed regions of the planet (Guha-Sapir et al., 2011). An important observation, however, is that the number of casualties is usually increased by lack of clean water, food, shelter, and adequate medical care during the aftermath (PAHO, 2000). An adequate logistics reply to a disaster is therefore crucial.

*Corresponding author: Pablo A. Maya Duque, ✉ Faculty of Engineering , University of Antioquia, Building 21, of. 435, Calle 70 No. 52-21, Medellín, Colombia, ☎ +57 4 219 55 75, ✉ pmayaduque@gmail.com

It has been conjectured that 80% of the disaster relief effort consists of logistics (Trunick, 2005). To a first approximation, therefore, disaster relief *is* logistics.

One of the main problems influencing the delivery of food, shelter, and medical supplies to affected regions is the state of the road network. In many situations, it is not a lack of supplies that kills people, but the impossibility to get those supplies to the people that need it. In Haiti, for example, extensive media coverage of the 2010 earthquake resulted in a large excess stock of relief supplies. However, distributing those supplies to the affected villages proved far more difficult as road infrastructure had been damaged or destroyed (Pedraza Martinez et al., 2010; Van Wassenhove et al., 2010). Further complicating the repair of the road network is the often limited availability of road repair capacity, especially in impoverished regions of the world. Therefore, it is crucial that road repair is planned and executed in the most efficient way possible.

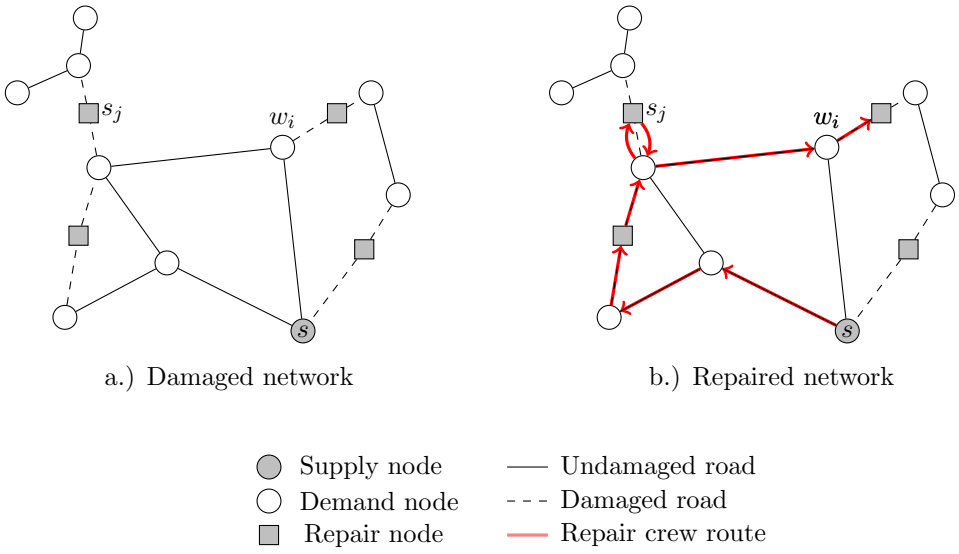
In this paper, we consider the problem of scheduling and routing the emergency repair crew of a rural road network that has been damaged by the occurrence of a natural disaster. We call this problem the *Network Repair Crew Scheduling and Routing Problem*, abbreviated as NRCSRP. The NRCSRP addresses the scheduling and routing of a single repair crew, starting from a single depot, while optimizing accessibility to the towns and villages that demand humanitarian relief. Extending this work to more than one repair crew and more than one depot is left for future research.

The problem is defined on an undirected and connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of which the nodes ($\mathcal{V}$) are either *demand* nodes ($\mathcal{V}_d$) or damaged nodes requiring *repair* ($\mathcal{V}_r$). There is one supply node that corresponds to the location in which the relief supplies are positioned and from which the repair crew initially departs. This node is called the *depot*. Demand nodes correspond to locations (usually villages) that demand humanitarian relief. The importance of a village (node i) is represented by a *demand*, i.e., a weight factor w_i, that might, e.g., correspond to the number of inhabitants. Damaged (repair) nodes represent the locations where the work of the repair crew is needed. Such nodes have a *repair time* s_j (for node j) that represents the time the repair crew spends on its first visit. Without loss of generality, we do not distinguish between demand nodes and transshipment nodes, i.e., cross points where two or more roads come together, since such nodes can be modeled as demand nodes with zero demand. Each edge $e_{ij} \in \mathcal{E}$ represents a road that connects two nodes $i, j \in \mathcal{V}$. A travel time t_{ij} is defined for each edge e_{ij} to represent the time it takes the repair crew to traverse it.

The aim of this problem is to determine the optimal sequence in which the crew should traverse the graph, starting from the depot node. Every time it encounters a damaged node that it has not visited before, it repairs the node and incurs the *repair time* of this node. On subsequent visits the crew can pass that node without incurring any additional time. When damaged nodes are repaired, demand nodes can become *accessible*. A demand node i is called accessible if there exists a path connecting this node to the depot that contains only undamaged and/or repaired nodes, and is not longer than a certain maximum distance D_i. The maximum distance D_i is node-specific and can be computed based on

pre-disaster conditions (e.g., the distance between a demand node and the depot should not be more than twice the distance it was before the disaster). Thus, in addition to the travel time t_{ij}, each edge e_{ij} has a distance measure, denoted d_{ij}, that is used to evaluate nodes' accessibility.

For each demand node, the schedule of the crew determines the moment in time at which this node becomes accessible. The objective function of the problem is the sum of the moments at which each demand node becomes accessible weighted by the node demand w_i. The objective of the network repair problem is to determine the schedule and route of the repair crew that minimizes this objective function. In general, it is not necessary for the repair crew to visit all damaged nodes. Figure 1 illustrates an example problem and solution for the NRCSRP with four damaged nodes.



a.) Damaged network b.) Repaired network

Figure 1: Example of a Damaged Network and Repair Crew Route for NRCSRP

The NRCSRP captures the most important aspects of the problem of routing a road repair crew in order to restore accessibility to demand points affected by damaged roads. To solve this problem, we develop two very different approaches. A dynamic programming (DP) model is able to find optimal solutions to small and medium-sized instances and provides insight into the fundamental structure of the problem. An efficient GRASP (greedy randomized adaptive search procedure) metaheuristic is able to solve medium- to large-scale instances in very small computing times. A comparison of the performance of both approaches yields interesting insight into the underlying problem structure.

The problem of scheduling the repair operations in a post-disaster situation can be modeled in many different ways and the exact formulation of the NRCSRP is the result of several design choices. As mentioned, one of the main causes of human casualties is the fact that a disaster tends to disconnect remote population centers from the main supply hub. In poorer countries around the world, people are usually dependent on a very limited

number of economic centers (usually larger cities) for sustenance. The demand nodes in the NRCSRP may therefore represent small villages or settlements, temporary medical facilities, shelters, or food distribution points. The supply node on the other hand will represent a city, an airport, or a port area where the relief goods arrive. When these demand points can no longer be reached by road from the supply point, the results are often dramatic. This is especially the case in rural areas, that lack the redundancy in the road network that characterizes cities. For this reason, the NRCSRP is explicitly defined on *sparse* networks, that are more common in rural than in urban situations, and the graphs underlying all generated test instances are sparse.

By explicitly considering the routing of the repair crew, the NRCSRP — unlike most previous contributions — adds a crucial time component that makes the problem more realistic and more suitable for real-life post-disaster situations. Like in real life, roads cannot be repaired until they can be reached by a repair crew. As mentioned, the objective function of the NRCSRP minimizes the demand-weighted moment in time at which all the demand nodes are connected to the supply node. The demand assigned to each demand node should be interpreted as a function of the importance of connecting this node, e.g., its population size. The objective function also takes the urgency of connecting the nodes into account, as even less important demand nodes (with small demand) may cause a large objective function increase when ignored for too long. In a sense, the NRCSRP models aspects of both the immediate response (routing of the repair crew) and the recovery (connecting the demand nodes as soon as possible) phases.

In a post-disaster immediate response situation, budget constraints are not usually a predominant issue, and the focus is on restoring connectivity of the affected regions with the limited physical means available. Nevertheless, an in-depth analysis of solutions produced by our solution approaches for the NRCSRP reveal interesting managerial results with considerable implications for further budgetary analysis. More specifically, our analysis shows that only a limited number of damaged roads (around 30%) needs to be repaired in order to restore full accessibility. Additionally, when studying the optimal or best-known solutions, we observe that the repair operations have, on average, a diminishing rate of return, connecting a lot of demand in the beginning and much less demand at the end. However, this simple observation hides a large amount of variation, and cannot be compressed into some simple rule-of-thumb that could replace an efficient algorithm. Moreover, we are able to demonstrate the importance of planning the repair several stages ahead by comparing our results to a greedy myopic heuristic.

The remainder of this paper is organized as follows. The next section reviews the literature on related network repair problems. In Sections 3 and 4 we develop two radically different solution approaches for the NRCSRP: a dynamic programming method and a GRASP (greedy randomized adaptive search procedure) metaheuristic, respectively. Both approaches are compared in Section 5, and the strong and weak points of each method are exposed. Section 5 also contains an analysis of the structure of optimal or high-quality solutions and managerial insights obtained from the numerical results. Conclusions and

suggestions for future research can be found in Section 6. A detailed list of symbols used in the paper can be found in Appendix Appendix A. A mixed-integer linear programming formulation of the problem, which we had to forgo as an inefficient solution method, is presented in Appendix Appendix B.

2. Literature Review

In recent years, the field of Operations Research and Management Sciences (OR/MS) has turned its attention to disaster management and humanitarian logistics in a big way. As pointed out by authors such as Ergun et al. (2010) and Van Wassenhove (2003), this subject is growing in importance due to the positive impact that the efficient planning of resources in humanitarian situations and the application of OR/MS techniques can have on alleviating the consequences of a disaster. However, several authors agree that disaster management is a topic in which the OR/MS community still has to strengthen its critical mass. Moreover, Altay and Green (2006) and Ergun et al. (2010) highlight that *disaster recovery*, i.e., the planning of actions taken during the *reconstruction phase*, is one of the main areas in which more research is needed. A recent survey by Kunz and Reiner (2012) confirms that "only ten papers specifically address the reconstruction phase" and stresses the importance of this stage by stating that "the quality of the logistical activities during this phase strongly impacts the success of the whole disaster recovery process, especially in terms of sustainability and long-term effectiveness (Beamon and Balcik, 2008; Benson et al., 2001; Besiou et al., 2011; Kovács and Spens, 2011)." Several authors such as Ergun et al. (2010) and Sheu (2007) point out the need for comprehensive models that: (1) integrate multiple stages of the disaster management process as a way to account for the interaction between the decisions made at different stages; (2) are adaptive to incorporate new information when a change occurs, due to the fact that in a disaster framework, information is usually limited at the beginning, and as time passes, more and more accurate information becomes available. Similarly, Holguín-Veras et al. (2013) highlight that single-period formulations cannot account for the inter-temporal effects due to the impacts of delivery actions that accrue over subsequent time periods. Therefore, there is a need for models that involve the timing in which the recovery actions are taken when evaluating the performance of this stage of the disaster management process.

The problem we address in this paper lies on the intersection of the aforementioned areas of research. It mainly focus on the recovery phase but also touches on the immediate response phase. Therefore, it integrates different phases of the disaster management process as it considers the emergency recovery of the road network while capturing the urgency of repairing a road to facilitate the relief distribution. The objective function of our problem accounts for the temporal impact of the repair decisions which is neglected in classical single-period models. Three different decisions are taken into account, on different managerial levels: (1) decide which damaged arcs have to be repaired; (2) assign the various repair activities to a repair team in a certain order; (3) determine the actual route or sequence of nodes to be visited by the repair crew. Most papers in the literature

address the first or second group of decisions and do not take into account the additional constraints that might be needed when routing the repair crew(s), such as precedence constraints to ensure a connected path that allows to traverse the network and reach the damaged arcs. Only a handfull of papers address the actual routing of the repair crews. However, these contributions rely on intricate approaches that are not easily scalable to larger problems.

Most of the papers that simultaneously tackle the three different decisions discussed here, use mathematical programming to formulate the network recovery problem. Due to the complexity of those models, they are usually tested only in few small sized instances or heuristic approaches are used to find good feasible solutions. Additionally, some of those papers take into account several objective functions which are, in general, handled through classical multi-objective methods such as weighted sum approaches and goal programming.

