

Lecture 5: Loans

1 Car Lease vs. Car Loan

Suppose, after getting your first permanent job, you want to buy a car. You find you can buy a good car for \$22,000, but you don't have that much in cash. You investigate your financing choices and find you can get a 5-year loan at 6% interest compounded monthly, or you can lease the car. To lease the car, you pay \$1,500 right away when you sign the lease, then \$265 per month for 5 years. You would also have the option to buy the car for \$12,000 in 5 years, when the lease is over, but you're not sure whether you want to do that. What's better, to get a loan and buy the car, or to lease it? See the Car Loan or Lease spreadsheet.

First, 6% interest compounded monthly means $6\%/12 = 0.5\%$ per month. (This is equivalent to 6.17% per year, see Luenberger p. 15.)

The monthly cashflow on the car loan is $-\$425 = \text{pmt}(\text{monthlyrate}, 60, \text{purchase})$. The present value of the "annuity" of 60 monthly cashflows of \$425 is $\text{pv}(\text{monthlyrate}, 60, \$425) = \$22,000$. So the point is that using 0.5% as a monthly discount rate makes \$22,000 now and 60 monthly cashflows of \$425 equal in value. The opportunity to borrow at 6% interest compounded monthly defines the monthly discount rate of 0.5% in this problem.

Next we consider leasing the car. The present value of the 60 monthly lease payments is $\text{pv}(\text{monthlyrate}, 60, \$265) = \$13,707$. We must add to this the \$1,500 due at signing, for a total of \$15,207. This is a *net present cost* i.e. the negative of net present value.

But we can't compare the \$22,000 and \$15,207 because if you buy the car, you still have it after 5 years, whereas if you merely lease the car, you don't. So let's consider the case where you keep the car as long as it lasts. Then to the \$15,207 we must add the present value of \$12,000 in 5 years, which is $12000(1.0617)^{-5} = 12000(1.005)^{-60} = 8896$. So the total is a net present cost of \$24,104. Thus, if you want to keep the car longer than 5 years, the loan is better.

But what if you only want to keep the car 5 years? Then the net present cost is \$15,207 for the lease. For the car loan, we need to figure in the result of selling the car in 5 years. You estimate that you would only be able to sell it for \$11,000 (less than what the dealer would charge you to buy your leased car after 5 years). The present value of \$11,000 in 5 years is $11000(1.0617)^{-5} = 11000(1.005)^{-60} = 8155$. So the net present cost is $\$22,000 - \$8155 = \$13,845$. So the loan is preferable to the lease also if you only want to keep the car 5 years.

2 Arizona Budget Crisis

An interesting example in the news involving what amounts to an annuity.¹

The state of Arizona faces a budget shortfall exceeding \$3 billion, due to the recession and collapse of the real estate market. The Arizona legislature is looking for ways to raise money without raising taxes. One idea: raise \$735 million by selling various buildings, including the state's executive office tower.

Problem: where will the governor, the treasurer, other executives, and all their staff work? Where will the prisoners go if the prisons are sold?

Solution: the contract includes terms for leasing the buildings back. The state will pay \$60 to \$75 million per year as a tenant (terms not yet determined, as they are soliciting bidders).

Problem: where will they go in 20 years?

Solution: the contract also specifies that the state will regain ownership of the buildings in 20 years.

So, in one sense, the deal is really just an annuity: the state of Arizona gets \$735 million now and pays \$60 to \$75 million per year for 20 years. It continues to use the buildings just as it always has and still owns them, in the end.

Question: is this a good deal? What kind of interest rate is the state paying?

Answer: We can solve

$$X = \frac{C}{r} \left(1 - \left(\frac{1}{1+r} \right)^n \right)$$

for r because we know X is \$735 million, $n = 20$ years, and C is somewhere between \$60 and \$75 million. Let's try it with both extreme values of C to see what interest rate we get.

This can be done in Excel with `rate(n,-C,X)`. (Once again we must use $-C$ instead of C .) The answer is that the interest rate is between 5.2% and 8%.

Is that good? What to compare to? Looking at arizona.municipalbonds.com shows that the interest rate for 20-year bonds of Arizona state agencies and cities is roughly 4.5% to 5%. These bonds are another way of borrowing over 20 years.²

Question: why isn't the Arizona legislature just looking to borrow?

¹Sources for this example: <http://www.azcentral.com/arizonarepublic/news/articles/2009/07/29/20090729assets0729.html> and http://www.nytimes.com/2009/09/25/us/25phoenix.html?_r=1&ref=us.

²Once you've studied bonds, you may realize this is not a fair comparison, because the annuity does not spread cashflows over time in the same way as a bond. See Section 3.5 of Luenberger on the concept of *duration* if you are interested. However, the qualitative conclusion here is not misleading.

The U.S. federal government pays an interest rate of 4.07% for 20-year bonds, less than Arizona. Would you rather lend to the federal government or the state of Arizona if you could get the same interest rate either way? Perhaps it's appropriate for Arizona to pay more due to considerations of *credit risk*: Arizona may be less likely to be able to pay, because Arizona's economy is in worse shape than the nation's, among other reasons.

Question: how is this not just an annuity?

The *sale-and-leaseback* contract mitigates credit risk: if the state of Arizona fails to make its payments, then the owners could sell some of the buildings to get the money they're owed. This *secured financing* generally has the purpose of mitigating credit risk: as the creditor, you have two possible ways to get paid what you're owed: if the borrower pays you back, or if the *collateral* used as security has enough value.

Problem that often appears in secured financing: the borrower's ability to repay and the value of the collateral can be highly correlated. The scenario in which Arizona real estate isn't worth much is a scenario in which the state of Arizona is more likely to be bankrupt!

So, from these considerations, it seems that the sale-and-leaseback should enable Arizona to borrow *cheaper* than by selling unsecured bonds, yet the opposite seems to be true! Some possible explanations:

- An interest rate of 4.5% to 5% reflects the state of Arizona's current level of debt. If the state borrows much more, its ability to repay all its debt may be reduced, leading to more credit risk and higher interest rates. Maybe the deal is the best the state can get, given how much more it needs to borrow.
- Maybe the legislators are panicking and not getting a good deal.