IEMS 326, Homework 5, Due 2/23/2011

1. Consider the oil well example we discussed in class. Now assume that the test for oil is not perfect but instead has a false positive probability of p and a false negative probability of q. That is, the probability of a positive test result if the oil field is poor is p. Similarly, the probability of a negative test result if the oil field is rich is q. Draw and solve the resulting decision tree assuming $p=30\%$ and $q=10\%$. For compactness you may replace the subtree for the alternative "don't test" with a single outcome node (whose value we calculated in class).

2. You own an unused silver mine in Colorado that will cost \$1.5m to reopen. If you open the mine you expect to be able to extract 12,000 ounces of silver a year for each of three years. After that the deposit will be exhausted. The silver price for the first year is \$250/oz. and each year the silver price is equally likely to rise or fall by \$80 from its level in the previous year. The extraction cost is \$200/oz and the discount rate is 10%. Assume costs and revenues occur at the end of each year. (To avoid rounding errors, I suggest doing the calculations on a spreadsheet. However, do not turn in the spreadsheet.)
a) Should you open the mine now? Assume that you cannot delay this decision. Draw and solve the resulting decision tree.
b) What if you could costlessly (but irreversibly) shut down the mine at any stage? Would you open the mine? When would you want to make use of this option? Draw and solve the resulting decision tree.

3. Suppose that you are the CEO of some big company. Your buddies on the board of directors have given you a million stock options for your brilliant leadership and to give you incentives for further success. The stock price of your company today is \$100. The stock price a year from now (according to typical financial models) is $S = \$100 \cdot e^Z$ where Z is normally distributed with mean 0.05 and standard deviation 0.2, $Z \sim N(0.05, 0.2^2)$. When the stock options mature a year from now, each will be worth $\max\{0, S - \$100\}$. If the stock price a year from now is below the current price, then the options will be worthless. Otherwise, their worth is equal to the increase in the stock price, $S - \$100$. Create a spreadsheet simulation with 100 randomly chosen scenarios to answer the following questions.
a) What is the expected value and standard deviation of the stock price a year from now?
b) What is the expected value and standard deviation of your options a year from now?
c) Since you're the CEO, you decide to invest a chunk of your company's money in mortgage CDOs. This changes the distribution of the company's stock returns. Now, $Z \sim N(-0.03, 0.4^2)$. How does this change the answers to parts a and b? Write a sentence or two describing the change.