Chen and Tzeng (1999) consider the problem of scheduling and routing a set of work-troops in order to repair a set of damaged points scattered throughout a road-network after a large-scale earthquake. Their model consists of an upper level that schedules the work teams and a lower level that is an asymmetric traffic assignment model. Three objectives are considered, and a fuzzy genetic algorithm multi-objective approach is applied to solve the model heuristically. The approach is tested on a single small instance based on the Northridge earthquake (USA, 1994). Feng and Wang (2003) focus on the road network repair activities that occur over the first 72 hours after the disaster. The authors propose a multi-objective programming model that assigns the repair tasks to the work teams and defines the route to be followed by each team while maximizing the performance of the emergency road repair activities, maximizing the number of people that benefit from it, and minimizing the risk for rescue crews. A case study presenting the Chi-Chi earthquake is used to assess the model on a small instance involving disruptions in 6 out of 62 arcs. Yan and Shih (2009) propose a mathematical bi-objective model based on two time-space networks, one for the emergency repair and another for the relief distribution, which considers multiple commodities and minimizes the length of time required for both emergency roadway repair and relief distribution.

The papers that do not explicitly consider the routing of the repair crew usually focus on prioritizing the elements of the network that should be upgraded or repaired while considering interdependencies among related infrastructure and budget constraints. Larger instances are solved as some of the more complicating constraints (e.g., precedence constraints) are not needed.

Karlaftis et al. (2007) develop a methodology to optimally prioritize bridge repair in an urban transport network following a natural disaster. The resource allocation problem is formulated as a three-stage model, and a genetic algorithm is implemented to solve it. An instance considering 400 bridges in the surroundings of Athens is used to test the approach and to draw useful insights regarding the magnitude of repairs and necessary funds for repairing a bridge network following an unforeseen event. A multi-objective network optimization model is proposed by Matisziw et al. (2010) to facilitate identification

and scheduling of potential recovery scenarios following disruptions involving substantial loss of network nodes and arcs. The network performance is optimized across a planning horizon restricted to the constraints on resources for recovery efforts. The developed model is applied to support recovery planning for a telecommunication backbone network, which differs from road networks in the sense that a constraint to enforce accessibility to the damaged nodes is not required. Cavdaroglu et al. (2013) consider the problem of restoring critical interdependent infrastructure after a non-routine event causes disruptions. The authors propose a mixed-integer program that integrates three different decisions: the set of components to be installed or repaired (i.e., restoration decisions), the assignment of selected components (tasks) to available work groups, and the order in which each work group will complete the tasks assigned to it. The latter set of decisions involves a network design phase that does not explicitly considers precedence constraints. The objective function of this problem measures how well the services are being restored over the horizon of the restoration plan. A heuristic solution method is proposed and tested on a data set that is a realistic representation of the power and telecommunication systems of a large portion of Manhattan. Maya Duque et al. (2013) propose a linear integer programming formulation and two heuristic solution approaches to prioritize and allocate resources on a network upgrading problem in order to improve the accessibility to a set of vertices in a network. This problem arises in the domain of rural road network planning when allocating resources to upgrade roads of a rural transport network, in order to improve the access that communities in small villages have to the regional centers. The problem addressed lies on the strategic and tactical level but does not involve the operational level as the actual scheduling and routing of the upgrading work is not considered. Tuzun Aksu and Ozdamar (2014) focus on the planning of road restoration efforts during disaster response and recovery. The road restoration work is scheduled with the goal of maximizing the total weighted earliness of all cleared paths as a measure of the network accessibility for all locations in the area during the restoration process. The authors propose a dynamic path based mathematical model in which a set of access paths are defined for each node in the network and the union of these paths compose the total set of pre-defined access paths to be cleared.

Stilp et al. (2012) study a related problem of managing debris collection and disposal operations following a disaster. While their work addresses various stages of debris management operations, the so-called *clearance problem with complete information*, focusing on clearing out the roads immediately following a disaster, is the most relevant to our problem. The authors consider a multi-period problem on a network with a given subset of supply and demand nodes. A subset of network arcs are blocked by debris and have to be cleared to connect supply to demand. Maximum budget caps how much debris (and correspondingly, how many arcs) can be cleared each time period. The objective of the clearance problem is to minimize total penalty for unmet demand incurred each period. The problem is modeled using mixed integer programming and solved using both CPLEX and a set of dedicated heuristics. Unlike Stilp et al. (2012), we do not have discrete time periods, and our model captures the actual time it takes for a repair crew to reach a

damaged point and to repair it, before the crew can move on to the next task. Therefore, our solution includes the explicit route of the repair crew in the time-spatial domain.

A number of classic optimization problems, such as prize collecting traveling salesman problem (Balas, 1989), the vehicle routing problem (e.g., Toth and Vigo (2002); Laporte (1992)), the prize collecting Steiner tree problem (Goemans and Williamson, 1995), and the orienteering problem (Golden et al., 1987), also have similarities with our work, where damaged points in our problem correspond to "customers" to be visited to collect a "reward". In this way, our problem can be thought of as an optimal routing problem between a subset of repair nodes. However, a number of significant distinctions between our problem and these classic optimization problems prevent the straightforward extension of existing solution approaches. First, our problem seeks to simultaneously find the repair schedule and the paths between the nodes to be repaired, since those two entities are interdependent. Furthermore, the order in which we visit the repair nodes has significant impact on the set of feasible paths between the nodes and on the "reward" collected when we visit them. Finally, the actual moment in time at which each node is repaired is also a factor in the total collected "reward", equating it to a time-dependent setting. Also note that unlike the Steiner tree problem setting, a tree solution structure is suboptimal in most instances of our problem.

3. Dynamic Programming Model for NRCSRP

First, we outline our dynamic programming (DP) model in words, then provide the mathematical formulation of the problem, and finally discuss the implementation. For a list of symbols used see Appendix Appendix A. A MIP formulation of our problem is given in Appendix Appendix B, but a direct implementation of this model in a commercial MIP solver resulted in an intractable solution method even for small problem instances.

Our DP model keeps track of the repair crew as it repairs one node in $\mathcal{V}_r$ after another, and makes sequential decisions on which node the crew should repair next. When a node is repaired, a subset of demand nodes that have not been accessible so far can become connected to the supply node, and a "cost" is incurred corresponding to the new satisfied demand weighted by the time when the connection is established. A constraint is implicitly included requiring to connect all demand nodes to the supply node, and the weighted time the algorithm connects all the nodes in $\mathcal{V}_d$ corresponds to the cost that is being minimized. Thus, the state of our model needs to keep track of the current time and location of the repair crew, as well as the subset of damaged nodes that have not been yet repaired, and the subset of demand nodes that are not yet accessible from the depot. Our DP formulation requires an additional auxiliary node $n+1$ to be added to the graph. This node corresponds to a dummy depot connected to all the damaged nodes (i.e., nodes in $\mathcal{V}_r$) through arcs with zero travel time. Note that the DP model explicitly tracks the crew as it moves from one repair node to the next, while the fastest feasible paths between the repair nodes are calculated at each state transition to compute the corresponding travel

time. Finally, we assume that the DP state is updated after repair of the current node is completed.

3.1. Formulation

Formally, this problem can be expressed as a dynamic programming model as follows.

DP state:. $s = (i, t, \bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r)$, where

$i \in (\mathcal{V}_r \setminus \bar{\mathcal{V}}_r) \cup \{0\} \cup \{n+1\}$	current repair node location of the crew,
$t \geq 0$	current time,
$\bar{\mathcal{V}}_d \subseteq \mathcal{V}_d$	demand nodes that are not yet accessible,
$\bar{\mathcal{V}}_r \subseteq \mathcal{V}_r$	damaged nodes that are not yet repaired.

DP action:. $a = j$, corresponding to the decision for the repair crew to move to node j, where, given the current state $s = (i, t, \bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r)$, the action space $\mathcal{A}(s)$ is

$$\mathcal{A}(s) = \begin{cases} \{j \in \bar{\mathcal{V}}_r : \tau(i, j, \bar{\mathcal{V}}_r) < \infty\}, & \text{if } \bar{\mathcal{V}}_d \neq \emptyset \\ \{n+1\}, & \text{if } \bar{\mathcal{V}}_d = \emptyset \end{cases}.$$

Here, $\tau(s, a) = \tau(i, j, \bar{\mathcal{V}}_r)$ is the function that returns minimum travel time between nodes i and j over all the paths that do not pass through nodes in $\bar{\mathcal{V}}_r$. The function returns infinity if no such paths exist. The fastest path problem from i to j for all $j \in \bar{\mathcal{V}}_r$ can be solved efficiently with one execution of a Dijkstra's algorithm (Dijkstra, 1959), similar to the discussion of the accessibility function below.

DP state transition:. $g(s, a)$ the state transition function that returns a state to which the system transitions when action a is chosen in state s. That is, $s = (i, t, \bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r) \longrightarrow_{(a=j)} (i', t', \bar{\mathcal{V}}'_d, \bar{\mathcal{V}}'_r) = s'$, where

$$\begin{aligned} i' &= j, \\ t' &= t + \tau(i, j, \bar{\mathcal{V}}_r) + s_j, \\ \bar{\mathcal{V}}'_d &= \bar{\mathcal{V}}_d \setminus v(\bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r, j), \\ \bar{\mathcal{V}}'_r &= \bar{\mathcal{V}}_r \setminus \{j\}. \end{aligned}$$

Here, $v(s, a) = v(\bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r, j)$ is an *accessibility function* that returns a subset of nodes in $\bar{\mathcal{V}}_d$ that become accessible when node $j \in \bar{\mathcal{V}}_r$ is repaired. Note that $v(\bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r, j)$ can return an empty set when no new nodes are connected to the supply node by repairing node j. A set of shortest (distance) path problems on the updated network (where node j can be passed without incurring the repair cost) needs to be solved in order to evaluate $v(.)$. An efficient way to do so is discussed later in this section.

DP action cost:. $c(s,a)$ is the instantaneous cost of action a when in state s, where for $s = (i, t, \bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r)$ and $a = j$ we have

$$c(i, t, \bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r, j) = (t + \tau(i, j, \bar{\mathcal{V}}_r) + s_j) \cdot \sum_{k \in v(\bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r, j)} w_k, \tag{1}$$

Recursive equation:. Let $f(s)$ denote the minimum cost incurred by the system reaching from the initial state $s_0 = (0, 0, \mathcal{V}_d, \mathcal{V}_r)$ to the current state $s = (i, t, \bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r)$. Then, we can write the following dynamic programming (Bellman's) recursive equation:

$$f(s') = \min_{\{s,\ s.t.,\ g(s,a)=s',\ a \in \mathcal{A}(s)\}} \{f(s) + c(s,a)\}, \tag{2}$$

where $f(s_0) = 0$. Recursively solving equation (2) we find the minimum value for

$$\min_{\{s=(i,t,\bar{\mathcal{V}}_d,\bar{\mathcal{V}}_r),\ s.t.,\ i=n+1\}} f(s),$$

which corresponds to the minimum value of our problem objective function.

Accessibility Function:. Recall, $v(s,a) = v(\bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r, j)$ is an *accessibility function* that returns a subset of nodes in $\bar{\mathcal{V}}_d$ that become accessible when node $j \in \bar{\mathcal{V}}_r$ is repaired. This function can be efficiently evaluated as follows.

First, let $\mathcal{G}(\bar{\mathcal{V}}_r) \subseteq \mathcal{G} = (\mathcal{V}, \mathcal{E})$, be a modified network where nodes in $\bar{\mathcal{V}}_r$ and the arcs adjacent to nodes in $\bar{\mathcal{V}}_r$ are omitted from the original graph $\mathcal{G}$. This accounts for the fact that the demand nodes cannot be connected to the supply node through the damaged nodes that have not been yet repaired. Then, observe that a previously unaccessible demand node $k \in \bar{\mathcal{V}}_d$ becomes accessible when node $j \in \bar{\mathcal{V}}_r$ is repaired if and only if there is a path from k to the depot node (node 0) in network $\mathcal{G}(\bar{\mathcal{V}}_r \setminus \{j\})$ that is not longer than D_k and passes through node j. If there was a path from k to 0 that is not longer than D_k and does not pass through node j, then that node would have been accessible prior to repairing node j, i.e., it would not have been in $\bar{\mathcal{V}}_d$. Thus, to evaluate the accessibility function $v(\bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r, j)$, we find the shortest distance path from each node in the set $\bar{\mathcal{V}}_d$ to node j and then add the shortest distance from j to 0 in network $\mathcal{G}(\bar{\mathcal{V}}_r \setminus \{j\})$. For each node $k \in \bar{\mathcal{V}}_d$ that has the sum of the two distances less than or equal to D_k, repairing node j would connect that node to the supply node, and the accessibility function $v(.)$ returns a set of those nodes.

