IEMS 326, Homework 6, Due 6/3/2011

1. This problem asks you to do some algebra related to the diversification spreadsheet. Suppose you can invest in n different assets where asset i has a random return R_i. Suppose you invest a fraction w_i of your portfolio in asset i. Let R be the return of your entire portfolio.
a) Write expressions for $E[R]$ and $\sigma[R]$ when you have $n=3$ assets.
b) Write an expression for $\sigma[R]$ when the assets are uncorrelated (i.e., $\text{corr}(R_i,R_j)=0$ for all $i \neq j$), have equal risk (i.e., $\sigma[R_i]=u$ for all i), and have equal weight in the portfolio, $w_i=1/n$.
c) (*Extra Credit*) Write an expression for $\sigma[R]$ when the assets all have equal risk (i.e., $\sigma[R_i]=u$ for all i); the same correlation (i.e., $\text{corr}(R_i,R_j)=c$ for all $i \neq j$); and have equal weight in the portfolio, $w_i=1/n$. What does $\sigma[R]$ tend to as n goes to infinity?

2. The spreadsheet with this homework shows mean returns and correlations for stock indices from seven countries (the data is from 1980-1993). The spreadsheet already has calculated the covariance matrix for you and the mean and variance of a portfolio giving equal weight to each stock index. *Do not turn in a spreadsheet.*
a) Use the Excel solver to determine the portfolio with the smallest variance. Do this both for the case when you allow the portfolio weights to be negative (i.e., allowing short-sales) and for the case where you restrict them to be nonnegative.
b) Now we look at the problem of finding the minimum variance portfolio that achieves some specified expected return. For each of the expected returns listed on the spreadsheet, find the portfolio with minimum variance that achieves this expected return. Enter the standard deviation of the minimum-variance portfolio in the neighboring column. Then plot the expected return (y-axis) versus the standard deviation (x-axis). You should have some sort of parabola. Do this both for the case when you allow the portfolio weights to be negative (i.e., allowing short-sales) and for the case where you restrict them to be nonnegative. Plot both cases on the same graph.

3. Luenberger, Exercise 6.1 (p. 170).

4. Luenberger, Exercise 6.5 (pp. 170-171).

5. Luenberger, Exercise 6.8 (p. 171).

6. Luenberger, Exercise 7.1 (p. 193).

7. Calculate the WACC for Citigroup assuming $r_f=2\%$ and a 4% market risk premium.