Note that the algorithm has to evaluate $v(\bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r, j)$ for all $j \in \bar{\mathcal{V}}_r$, each of which would require to find a minimum path from j to all the nodes in $\bar{\mathcal{V}}_d$. We can improve the implementation efficiency by intelligently sequencing these operations. To this end, the algorithm uses Dijkstra's algorithm, which efficiently finds a minimum cost path in a static network from one node to all other nodes. At each iteration of our dynamic programming functional equation (2), we implement Dijkstra's algorithm on $\mathcal{G}$ where every node in $\bar{\mathcal{V}}_r$ is terminal. That is, as soon as the algorithm reaches one of the nodes that have note

been previously repaired (i.e., in set $\bar{\mathcal{V}}_r$), the paths are not allowed to continue through that node. Thus, we find a shortest path from supply node 0 to all nodes in $\bar{\mathcal{V}}_r$, and let β_j for $j \in \bar{\mathcal{V}}_r$ denote such distance. Then, for each $j \in \bar{\mathcal{V}}_r$ such that $\beta_j \leq \max_{k \in \bar{\mathcal{V}}_d} D_k$ we find a shortest path in network $\mathcal{G}(\bar{\mathcal{V}}_r \setminus \{j\})$ from j to all nodes in $\bar{\mathcal{V}}_d$, and let δ_{jk} for $k \in \bar{\mathcal{V}}_d$ denote such distance. Then, $v(\bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r, j) = \{k \in \bar{\mathcal{V}}_d,\ s.t.,\ \beta_j + \delta_{jk} \leq D_k\}$. Observe, that our dynamic programming model requires solving a set of smaller dynamic programming models (shortest paths from node 0 to nodes in $\bar{\mathcal{V}}_d$ on a partially repaired network) to evaluate the cost associated with each decision in the action space.

3.2. Implementation

There are a number of observations and problem structure properties we use to facilitate efficient implementation of the DP model.

3.2.1. Initialization.

To initialize the DP recursive equation, the algorithm preprocesses the problem instance to find a more compact initial state than $s_0 = (0, 0, \mathcal{V}_d, \mathcal{V}_r)$. We note that in a given instance, a number of demand nodes can maintain their accessibility despite the damage to the road network, and s_0 does not need to include all the nodes in $\mathcal{V}_d$. Thus, a shortest path algorithm is run on the damaged network (i.e., no nodes in $\mathcal{V}_r$ are assumed to be repaired) to find the minimum distance between the supply node 0 and all the demand nodes. Then, the distances found are compared to the values of D_i's, and nodes in $\mathcal{V}_d$ that are already accessible are omitted from the initial state s_0. In addition, the demand nodes with $w_i = 0$ are omitted, since these nodes correspond to transshipment (intersection) nodes and, regardless of their accessibility status, do not contribute to the value of the objective function.

3.2.2. Cost Function Reformulation.

The cost function defined in equation (1) assigns a positive value to an action corresponding to repairing a damaged node that connects new demand nodes to the supply and a zero value when a repair action does not establish accessibility for any demand nodes. When a network is severely damaged requiring to repair multiple nodes to improve accessibility, our DP state network has a significant number of action arcs correspond to zero cost, and only a small subset of actions has value larger than zero. Implementation of minimum cost algorithms on such networks is inefficient since a lot of actions and nodes have identical values (i.e., equal contribution to the objective function). To facilitate more efficient implementation of the DP model, we redefine the cost function as follows,

$$c'(i, t, \bar{\mathcal{V}}_d, \bar{\mathcal{V}}_r, j) = (\tau(i, j, \bar{\mathcal{V}}_r) + s_j) \cdot \sum_{k \in \bar{\mathcal{V}}_d} w_k. \tag{3}$$

Thus, equation (3) assigns a cost of going from node i to node j equal to the penalty accrued during that move (plus repair time at node j) for all the demand nodes that have not been accessible at that time. Then, the original recursive equation (2) is solved while substituting cost function $c(.)$ with the new cost function $c'(.)$. Observe that the modified formulation results in exactly the same objective function value, the only difference being how we sum the cost: all at once at the time of connecting a demand node to the supply (1), or progressively as crew moves along the path (3).

4. A GRASP Metaheuristic for the NRCSRP

Due to the limitation on the size of problems that can be solved by our DP model, we present an alternative approach to tackle the NRCSRP: a GRASP metaheuristic. GRASP is a constructive metaheuristic introduced by Feo and Resende (1995) that generally consists of two phases: construction of a feasible solution and improvement of the constructed solution. Both phases are usually iterated several times, after which the best solution is reported. The construction phase uses a controlled amount of randomness in order to overcome the myopic behaviour of a greedy constructive heuristic. The improvement phase uses, in most cases, a local search strategy that iteratively improves the initial solution by making small changes to it. The choice to develop a GRASP metaheuristic is not arbitrary. Besides its simplicity and the fact that its effectivity has often been demonstrated, the nature of the problem invites the use of a constructive heuristic that starts from an empty solution and adds one element (the next road to repair) at a time. Moreover, the DP method can be seen as an advanced constructive procedure that uses backtracking.

In Section 4.1, we present a set of basic definitions for our GRASP metaheuristic. The construction phase is described in Section 4.2, while the improvement phase is presented in Section 4.3. Finally, the technical details related to the implementation are discussed in Section 4.4.

4.1. Heuristic Notation and Definitions

We consider the undirected and connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ that defines the NRCSRP, introduced in Section 1, and refer to it as the *original graph*. Additionally, we define a *reduced graph* $\mathcal{G}'_0 = (\mathcal{V}'_0, \mathcal{E}'_0)$, in which only the repair nodes and the depot node appear. In this reduced graph the set of vertices is $\mathcal{V}'_0 = \mathcal{V}_r \cup \{0\}$. The set of edges $\mathcal{E}'_0$ consists of those node pairs $(i, j) \in \mathcal{E}'_0$ between which a path in $\mathcal{G}$ exists that does not contain any other nodes in $\mathcal{V}'_0$ (i.e., that does not pass through nodes in $\mathcal{V}_r$ that have not been repaired). The cost of edge $(i, j) \in \mathcal{E}'_0$ is equal to the minimum travel time between nodes i and j in $\mathcal{G}$. Note that, for a given pair of nodes i and j, such a path might not exist. If that is the case, no edge is present in the reduced graph $\mathcal{G}'_0$. Figure 2 presents an example of an original graph and the associated reduced graph.

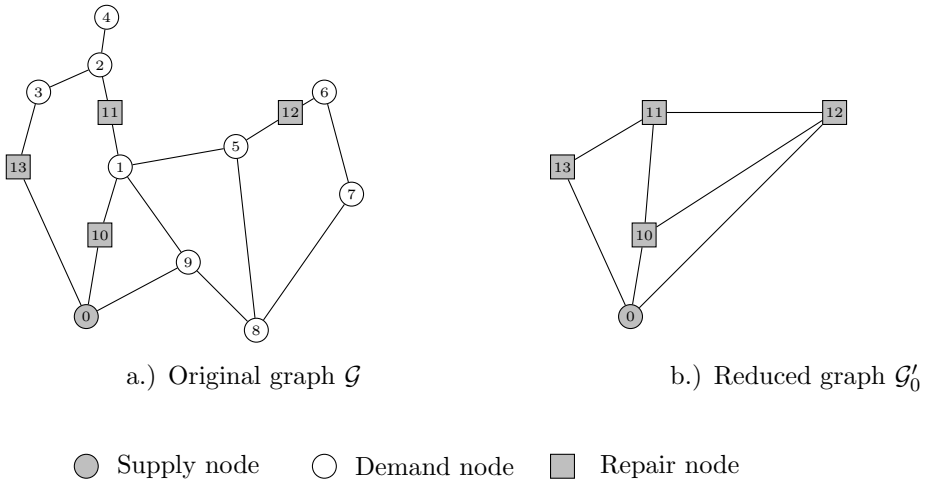


Figure 2: Graph Reduction Process

Let $\mathbf{x} = [x_1, x_2, \ldots, x_k]$ where $k \leq |\mathcal{V}_r|$ and $x_n \in \mathcal{V}_r$ for $n = 1, \ldots, k$ be a candidate solution of the NRCSRP. Such a candidate solution is a permutation of a subset of the damaged nodes, and corresponds to the sequence to be followed by the repair crew, where x_n is the n^{th} repair node on the crew's schedule (for $n = 1, \ldots, k$). Note, however, that not all permutations might be feasible due to existing precedence relationships. These precedence relationships are due to the fact that there might be some damaged nodes that can be reached only after another damaged node has been repaired. Therefore, we define a solution $\mathbf{x}$ to be *feasible* if the sequence that it represents makes all the demand nodes accessible to the depot and all precedence relationships hold.

For a given permutation $\mathbf{x} = [x_1, \ldots, x_k]$ where $k \leq |\mathcal{V}_r|$, $\mathcal{G}'_n = (\mathcal{V}'_n, \mathcal{E}'_n)$ for $1 \leq n \leq k$ denotes the updated reduced graph after nodes $x_1, \ldots, x_n$ have been repaired and the repair crew is located at node x_n. Then, $\mathcal{V}'_n = \mathcal{V}_r \setminus \{x_1, \ldots x_{n-1}\}$ and $\mathcal{E}'_n$ is the set of edges such that $(i, j) \in \mathcal{E}'_n$ for $i, j \in \mathcal{V}'_n$ if there exists a path in $\mathcal{G}$ between nodes i and j that does not contain any other nodes in $\mathcal{V}'_n$.

Then, for a given $\mathbf{x}$ we define,

$t(x_n)$	Time at which node x_n is repaired;
$\bar{\mathcal{V}}_d^n \subseteq \mathcal{V}_d$	Demand nodes that remain inaccessible after repairing nodes $x_1, ..., x_n$ in sequence $\mathbf{x}$;
$\bar{\mathcal{V}}_r^n \subseteq \mathcal{V}_r$	Damaged nodes that have not been repaired after repairing nodes $x_1, ..., x_n$ in sequence $\mathbf{x}$;
$\mathcal{G}'_n = (\mathcal{V}'_n, \mathcal{E}'_n)$	Reduced graph after repairing nodes $x_1, ..., x_n$ in sequence $\mathbf{x}$;
$\delta(x_n, \mathcal{G}'_n)$	Set of nodes adjacent to x_n in the graph $\mathcal{G}'_n$, that is $\delta(x_n, \mathcal{G}'_n) := \{i \in \bar{\mathcal{V}}_r^n : (x_n, i) \in \mathcal{E}'_n\}$.

Similarly to the description of our DP model, we define an *accessibility function* $v(\mathbf{x}, i)$, for $\mathbf{x} = [x_1, \ldots, x_k]$ and $i \in \bar{\mathcal{V}}_r^k$. Function $v(\mathbf{x}, i)$ returns the subset of nodes from $\bar{\mathcal{V}}_d^k$

that become accessible when node i is repaired after first having repaired nodes $x_1, \ldots, x_k$ (that is, when the damaged node i is added in the position $k+1$ to the repair sequence). In Section 4.4.2 we present an efficient way to evaluate the accessibility function $v(\mathbf{x}, i)$ in this context.

Finally, solution $\mathbf{x} = [x_1, \ldots, x_k]$ has an associated objective value $f(\mathbf{x})$ that is equal to the demand-weighted sum of the time when each demand node in $\mathcal{V}_d$ becomes accessible. Thus, the objective value $f(\mathbf{x})$ of a solution $\mathbf{x} = [x_1, \ldots, x_k]$ is

$$f(\mathbf{x}) = \sum_{n=1}^{k} \left\{ t(x_n) \cdot \sum_{\substack{i \in v(\mathbf{x}', x_n): \\ \mathbf{x}' = [x_1, \ldots, x_{n-1}]}} w_i \right\}. \tag{4}$$

4.2. Construction Phase

The construction phase of our GRASP algorithm aims to generate an initial feasible solution for the NRCSRP. Although some randomness is needed to generate a diverse set of initial solutions during different iterations of the algorithm, a completely random procedure (e.g., a random permutation of the damaged nodes) is generally not advised. This is due to the fact that such procedure might either fail to generate a feasible solution or only generate low quality initial solutions. Therefore, we propose a procedure that generates feasible solutions for the NRCSRP taking into account the pre-disaster network conditions.

The repair sequence is initialized at time 0 with the depot node (i.e., $n = 0$, $x_0 = 0$, $\mathbf{x} = \emptyset$) and the following steps are iterated until either all damaged nodes are repaired or all demand nodes have became accessible, whichever comes first.

Step 1. Choose the next node to be repaired (x_{n+1}) from the set of nodes adjacent to x_n ($\delta(x_n, \mathcal{G}'_n)$). (Section 4.2.1 describes the procedure that is used to select node x_{n+1}.)

Step 2. Update the repair time for node x_{n+1}:

$$t(x_{n+1}) = t(x_n) + t'_{x_n x_{n+1}} + s_{x_{n+1}}, \tag{5}$$

where $t'_{x_n x_{n+1}}$ is the travel time between damaged nodes x_n and x_{n+1} in the reduced graph $\mathcal{G}'_n$ (i.e., the time to traverse the edge that connects nodes x_n and x_{n+1} in the reduced graph $\mathcal{G}'_n$).

Step 3. Evaluate the accessibility function $v(\mathbf{x}, x_{n+1})$ where $\mathbf{x} = [x_1, \ldots, x_n]$. (Section 4.4.2 describes this procedure.) Use the set of nodes returned to update $\bar{\mathcal{V}}_d^{n+1}$ and $\bar{\mathcal{V}}_r^{n+1}$, as follows,

$$\bar{\mathcal{V}}_d^{n+1} = \bar{\mathcal{V}}_d^n \setminus v(\mathbf{x}, x_{n+1}), \tag{6}$$
$$\bar{\mathcal{V}}_r^{n+1} = \bar{\mathcal{V}}_r^n \setminus \{x_{n+1}\}. \tag{7}$$

Update the value of the objective function,

$$f(x_1, ..., x_{n+1}) = f(x_1, ..., x_n) + b(\mathbf{x}, x_{n+1}), \tag{8}$$

where $b(\mathbf{x}, x_{n+1})$ is the marginal contribution to the objective function value of repairing node x_{n+1} after having repaired the nodes $\mathbf{x} = [x_1, \ldots, x_n]$. That is,

$$b(\mathbf{x}, x_{n+1}) = t(x_{n+1}) \cdot \sum_{i \in v(\mathbf{x}, x_{n+1})} w_i. \tag{9}$$

Step 4. Update the new repair sequence $\mathbf{x} \longleftarrow [\mathbf{x}, x_{n+1}]$.

Set $n \longleftarrow n + 1$, update the reduced graph $\mathcal{G}'_n = (\mathcal{V}'_n, \mathcal{E}'_n)$ and identify the set of adjacent nodes $\delta(x_n, \mathcal{G}'_n)$.

In Algorithm 1, we present a schematic overview of this procedure.

Algorithm 1 Construction Phase

```
Initialization: n = 0, x_n = 0, G'_0 = (V'_0, E'_0), x = ∅
while V̄^n_r ≠ ∅ and V̄^n_d ≠ ∅ do
    Select a node x_{n+1} from δ(x_n, G'_n) to be repaired
    Update t(x_{n+1})
    Evaluate the accessibility function v(x, x_{n+1})
    Update the sets V̄^{n+1}_d and V̄^{n+1}_r
    Update the objective function f([x, x_{n+1}])
    Update the repair sequence x ⟵ [x, x_{n+1}]
    Set n ⟵ n + 1
    Update the reduced graph G'_n = (V'_n, E'_n) and set δ(x_n, G'_n)
end while
```

4.2.1. Selection of the Next Node to Be Repaired.

Algorithm 1 (Step 1) requires that, at each iteration and for a given current candidate solution $\mathbf{x} = [x_1, \ldots, x_n]$, we choose the next node to be repaired (x_{n+1}) from the list of adjacent nodes $\delta(x_n, \mathcal{G}'_n)$. Depending on how this selection is performed, different solutions might be obtained. We propose a selection procedure that encompasses two basic principles: (1) it uses information about the network before the disaster (i.e., in which all nodes in $\mathcal{V}_r$ are non-damaged); and (2) it includes randomness such that different solutions might be generated every time the constructive phase is executed. This procedure is carried out as follows.

Given the original network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and assuming that all damaged nodes are repaired (corresponding to the network before the disaster), we solve a minimum cost flow problem in which the cost corresponds to the travelled distance; each demand node i has a demand

w_i; each damaged node has a demand equal to zero; and the depot has a supply capacity equal to $\sum_{i \in \mathcal{V}_d} w_i$. Based on the solution of this minimum cost flow problem, parameter u_i is computed for each damaged node. The value of u_i is equal to the amount of flow that passes through node i and reflects the importance of node i in the pre-disaster conditions.

Next, given a candidate solution $\mathbf{x} = [x_1, \ldots, x_n]$, the next node to be repaired (x_{n+1}) has to be selected from the set of nodes adjacent to x_n ($\delta(x_n, \mathcal{G}'_n)$). To accomplish this, for each node $k \in \delta(x_n, \mathcal{G}'_n)$ we compute a probability p_k of selecting that node to be the next node x_{n+1}. Probability p_k is computed using equation (10). The first component of equation (10) represents the importance of node k relative to all the nodes in $\delta(x_n, \mathcal{G}'_n)$. This component is multiplied by $\lambda \in [0, 1]$ that weighs the importance of the pre-disaster conditions. The second component of equation (10) distributes the remaining (i.e., $(1-\lambda)$-weighted) probability of selecting node k equally among all the nodes in $\delta(x_n, \mathcal{G}'_n)$.

$$p_k = \lambda \frac{u_k}{\sum_{j \in \delta(x_n, \mathcal{G}'_n)} u_j} + (1 - \lambda) \frac{1}{|\delta(x_n, \mathcal{G}'_n)|}. \tag{10}$$

Using the presented procedure to generate the initial solution guides our algorithm to solutions that resemble the network before the disaster while simultaneously introducing diversity to the set of initial solutions. This random diversity of the initial solutions facilitates better exploration of the solution space.

4.3. Improvement Phase

The improvement phase of our GRASP metaheuristic uses a local search algorithm to improve the initial solution generated during the construction phase. The local search improves this solution by iteratively making small changes, called *moves* to the current solution. The set of solutions that can be reached from the current solution by applying a single move, is called the *neighborhood* of that solution. In each iteration, a solution from the neighborhood of the current solution is selected to become the new current solution according to a pre-specified move *strategy*. The local search stops when all solutions in the neighborhood of the current solution have a worse objective function value, or when a given maximum number of iterations, θ, is reached.

The first step of the local search algorithm is to clean up the solution generated during the construction phase, $\mathbf{x} = [x_1, \ldots, x_k]$ for $k \leq |\mathcal{V}_r|$. More specifically, the algorithm removes all the repair nodes in the sequence that do not improve connectivity nor accessibility of any other node. To do this, for each node $x_i \in \mathbf{x}$, $\bar{\mathcal{V}}_d^{i-1}$ is compared to $\bar{\mathcal{V}}_d^{i}$. If both sets are the same, repairing node x_i has not improved the *accessibility* of the network. Equivalently, the algorithm evaluates the accessibility function discussed in Section 4.4.2 to check whether $v(\mathbf{x}', x_i) \neq \emptyset$, for $\mathbf{x}' = [x_1, \ldots\ldots x_{i-1}]$. It also compares sets of repair nodes that can be reached before and after repairing node x_i, i.e., it compares $\delta(0, \mathcal{G}'_{i-1})$ and $\delta(0, \mathcal{G}'_i)$. If x_i itself is the only difference between those two sets, the *connectivity* of the network has not been improved. Thus, we define a boolean function $h(x_i)$ for

all $x_i \in \mathbf{x}$ that is equal to 1 if repairing node x_i after nodes $x_1, ..., x_{i-1}$ in $\mathbf{x}$ have been already repaired improves at least one of either *connectivity* or *accessibility* of the network. Otherwise, $h(x_i)$ is set to 0. The cleanup step of the improvement phase evaluates $h(x_i)$ for all $x_i \in \mathbf{x}$ and removes all x_i for which $h(x_i) = 0$ from the repair sequence $\mathbf{x}$. We denote the new solution by $\mathbf{x}_0 = [x_1, ..., x_{k_0}]$. Note that the new solution $\mathbf{x}_0$ corresponds to a feasible solution with an objective function value that is guaranteed to be no worse than the initial solution $\mathbf{x}$.

After the initial solution has been cleaned up, the second stage of the improvement phase is executed, which reorders the repair nodes in $\mathbf{x}_0$ in order to improve the solution. This is carried out by iteratively relocating elements in $\mathbf{x}_0$ one node at a time. To perform a *relocation move* three distinct steps are executed:

Step 1. Select node $x_i \in \mathbf{x}_0$ to be relocated. The algorithm chooses node x_i that has the largest marginal contribution to the objective function, i.e.,

$$i = \arg\max_l \{b(x_l) : l \in (1, ..., k_0)\}. \tag{11}$$

Step 2. Choose the position j to which node x_i will be relocated. Starting with $j = 1$, the algorithm checks whether inserting x_i into the j^{th} position is feasible and decreases the objective value $f(.)$. The algorithm iteratively increases j by one until it finds such a position or reaches the current position of node x_i, i.e., $j = i$. (See Section 4.4.1 for details on how the feasibility check is performed.)
If no such j is found, the algorithm goes back to step 1 and selects the x_i with the next largest value of $b(x_i)$.

Step 3. Update the new solution. Let the *relocation function* $r(\mathbf{x}_0, i, j)$ return the solution resulting from relocating $x_i \in \mathbf{x}_0$ into the j^{th} position and update $b(x_i)$ values for $i \in (1, ..., k_0)$ for the new repair sequence. That is

$$r(\mathbf{x}_0, i, j) = [x_1, ..., x_{j-1}, x_i, x_j, ..., x_{i-1}, x_{i+1}, ..., x_{k_0}]. \tag{12}$$

The intuition behind *Step 1* is to select the damaged node from the repair sequence that has the largest impact on the objective function (possibly, due to its connectivity to a large number of demand nodes or to the demand nodes with high demand values) and attempt to decrease its marginal value by inserting it as early as possible into the repair sequence. Note that another variation of the presented metaheuristic is to implement the cleanup step on the updated solution after each reordering iteration, but our numerical tests demonstrate no benefit of doing so.

In order to efficiently explore the neighborhood of the current solution $\mathbf{x}_0 = [x_1, \ldots, x_{k_0}]$ we form a list $\mathcal{L}(\mathbf{x}_0)$ containing the nodes to be repaired, $(x_1, \ldots, x_{k_0})$. This list is sorted in decreasing order with respect to the values $b(x_i)$ associated with each damaged node $x_i \in \mathbf{x}_0$. The sorted list is explored from the beginning until we find the first node to be relocated that improves the solution. When a better solution is found, we update the

solution and resort the list. Algorithm 2 presents a schematic overview of the improvement phase.

Algorithm 2 Improvement Phase

1: Pre-processing: clean up solution deleting nodes with $h(x_i) = 0$
2: **repeat**
3: Sort $\mathcal{L}(\mathbf{x}_0)$ in descending order of $b(x_i)$
4: **for** $i = 1 \dots k_0$ **do**
5: **for** $j = 1 \dots i$ **do**
6: $\hat{\mathbf{x}} \longleftarrow r(\mathbf{x}_0, i, j)$
7: **if** $\hat{\mathbf{x}}$ is feasible AND $f(\hat{\mathbf{x}}) < f(\mathbf{x}_0)$ **then**
8: $\mathbf{x}_0 \longleftarrow \hat{\mathbf{x}}$
9: Return to line 3
10: **end if**
11: **end for**
12: **end for**
13: **until** Stopping criterion is met

4.4. Implementation

Here, we discuss the nuances of implementation of the presented GRASP algorithm. First, in Section 4.4.1, we describe the procedure to determine whether a solution obtained by a relocation move is feasible. Then, in Section 4.4.2, we present the procedure to evaluate the accessibility function $v(\mathbf{x}, i)$.

4.4.1. Feasibility Check.

Each iteration of the improvement phase, described in Section 4.3, requires the algorithm to explore the neighborhood of the current solution $\mathbf{x}_0$. To this end, for each neighboring solution $\hat{\mathbf{x}}$ obtained by relocating a repair node $x_i \in \mathbf{x}_0$, the algorithm first evaluates whether $\hat{\mathbf{x}}$ is feasible, and then whether it improves the current solution $\mathbf{x}_0$. In case a node x_i is relocated to position j that precedes its current position (i.e., $j < i$), the solution $\hat{\mathbf{x}}$ obtained is feasible if node $x_i \in \delta(x_{j-1}, \mathcal{G}'_{j-1})$, where node x_{j-1} is the node in the position $j - 1$ of the repair sequence.

4.4.2. Accessibility Function Evaluation.

Each time a node i is repaired and added to the repair sequence in solution $\mathbf{x}$, the accessibility function $v(\mathbf{x}', x_n)$, for $x_n = i$ and $\mathbf{x}' = [x_1, \dots x_{n-1}]$, has to be evaluated in order to determine the demand nodes that become accessible. This function requires to compute, for each demand node $j \in \overline{\mathcal{V}}_d^n$ (i.e., each demand node j that is not yet accessible), the shortest path from j to the depot and to compare the length of that shortest path with the maximum acceptable distance D_j.

To improve the efficiency of the algorithm, we formulate the problem of finding the shortest path from the depot to all the demand nodes as a minimum cost flow problem (Maya Duque et al., 2013).

$$\min \sum_{(i,j)\in\mathcal{E}} l_{ij} y_{ij} \tag{13}$$

s.t.

$$\sum_{j:(i,j)\in\mathcal{E}} y_{ij} - \sum_{j:(j,i)\in\mathcal{E}} y_{ij} = d_i \qquad \forall i \in \mathcal{V} \tag{14}$$

$$y_{ij} \geq 0 \qquad \forall (i,j) \in \mathcal{E} \tag{15}$$

In this model, for each edge e_{ij} connecting nodes i and j, two decision variables are defined: y_{ij} corresponding to the edge being traversed from i to j, and y_{ji} corresponding to the traversal from j to i. Each demand node i has a demand $d_i = w_i$; each damaged node has a zero demand; and the depot has a supply capacity d_i equal to $\sum_{i\in\mathcal{V}_d} w_i$. The parameter l_{ij} depends on the length of edge e_{ij} and the status of the nodes that it connects, i.e., whether i and j are damaged or repaired. Thus, when evaluating the accessibility function after repairing node x_n, the value of the parameter l_{ij} would depend on the set of damaged nodes that remain unrepaired, $\bar{\mathcal{V}}_r^n$. If, at least, either i or $j \in \bar{\mathcal{V}}_r^n$, $l_{ij} = l_{ji} = d_{ij} + M$, otherwise $l_{ij} = l_{ji} = d_{ij}$, where M is a large constant value.

Based on the optimal value of decision variables y_{ij} of the minimum cost flow problem, the shortest path from the depot to a given node j can be obtained by tracking back the edges that are used to pass the flow. Thus, the shortest path from the depot to j is equal to the sum of the lengths of the edges that carry the flow between these two nodes. For each node j that was not accessible after having repaired nodes $x_1, \ldots, x_{n-1}$ (i.e., for all $j \in \bar{\mathcal{V}}_d^{n-1}$), the shortest paths obtained from the solution of (13)-(15) are compared to the maximum acceptable distances D_j's. The function $v(\mathbf{x}', x_n)$, for $\mathbf{x}' = [x_1 \ldots x_{n-1}]$, returns a subset of nodes $j \in \bar{\mathcal{V}}_d^{n-1}$ for which the obtained shortest paths are not greater than D_j.

Note that, since at each iteration only one additional damaged node is repaired, only a few of the coefficients l_{ij} have to be updated from one iteration to the next. Therefore, the minimum cost flow problem can be efficiently reoptimized at each iteration.

5. Computational Experiments

In this section, we evaluate the two solution approaches we propose for the NRCSRP. To this end, we use a set of randomly generated instances of the problem which are available from `http://antor.ua.ac.be/downloads/NRCSRP`. The dynamic programming algorithm was coded in MATLAB while the GRASP metaheuristic was implemented in Java and CPLEX using Concert Technology (IBM ILOG CPLEX Optimization Studio Academic Research Edition V12.2).

5.1. Instance Generation

To generate the instances necessary for the experiments, we use the network generator GNETGEN, which is a modification of the widely used NETGEN generator proposed by Klingman et al. (1974). The generator creates a minimum cost flow network for a given number of nodes $n = |\mathcal{V}|$ and edges $|\mathcal{E}|$. In the generated network, each node has associated a supply/demand value, being the supply equals to total demand, and each edge has a cost for being traversed. In our case, this cost represents the travel distance d_{ij} between the two nodes i and j connected by edge e_{ij}, and it is a variable within the interval [0, 10]. This network is transformed into a network repair instance using the following procedure.

For each edge e_{ij}, we generate the parameter t_{ij} that represents the travel time. This parameter is defined as a function of d_{ij} and an average travel speed υ as shown in equation (16), where r is a uniformly distributed value in the interval $[0, 1]$.

$$t_{ij} = (1 + r)\frac{d_{ij}}{\upsilon} \tag{16}$$

Next, a number of damaged nodes $n_d = |\mathcal{V}_d|$ is determined. We use a parameter α that specifies the percentage of damage of the network, i.e., the percentage of the edges in the network that are damaged by the disaster. Thus, the number of damaged edges is $n_d = \lceil \alpha |\mathcal{E}| \rceil$.

We randomly select n_d edges. For each selected edge e: (1) an intermediate point on the edge is randomly chosen, and (2) a damaged node j is created corresponding to that intermediate point. The repair time s_j for the damaged node is set as a random variable uniformly distributed in the interval [10, 60]. (3) The edge e is replaced by two new edges, each of them connecting one of the extremes of edge e and node j. (4) The distance and travel time for each of the new edges depend on the proportion that it represents of the original edge e.

Finally, for each node i, we must generate the maximum acceptable distance D_i to the depot. That is, the maximum acceptable length of an undamaged or repaired path from i to the depot. The parameter D_i is defined as a function of SP_i, the shortest path from i to the depot on the network in which all damaged nodes are repaired (i.e., the pre-disaster conditions). In order to compute D_i, we use a parameter β that represents the maximum tolerable percentage by which the path connecting i to the depot can increase. Thus, for each node i the maximum acceptable distance D_i is defined as $(1 + \beta)SP_i$.

Two sets of instances are considered for the computational experiments. First, we consider a set $\mathcal{S}_1$ of small instances, a majority of which can be solved exactly by our dynamic programming model. We use these instances to analyze the performance of the DP approach, evaluate the components of the GRASP metaheuristic, and compare the two solution methods. Second, we analyze a set $\mathcal{S}_2$, containing medium and large size instances, that is used to test the performance of the GRASP metaheuristic. Note that using each one of the minimum cost flow networks generated by GNETGEN, several instances can be created

by using different combinations of the parameters α and β. Table 1 shows the number of nodes considered in each instance, the number of instances generated for each number of nodes, the values used for parameters α and β, and the total number of instances in each set.

Table 1: Sets of Instances Generated for the Computational Experiments

	$\mathcal{S}_1$	$\mathcal{S}_2$
Number of nodes	21, 26, 31, 36, 41	61, 81, 101, 201, 301, 401
Instances per network size	3	3
Values of α (%)	5, 10, 25, 30, 50	5, 10, 25, 30, 50
Values of β (%)	5, 10, 25, 50	5, 10, 25, 50
Total instances	300	360

5.2. Dynamic Programming

The performance of our dynamic programming model is evaluated using set $\mathcal{S}_1$, which consists of 300 small instances. Limiting the maximum computing time to 24 hours for each instance, the algorithm found optimal solutions for 225 of those instances. Table 2 presents the number of instances solved to optimality for each combination of number of nodes in the network and value of the parameter α, while Table 3 shows the number of optimal solutions found for each combination of number of nodes and value of the parameter β. The number of optimal solutions found decreases when the number of nodes and the level of damage of the network (α) increase. On the other hand, the number of instances solved to optimality slightly increases when β, the maximum tolerable percentage in which the path connecting i to the depot can augment, increases.

Table 2: Number of Optimal Solutions Found by the DP for Each Combination of Number of Nodes and Values of α, out of 12 ($\mathcal{S}_1$ Set of Instances)

	α				
No. nodes	5	10	25	30	50
21	12	12	12	12	9
26	12	12	12	12	12
31	12	12	12	11	1
36	12	12	5	3	0
41	12	12	2	2	0

Table 4 presents the average computing time for each combination of number of nodes and value of parameter α that were solved within 24 hours. It increases when both, the value of α and the number of nodes, increase. However, the effect of the parameter α is considerably stronger. The average computing time for the instances with 31 nodes and

Table 3: Number of Optimal Solutions Found by the DP for Each Combination of Number of Nodes and Values of β, out of 15 ($\mathcal{S}_1$ Set of Instances)

	β			
No. nodes	5	10	25	50
21	14	14	14	15
26	15	15	15	15
31	11	12	12	13
36	6	7	9	10
41	6	6	8	8

$\alpha = 25\%$ is about 4 hours. From this point on, the increasing pattern of the computing times changes due to the fact that the number of instances that DP is able to solve to optimality within the 24 hours decreases.

Table 4: CPU Time (in seconds) with Respect to the Instance Size and Values of α ($\mathcal{S}_1$ Set of Instances)

	α				
No. nodes	5	10	25	30	50
21	0.03	0.03	1.73	34.88	10243.21
26	0.02	0.03	1.26	15.94	19251.06
31	0.03	0.12	1657.24	14542.48	232.74
36	0.06	0.88	7642.19	205.56	-
41	0.19	23.85	5640.91	24334.51	-

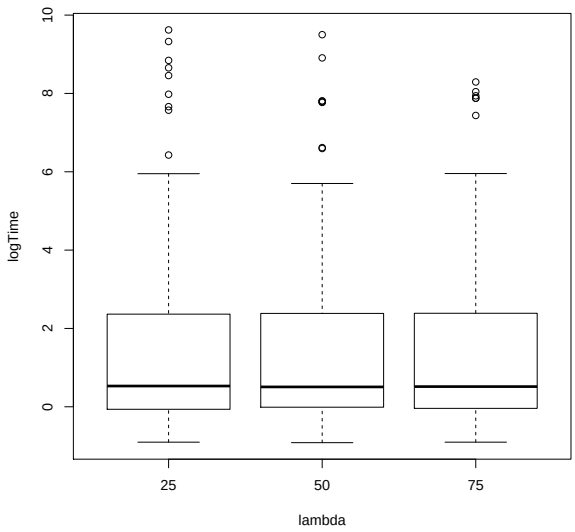
5.3. GRASP Metaheuristic

We first use a subset of the small instances in $\mathcal{S}_1$ and the corresponding optimal solutions found by dynamic programming model to evaluate the significance of the different parameters of the GRASP algorithm. Then, using the optimal parameter setting, we evaluate the effectiveness of our GRASP algorithm to generate good feasible solutions and discuss its general performance in terms of computing time and the effect of the different components and parameters of the algorithm when solving the larger instances ($\mathcal{S}_2$).

5.3.1. Parameter Tuning.

The GRASP algorithm we propose in this paper has three parameters that can be tuned: the importance granted to the pre-disaster conditions when generating the initial solution (λ), the maximum number of relocation moves that can be performed in the local search (θ), and the number of times that the two phases of the algorithm are iterated (τ) (i.e., the number of feasible solutions that are generated initially).

Figure 3: Boxplot of the log(CPUtime) and λ



In a statistical experiment we determine the significance of the effect of the three parameters on the quality of the solutions and computing time. To this end, we use three different values for the parameters λ (25%, 50% and 75%) and θ (10, 50 and 100), and two values for the parameter τ (1 and 5). The analysis of variance (ANOVA) indicates that the parameters θ and τ have significant effect on the quality of the solution and computing time. Thus, when θ and τ increase, the value of the objective function decreases while the computing time increases. At the same time, parameter λ does not have significant effect on the GRASP performance. However, as it is shown in Figure 3, the computing times for the most difficult instances (outliers in the figure) decrease when the value of λ increases. Based on the results of the designed experiment, the parameters λ, θ and τ are set to 50%, 100 and 5, respectively.

5.3.2. Numerical Results for the Set of Small Instances, $\mathcal{S}_1$.

To evaluate the effectiveness of our GRASP algorithm in generating good solutions, we solve the 300 small instances (set $\mathcal{S}_1$) and compare the solutions found by GRASP with those obtained using the DP approach. Our GRASP algorithm found 219 out of the 225 optimal solutions obtained by the DP model. For these instances, the average computing time was less than 2.10 seconds. The maximum gap to optimality for these 225 instances is 3.94%. When the analysis is extended to the complete set of instances, i.e., including the instances that were not solved to optimality by our DP algorithm, the average computing time increases considerably. In addition, the computing time increases for larger values of α and smaller values of β, i.e., when the network is severely affected by the disaster and the maximum tolerable increase in the shortest path from a node i to the depot to be considered accessible is small. Note, however, that the effect of the parameter α is

significantly larger than the effect of the parameter β. Tables 5 and 6 present the average computing time for each combination of the number of nodes and values of the parameters α and β, respectively.

Table 5: CPU Time (in seconds) with Respect to the Instance Size and Values of α ($\mathcal{S}_1$ Set of Instances)

	α				
No. nodes	5	10	25	30	50
21	0.62	0.73	1.42	2.02	6.66
26	0.77	0.88	1.63	2.35	6.00
31	0.97	1.11	2.52	3.80	46.10
36	1.03	1.54	6.66	12.60	128.42
41	1.35	1.98	31.78	43.83	613.28

Table 6: CPU Time (in seconds) with Respect to the Instance Size and Values of β ($\mathcal{S}_1$ Set of Instances)

	β			
No. nodes	5	10	25	50
21	3.26	2.87	1.77	1.27
26	3.31	2.68	1.80	1.51
31	20.40	13.38	6.27	3.54
36	54.51	36.57	17.84	11.28
41	250.95	163.31	118.46	21.06

5.3.3. Numerical Results for the Set of Medium and Large Instances, $\mathcal{S}_2$.

We use instance set $\mathcal{S}_2$ to evaluate the performance of our algorithm when solving medium and large size instances. Note that DP is unable to solve these instance within the 24 hours deadline. For each of the instances we run the GRASP algorithm with parameters λ, θ and τ set to 50%, 100 and 5, respectively. Table 7 presents the average computing time for the combinations of number of nodes and α values for which all the instances were solved in less than 10 hours. Parameter α, the percentage of damaged network, has a higher impact on the computing time than the number of nodes. In addition, our results show that parameter β, the maximum tolerable percentage in which the path connecting each node i to the depot is allowed to increase, also affects the computing time such that when the value of β increases the computing time decreases. However, this effect is less significant than the one generated by the value of parameter α and the number of nodes.

Table 8 presents the average number of iterations it took GRASP algorithm to find the best solution. We observe that as the value of α increases, the number of iterations also increases. That is, finding a good solution is more difficult for the instances in which the network is severely damaged.

Table 7: CPU Time (in seconds) with Respect to the Instance Size and Values of α ($\mathcal{S}_2$ Set of Instances)

	α				
No. nodes	5	10	25	30	50
61	1.50	2.77	173.93	331.95	5708.91
81	2.53	8.59	615.72	1531.73	22907.68
101	5.06	31.79	1624.17	-	-
201	10.76	355.83	-	-	-
301	373.18	5818.85	-	-	-
401	1684.75	13447.40	-	-	-

Table 8: Average Number of Iterations in Which the Best Solution is Found with Respect to the Instance Size and Values of α ($\mathcal{S}_2$ Set of Instances)

	α				
No. nodes	5	10	25	30	50
61	1.0	1.0	2.8	1.9	3.3
81	1.3	1.2	2.0	3.3	2.8
101	1.3	1.5	2.5	-	-
201	1.0	2.1	-	-	-
301	1.5	2.3	-	-	-
401	1.0	2.2	-	-	-

5.4. Solution Analysis and Insights

In this section, we analyze the structure of the solutions found with the goal to validate our optimization model and obtain some managerial insights. The discussion in this section is restricted to the solutions found by the GRASP heuristic for instance set S_1. While not all solutions found using GRASP are guaranteed to be optimal (see Section 5.3), an analysis of the subset of S_1 instances for which GRASP does find an optimal solution yields analogous results.

5.4.1. Comparison to Myopic Heuristic.

As discussed earlier, one important distinction between our model and the existing literature is the fact that we enforce predecessor constraints and ensure feasible repair schedules, where a damaged node cannot be repaired until the crew has repaired all the damaged nodes on a path between that node and the depot. The effect of these constraints is especially important when the crew needs to repair a sequence of nodes before a large demand can be connected. In such situations, repairing any of the intermediate nodes will have a small effect on the objective function value at best, and good solution strategies can only be found by considering several repairs into the future. The importance of this strategy

is demonstrated by comparing our solutions to those found by a greedy heuristic that myopically selects the next node to repair without considering the effect of future repairs (we call such heuristic "myopic"). To this end, we considered 280 out of 300 instances in $\mathcal{S}_1$ for which two or more nodes needed to be repaired to obtain full accessibility.

For 127 out of 280 instances, GRASP found a better solution than the myopic heuristic, with an average difference between the two solutions of 54.52% and a maximum difference of almost 300%. The myopic heuristic found a better solution in 8 cases, with an average difference of 1.64% and a maximum difference of 3.19%. For the instances in which GRASP found a better solution, the myopic heuristic solution always repairs a larger number of nodes. On average, the myopic heuristic repairs 58.78% of the damaged nodes while our GRASP algorithm repairs only 34.04%. Moreover, for the instances with 10 or more damaged nodes, the myopic heuristic repairs, on average, 18.1 nodes while the GRASP algorithm repairs 8.5 nodes. This demonstrates that failing to look ahead and, by doing so, ignoring the effects of repairing a given node on future demand accessibility generally results in a repair operation that takes much more time and consumes more resources, by repairing more roads, than strictly necessary.

5.4.2. Solution Structure and Managerial Insights.

The first important observation we make from a policy and planning perspective is that only a fraction of the damaged roads (i.e., repair nodes) need to be repaired to ensure accessibility for the entire network (see Tables 9 and 10). As expected, that percentage decreases as β values increase, since this corresponds to more relaxed requirements for a demand node to be accessible. The correlation with α values is not as clear. While there is an overall positive trend for the percentage of repaired nodes, instances with $\alpha = 5\%$ correspond to a particularly high number of repaired nodes. This can be explained by an observation that for such low value of α a number of instances have only one or two damaged nodes, and the solution requires to repair all of them.

Table 9: Average Percentage of Nodes Repaired for Various α Values.

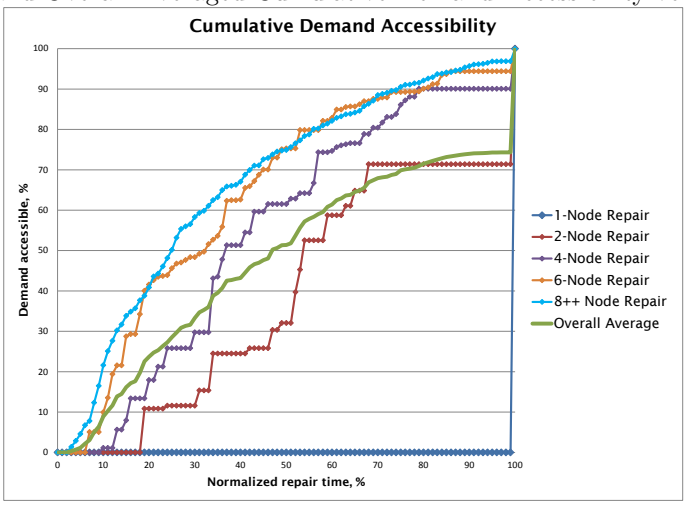
Values of α (%)	5	10	25	30	50
Nodes repaired (%)	37.24	28.01	29.72	29.69	31.82

Table 10: Average Percentage of Nodes Repaired for Various β Values.

Values of β (%)	5	10	25	50
Nodes repaired (%)	39.88	34.81	26.02	19.55

By analyzing the cumulative demand that becomes accessible over the duration of the repair operation we observe that, on average, the percentage of accessible demand follows a law of diminishing returns. Figure 4 illustrates how cumulative demand accessibility changes over the course of the entire repair operation, for different numbers of nodes repaired. The figure graphs demand averaged over all $\mathcal{S}_1$ instances and a few examples

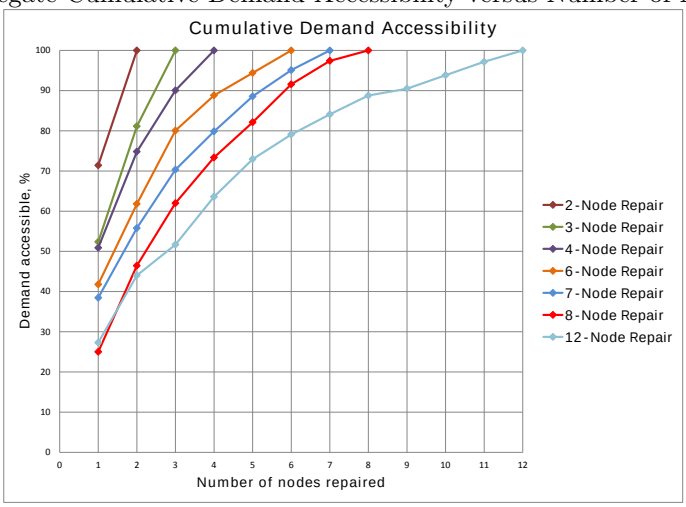
Figure 4: Aggregate and Overall Averaged Cumulative Demand Accessibility versus Normalized Time



of demand averaged over subsets of instances with the same number of nodes repaired. To account for different repair times between the instances, the x-axis corresponds to normalized repair time. From the graphs, we observe an initial delay until the first set of demand nodes become accessible, which corresponds to the travel and repair times of the crew to complete its first assignment. Then, on average, the percentage of marginal demand connected decreases with the passing of time and with each additional repair of a node, resulting in a concave cumulative demand curve. We make the same observation in Figure 5, illustrating accessible demand versus number of nodes repaired. Towards the end of the repair operation (Figure 4), the curves become flat, corresponding to the repair crew traveling father away to repair the subsequent node or repairing multiple nodes before the next improvement in demand accessibility. At the very end, we observe a jump in percent of accessible demand, which is explained by a considerable amount of demand that becomes accessible after the last repair is carried out by the crew at the completion of the repair operation.

While, on average, the rule of diminishing returns is clearly illustrated, there is a lot of variation from instance to instance with regards to the percentage demand connected by each consecutive node repaired. This fact highlights the importance of solving the optimization model for each individual instance, and further explains poor performance of the myopic heuristic. Boxplots in Figures 6-8 illustrate the distribution of cumulative demand accessible after each repaired node for all instances that require four, eight and twelve nodes to be repaired, respectively. While all instances connect a progressively higher percentage of demand as more nodes are repaired, we observe high variance earlier in the operation. Figure 9 illustrates individual examples of the cumulative demand accessibility over the duration of the repair operation. Here, we see that Examples 1, 3, 5 and 6 have a significantly lower percentage of demand connected early on in comparison to Examples 2 and 4. At the same time, Examples 2, 3, 4 and 6 illustrate instances where multiple nodes need to be repaired before the next improvement in network accessibility. More

Figure 5: Aggregate Cumulative Demand Accessibility versus Number of Nodes Repaired



specifically, in case of Example 3 the sixth node repaired does not directly contribute to the cumulative accessible demand, but its repair is necessary in order for the seventh repaired node to connect a substantial percentage of demand. Examples 4 and 6 illustrate the cases where multiple consecutive repairs are required before the next improvement in accessible demand. This diverse set of examples highlights the importance of finding a close-to-optimal solution of each individual problem instance.

6. Conclusion and Future Research

In this paper, we study the problem of emergency repair of a rural road network that has been damaged by the occurrence of a natural disaster. We address the scheduling and routing of a repair crew, starting from a single depot, while optimizing accessibility to the towns and villages that demand humanitarian relief. This is an important problem given the impact that a functioning road network has on providing the opportune relief to the affected population. This paper extends the traditional network upgrading problems by considering not only the selection of the edges to be repaired or upgraded, but also the time dependency and crew routing associated with the required repair work.

While the NRCSRP was developed mainly to be used as a support tool in the initial response stage, our problem setting and solution methods could be directly extended to the later recovery stage, as long as the time scale of travel times and repair times are equivalent. Problems such as connecting remote villages to a central medical facility would be applicable here, where moving the repair equipment from one node to the next requires significant time that is comparable in scale to the actual repair time.

We develop a dynamic programming model and discuss efficient implementation techniques that allows us to find optimal solutions for a range of instance sizes of the problem. However, the DP approach limits the size of the road networks that can be solved within a

Figure 6: Boxplot of Cumulative Demand Accessibility for All Instances with 4 Nodes Repaired.

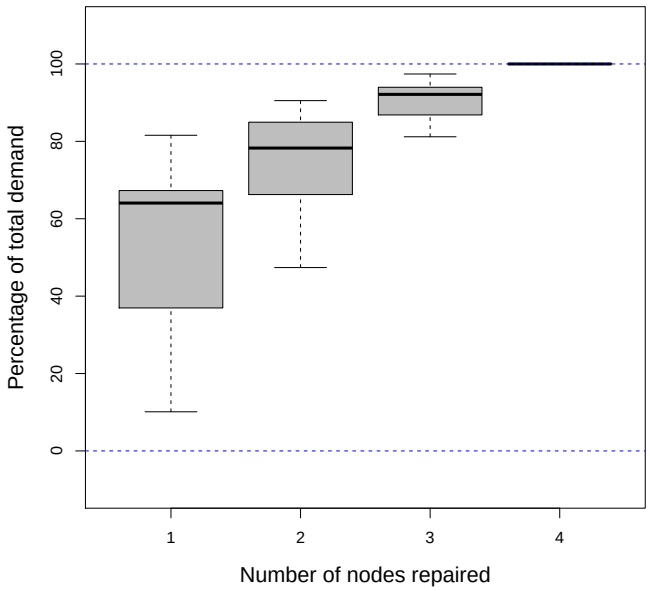


Figure 7: Boxplot of Cumulative Demand Accessibility for All Instances with 8 Nodes Repaired.

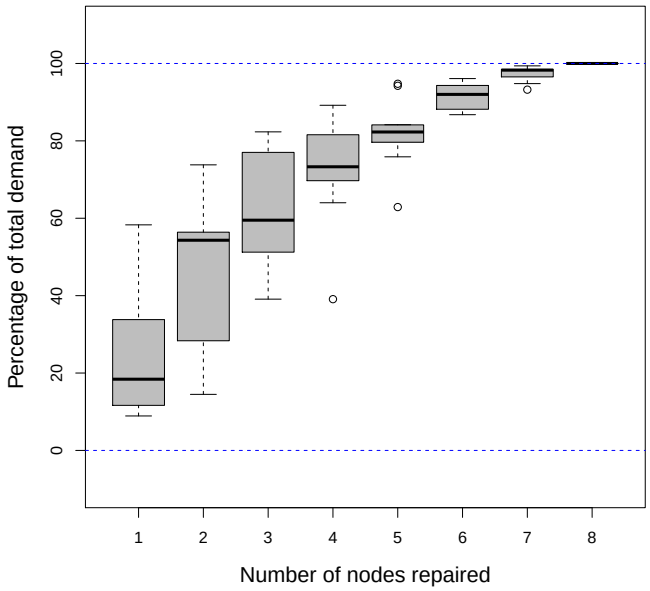


Figure 8: Boxplot of Cumulative Demand Accessibility for All Instances with 12 Nodes Repaired.

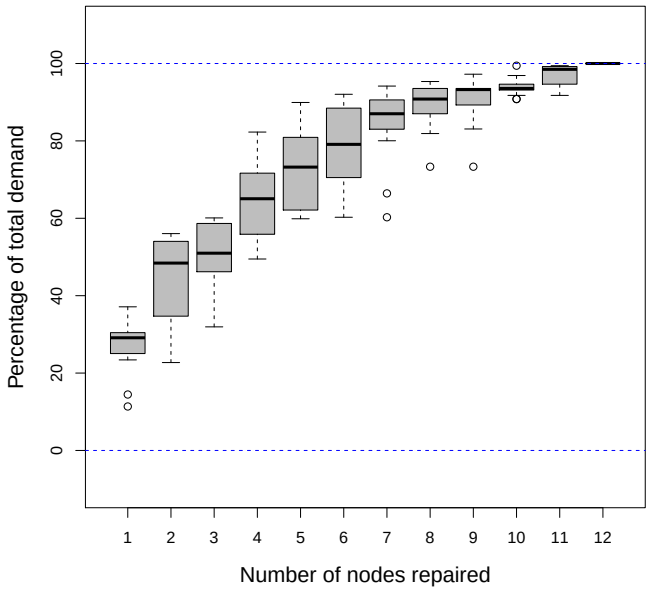
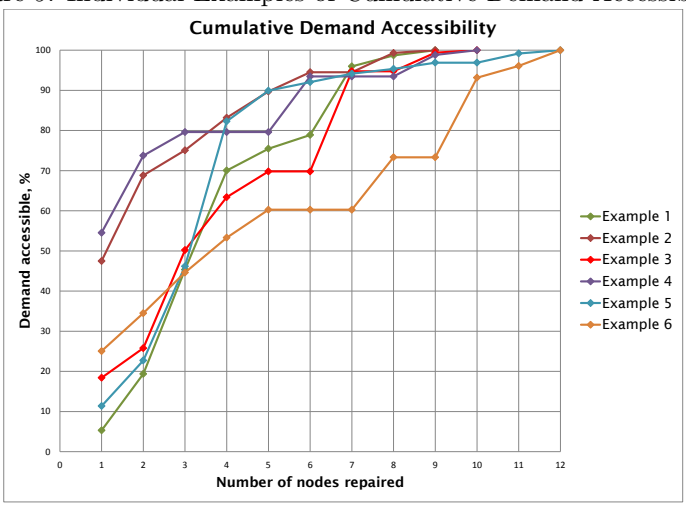


Figure 9: Individual Examples of Cumulative Demand Accessibility



24-hour time limit. This fact motivated the development of a metaheuristic (GRASP) that we develop for our problem. We run extensive computational experiments to analyze the difficulty of solving instances of the NRCSRP depending on the values of three distinct parameters. Two of the parameters are beyond the control of the decision maker: the number of nodes or population centres in a network and the level of network damage caused by the disaster, which is denoted by parameter α and determines the number of edges requiring repair. The third parameter, β, is the maximum tolerable percentage by which a path connecting each demand node i to the depot is allowed to increase in order to be considered accessible. This parameter can be controlled by the decision maker and depends on the specifics of each setting.

Our results show that the factor that has the highest impact on the difficulty of solving an NRCSRP instance is the α parameter. That is, the most severely affected networks correspond to the most difficult instances to solve. This is mainly due to the fact that the NRCSRP is, in part, a combinatorial problem for which a solution is represented by a permutation of the damaged nodes that have to be repaired. Therefore, as the level of network damages increases, so does the number of nodes that have to be repaired, which, in turn, increases the number of possible permutations to be considered. An instance of the NRCSRP also becomes more difficult to solve when the size of the network increases. However, the effect of this parameter is not as significant as it is for parameter α. Finally, when the value of the parameter β increases, the time required to solve each instance decreases. This phenomenon is explained by the fact that increasing the value of β implies that the decision maker is willing to accept a longer path between a demand node and the depot in order to consider that demand point accessible.

When comparing the difficulty of solving an instance of the NRCSRP using the dynamic programming model versus the GRASP algorithm, the results show that the two algorithms experience similar trends in the levels of difficulty. That is, the instances that are easily solved by the DP are also easily solved by the GRASP. Similarly, for the instances that could not be solved to optimality by DP within the time limit, the computational time of the GRASP algorithm is also higher. However, the DP methods performs better than the GRASP for instances with few nodes and a small percentage of network damage (α), while the GRASP performs better than the DP when both the size of the instance network and the level of damage increase. We use the GRASP heuristic to solve instances containing up to 401 nodes, and the results confirm that parameter α strongly correlates with the level of difficulty of an NRCSRP instance.

Analysis of the solution structure provides important managerial insights for our problem. We observe that only a fraction of damaged nodes (approximately 30%, on average) needs to be repaired to establish accessibility for the entire network. In addition, while, on average, each subsequent repair of a damaged node follows a law of diminishing return, that property does not necessarily hold true for all instances. This fact, along with poor performance of the tested myopic heuristic, establish the need for optimization of the repair crew schedule and rout for each individual instance. This is especially the case

when a sequence of multiple nodes need to be repaired before observing improvement in the network accessibility.

In this paper, we have designed an objective function to accurately capture and evaluate the accessibility measure, since it is closely tied to the performance of our solution strategies. An interesting open research direction to pursue in future work is to study alternative measures and objective functions for estimating network accessibility. Moreover, the efficiency of the GRASP algorithm can be potentially improved by restricting the complete exploration of the current solution neighborhood, at each iteration, and instead, focusing on specific regions of the feasible solution space. Note, however, that from the perspective of the quality of the solutions, the suggested measures might not have any positive effect on the value of the objective function itself, just the algorithm run time.

From our extensive study of the NRCSRP, we are able to identify problem properties that can be useful in extending this work to more complex and realistic settings. The first promising research direction is to extend this problem to more than one repair crew and more than one depot node. This extended problem would model real-life situations where different organizations or relief entities are involved in the emergency reconstruction operations of the region's road network. In addition, the collaboration and coordination aspects of such problem settings would have to be considered. For example, one would have to answer the question of what happens when two or more crews are able to work simultaneously on repairing a single damaged node. Another interesting extension that emerges from considering multiple depots is the distribution allocation decisions. In such cases, one has to consider the limited amount of resources available at each depot, and the repairing decisions are restricted by the need of fulfilling the relief demand of the towns that are made accessible.

Appendix A. List of Symbols

$\mathcal{V}$	Set of nodes $\{0, 1, \ldots, n+1\}$; node 0 is the depot; node $n+1$ is a dummy node; $\mathcal{V} = \{0\} \cup \mathcal{V}_d \cup \mathcal{V}_r \cup \{n+1\}$
$\mathcal{V}_d$	Set of demand nodes; $\mathcal{V}_d \subset \mathcal{V}$
$\mathcal{V}_r$	Set of damaged nodes requiring repair; $\mathcal{V}_r \subset \mathcal{V}$
$\mathcal{E}$	Set of edges or roads
e_{ij}	edge connecting nodes i and j; $e_{ij} \in \mathcal{E}$
$\mathcal{E}(k)$	Set of arcs adjacent to node $k \in \mathcal{V}$, $\mathcal{E}(k) \subset \mathcal{E}$
w_i	Relief demand of node $i \in \mathcal{V}$
t_{ij}	Time required to travel between nodes i and j on edge e_{ij}
s_j	Time required to repair damaged node $j \in \mathcal{V}_r$
d_{ij}	Length of edge e_{ij} connecting nodes i and j, i.e., distance between nodes i and j
d_e	Length of edge e
D_i	Maximum acceptable distance from node $i \in \mathcal{V}_d$ to the depot in order for i to be accessible to it.

Appendix B. Mixed Integer Programming Model for NRCSRP

For completeness, we present an alternative formulation for NRCSRP in addition to the dynamic programming model. The mixed integer programming formulation developed in this section requires an additional auxiliary node $n+1$ to be added to the graph. This node corresponds to a dummy depot connected to all the damaged nodes (i.e., nodes in $\mathcal{V}_r$) through arcs with zero travel time. The supply capacity of the depot w_0 is set to the total relief demand in the network. The notation used in that model is presented in Appendix Appendix A.

The model we propose involves three different kinds of decisions. First, the model has to decide the order in which the damaged nodes are repaired. The second set of decisions determines the route to be followed by the repair crew. That is, the paths connecting each pair of consecutive damaged nodes in the repair sequence. Those paths indicate the nodes that have to be visited when travelling between two consecutive damaged nodes and might include non-damaged nodes and other damaged nodes that have been previously repaired. Any node can be visited more than once, however, a damaged node has to be repaired the first time it is visited. The third set of decisions defines the shortest path that connects each demand node to the depot. The length of such path is used to determine whether a node that demands humanitarian relief can be considered accessible or not.

We let binary variables x_{ij} keep record of the order in which the damaged nodes are repaired (i.e., visited by the repair crew for the first time). A variable is defined for each pair of nodes in $\mathcal{V}_r \cup \{0, n+1\}$ and it takes value 1 when node j is repaired right after node i. Additionally, the variable z_j keeps the time at which the damaged node j is repaired, that is, the time at which node j is visited for the first time by the repair crew plus the actual repair time s_j. Two binary variables are defined to track the path that connects any pair of nodes in the tour followed by the repair crew. Variable r_e^{ij} is assigned value 1 when edge e is used on the path from i to j and 0 otherwise, while variable a_k^{ij} is given value 1 when node k is visited on the path from i to j. Similarly, two other binary variables are considered to track the path that connects each demand node to the supply node. Variable y_e^{0i} takes value 1 when edge e is used on the path from the depot to node i, and variable b_k^{0i} is assigned value 1 when node k is visited on the path from the depot to node i. A mixed integer programming model for the network emergency repair problem is the following:

$$\min \sum_{i \in \mathcal{V}_d} w_i \max_{k \in \mathcal{V}_r} \left\{ z_k b_k^{0i} \right\} \tag{B.1}$$

s.t.

$$\sum_{j \in \mathcal{V}_r \cup \{0, n+1\}} x_{ij} = 1 \qquad \forall i \in \mathcal{V}_r \cup \{0, n+1\} \tag{B.2}$$

$$\sum_{i \in \mathcal{V}_r \cup \{0,n+1\}} x_{ij} = 1 \qquad \forall j \in \mathcal{V}_r \cup \{0, n+1\} \tag{B.3}$$

$$\sum_{i,j \in \mathcal{S}} x_{ij} = |\mathcal{S}| - 1 \qquad \begin{array}{r} \forall \mathcal{S} \subset \mathcal{V}_r \cup \{0, n+1\} \\ |S| > 1 \end{array} \tag{B.4}$$

$$\sum_{e \in \mathcal{E}(i)} r_e^{ij} = x_{ij} \qquad \forall i, j \in \mathcal{V}_r \cup \{0\} \tag{B.5}$$

$$\sum_{e \in \mathcal{E}(j)} r_e^{ij} = x_{ij} \qquad \forall i, j \in \mathcal{V}_r \cup \{0\} \tag{B.6}$$

$$\sum_{e \in \mathcal{E}(k)} r_e^{ij} = 2a_k^{ij} \qquad \begin{array}{r} \forall i, j \in \mathcal{V}_r \cup \{0\} \\ \forall k \in \mathcal{V} \setminus \{i, j\} \end{array} \tag{B.7}$$

$$\sum_{e \in \mathcal{E}(0)} y_e^{0j} = 1 \qquad \forall j \in \mathcal{V}_d \tag{B.8}$$

$$\sum_{e \in \mathcal{E}(j)} y_e^{0j} = 1 \qquad \forall j \in \mathcal{V}_d \tag{B.9}$$

$$\sum_{e \in \mathcal{E}(k)} y_e^{0j} = 2b_k^{0j} \qquad \forall j \in \mathcal{V}_d, \forall k \in \mathcal{V} \setminus \{0, j\} \tag{B.10}$$

$$\sum_{e \in \mathcal{E}} y_e^{0j} d_e \leq D_j \qquad \forall j \in \mathcal{V}_d \tag{B.11}$$

$$z_j = \sum_{i \in \mathcal{V}_r} (z_i + \sum_{e \in \mathcal{E}} r_e^{ij} t_e) x_{ij} + s_j \qquad \forall j \in \mathcal{V}_r \tag{B.12}$$

$$z_k a_k^{ij} \leq z_j \qquad \begin{array}{r} \forall i, j \in \mathcal{V}_r \cup \{0\} \\ \forall k \in \mathcal{V}_r \end{array} \tag{B.13}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in \mathcal{V}_r \cup \{0, n+1\} \tag{B.14}$$

$$r_e^{ij} \in \{0, 1\} \qquad \begin{array}{r} \forall i, j \in \mathcal{V}_r \cup \{0\} \\ \forall e \in \mathcal{E} \end{array} \tag{B.15}$$

$$a_k^{ij} \in \{0, 1\} \qquad \begin{array}{r} \forall i, j \in \mathcal{V}_r \cup \{0\} \\ \forall k \in \mathcal{V} \end{array} \tag{B.16}$$

$$y_e^{0i} \in \{0, 1\} \qquad i \in \mathcal{V}_d, \forall e \in \mathcal{E} \tag{B.17}$$

$$b_k^{0i} \in \{0, 1\} \qquad i \in \mathcal{V}_d, \forall k \in \mathcal{V} \tag{B.18}$$

$$z_j \geq 0 \qquad \forall j \in \mathcal{V}_r \tag{B.19}$$

The objective function (B.1) minimizes the weighted sum of the times at which each node j in $\mathcal{V}_d$ becomes accessible. That is, the time when the last damaged node $k \in \mathcal{V}_r$ visited by the path connecting j to the depot is repaired. Constraints (B.2)-(B.4) establish the order in which the damaged nodes are repaired; they define an open travelling salesman problem restricted to the set of nodes formed by the damaged nodes and the depots (i.e., $\mathcal{V}_r \cup \{0, n+1\}$). Therefore, constraints (B.2) and (B.3) ensure that each damaged

node is repaired exactly once, and that the repair sequence starts and ends at the initial node 0 and the auxiliary node $n+1$, respectively. The sub-tour elimination constraints are taken into account through equations (B.4). In addition to determining the order in which the damaged nodes are repaired, it is also necessary to track the path to be followed by the repair crew when travelling between damaged nodes i and j. That path exists only if node i immediately precedes node j in the repair sequence and might involve visiting damaged nodes that have been previously repaired. Thus, constraints (B.5) and (B.6) enforce, respectively, that there is exactly one edge leaving i on the path from i to j, and that there is exactly one edge entering j on the path from i to j. That is, only one path between i and j is tracked. Additionally, the constraints (B.7) ensure that the path from i to j is connected. Similarly, constraints (B.8)-(B.10) help to track the paths that connect every demand node to the depot. Constraints (B.11) guaranty that the length of each of those paths does not exceed the permitted accessible parameter D_i. Constraints (B.12) define the time z_j at which damaged node j is repaired. That time is the result of adding the time in which the predecessor node i is repaired, the travel time on the path from i to j, and the time it takes to repair node j. Additionally, constraints (B.13) ensure that if there is a damaged node on the path that connects the damaged nodes i and j, it is repaired before repairing node j. Finally, constraints (B.14)-(B.19) define the type and bounds for the decision variables.

The objective function (B.1) and the constraints (B.12) and (B.13) involve the product of decision variables. However, these expressions can be reformulated to avoid non-linearity by introducing a big constant M, as shown in expressions (B.20), (B.21) and (B.22), respectively.

$$\min \sum_{i \in \mathcal{V}_d} w_i \max_{k \in \mathcal{V}_r} \left\{ z_k + (b_k^{0i} - 1)M \right\} \tag{B.20}$$

$$z_j = \max_{i \in \mathcal{V}_r} \left\{ z_i + \sum_{e \in \mathcal{E}} r_e^{ij} t_e + (x_{ij} - 1)M \right\} + s_j \qquad \forall j \in \mathcal{V}_r \tag{B.21}$$

$$z_k + (a_k^{ij} - 1)M \leq z_j \qquad \forall i, j \in \mathcal{V}_r \cup \{0\}, \quad \forall k \in \mathcal{V}_r \tag{B.22}$$

Finally, the function maximum that is considered in expressions (B.20) and (B.21) can be also reformulated through linear expressions by defining auxiliary decision variables and constraints.

References

N. Altay and W. Green. OR/MS research in disaster operations management. *European Journal of Operational Research*, 175(1):475–493, 2006.

E. Balas. The prize collecting traveling salesman problem. *Networks*, 19(6):621–636, 1989.

B. Beamon and B. Balcik. Performance measurement in humanitarian relief chains. *International Journal of Public Sector Management*, 21(1):4–25, 2008.

C. Benson, J. Twigg, and M. Myers. NGO initiatives in risk reduction: an overview. *Disasters*, 25(3):199–215, 2001.

M. Besiou, O. Stapleton, and L. Van Wassenhove. System dynamics for humanitarian operations. *Journal of Humanitarian Logistics and Supply Chain Management*, 1(1): 78–103, 2011.

B. Cavdaroglu, E. Hammel, J. E. Mitchell, T. C. Sharkey, and W. A. Wallace. Integrating restoration and scheduling decisions for disrupted interdependent infrastructure systems. *Annals of Operations Research*, pages 1–16, 2013.

Y. Chen and G. Tzeng. A fuzzy multi-objective model for reconstructing post-earthquake road-network by genetic algorithm. *International Journal of Fuzzy Systems*, 1(2):85–95, 1999.

E. W. Dijkstra. A note on two problems in connexion with graphs. *Numerische Mathematik*, 1(1):269–271, December 1959.

O. Ergun, G. Karakus, P. Keskinocak, J. Swann, and M. Villarreal. Operations Research to improve disaster supply chain management. In *Wiley Encyclopedia of Operations Research and Management Science*, number 2. John Wiley & Sons, Inc., 2010.

C. Feng and T. Wang. Highway emergency rehabilitation scheduling in post-earthquake 72 hours. *Journal of the Eastern Asia Society for Transportation Studies*, 5:3276–3285, 2003.

T. Feo and M. Resende. Greedy randomized adaptive search procedures. *Journal of global optimization*, 6(2):109–133, 1995.

M. X. Goemans and D. P. Williamson. A general approximation technique for constrained forest problems. *SIAM Journal on Computing*, 24(2):296–317, 1995.

B. L. Golden, L. Levy, and R. Vohra. The orienteering problem. *Naval Research Logistics (NRL)*, 34(3):307–318, 1987.

D. Guha-Sapir, F. Vos, R. Below, and S. Ponserre. Annual disaster statistical review. Technical report, Centre for Research on the Epidemiology of Disasters (CRED), Institute of Health and Society (IRSS), and Université Catholique de Louvain, 2011.

J. Holguín-Veras, N. Pérez, M. Jaller, L. N. V. Wassenhove, and F. Aros-Vera. On the appropriate objective function for post-disaster humanitarian logistics models. *Journal of Operations Management*, 31(5):262 – 280, 2013.

M. Karlaftis, K. Kepaptsoglou, and S. Lambropoulos. Fund allocation for transportation network recovery following natural disasters. *Journal of Urban Planning and Development*, 133(1):82–89, 2007.

D. Klingman, A. Napier, and J. Stutz. NETGEN: A program for generating large scale capacitated assignment, transportation, and minimum cost flow network problems. *Management Science*, 20(5):814–821, 1974.

G. Kovács and K. Spens. Trends and developments in humanitarian logistics–a gap analysis. *International Journal of Physical Distribution & Logistics Management*, 41(1): 32–45, 2011.

N. Kunz and G. Reiner. A meta-analysis of humanitarian logistics research. *Journal of Humanitarian Logistics and Supply Chain Management*, 2(2):116–147, 2012.

G. Laporte. The vehicle routing problem: An overview of exact and approximate algorithms. *European Journal of Operational Research*, 59(3):345–358, 1992.

T. C. Matisziw, A. T. Murray, and T. H. Grubesic. Strategic network restoration. *Networks and Spatial Economics*, 10(3):345–361, 2010.

P. Maya Duque, S. Coene, P. Goos, K. Sörensen, and F. Spieksma. The accessibility arc upgrading problem. *European Journal of Operations Research*, 224(3):458–465, 2013.

PAHO. *Natural disasters: Protecting the public's health*. Pan American Health Organization, 2000.

A. Pedraza Martinez, O. Stapleton, and L. Van Wassenhove. Using OR to support humanitarian operations: Learning from the haiti earthquake. 2010.

J.-B. Sheu. Challenges of emergency logistics management. *Transportation Research Part E: Logistics and Transportation Review*, 43(6):655 – 659, 2007.

K. Stilp, O. Ergun, and P. Kekinocak. Managing debris collection and disposal operations. Technical report, Center for Humanitarian Logistics, Georgia Institute of Technology, 2012.

P. Toth and D. Vigo. *The vehicle routing problem*, volume 9. Siam, 2002.

P. Trunick. Delivering relief to tsunami victims. *Logistics Today*, 46(2):1–9, 2005.

D. Tuzun Aksu and L. Ozdamar. A mathematical model for post-disaster road restoration: Enabling accessibility and evacuation. *Transportation Research Part E: Logistics and Transportation Review*, 61:56–67, 2014.

L. Van Wassenhove. New interesting POM cases from Europe. *POMSChronicle*, 10:19, 2003.

L. Van Wassenhove, A. Martinez, and O. Stapleton. An analysis of the relief supply chain in the first week after the haiti earthquake. INSEAD humanitarian research group, 2010.

S. Yan and Y. Shih. Optimal scheduling of emergency roadway repair and subsequent relief distribution. *Computers & Operations Research*, 36(6):2049–2065, 2009.