Formula Sheet

1 Time Value of Money

1.1 Future Value

The future value of x after n periods of growth at (annual) interest rate a compounded m times per year is

$$x(1+r)^n$$

where $r = a/m$ is the per-period interest rate.

The effective annual interest rate is

$$i = (1 + a/m)^m - 1.$$

The future value of x after t years of growth at annual growth rate d is

$$x(1+d)^t.$$

1.2 Present Value

In the following, r is the per-period discount rate, d is the annual discount rate, and there are m periods per year.

The present value of y to be received n periods later is

$$y(1+r)^{-n} = \frac{y}{(1+r)^n}.$$

The present value of y to be received t years later is

$$y(1+d)^{-t} = \frac{y}{(1+d)^t}.$$

The relationship between r and d is

$$d = (1+r)^m - 1 \quad \text{and} \quad r = (1+d)^{1/m} - 1.$$

1.3 Present Value: Perpetuities and Annuities

When the discount rate is r per period, the present value P of an annuity making n payments of C, each one period apart, starting in one period:

$$P = \frac{C}{r}(1 - (1 + r)^{-n}), \qquad C = \frac{Pr}{1 - (1 + r)^{-n}}$$

Present value of a perpetuity of C per period, starting in one period:

$$\frac{C}{r}.$$

2 Inflation

When p is a nominal cost that grows at rate h per year, the nominal cost after t years is

$$p(1 + h)^t.$$

When i is an inflation rate and p is a nominal cost occurring at time u, the real cost as measured in time s dollars is

$$p(1 + i)^{s-u}.$$

The real cost, as measured in base-b dollars, of an actual cost A at time t, is

$$R = A(1 + f)^{b-t},$$

where f is the annual rate of inflation. If the actual cost of something at time t is A_t, and its actual cost changes at an annual rate g, then its actual cost at time u is

$$A_u = A_t(1 + g)^{u-t}.$$

The relationship between the inflation rate f, the actual discount rate d_A, and the real discount rate d_R is

$$(1 + f)(1 + d_R) = 1 + d_A.$$

3 Probability

If A and B are two evens, then Bayes' rule is

$$\Pr[A|B] = \frac{\Pr[A \text{ and } B]}{\Pr[B]} = \frac{\Pr[B|A]\Pr[A]}{\Pr[B]}$$

Let X be a random variable. If there are n total scenarios with probabilities $p_1, \ldots, p_n$, and X_i is the value of X in scenario i, then the mean of X is

$$\mathrm{E}[X] = \sum_{i=1}^{n} p_i X_i.$$

Regardless of how many scenarios there are, the variance

$$\mathrm{Var}[X] = \mathrm{E}[(X - \mathrm{E}[X])^2] = \mathrm{E}[X^2] - (\mathrm{E}[X])^2$$

and the standard deviation $\sigma[X] = \sqrt{\mathrm{Var}[X]}$.

Let Y be another random variable. The covariance between X and Y is

$$\mathrm{Cov}[X, Y] = \mathrm{E}[(X - \mathrm{E}[X])(Y - \mathrm{E}[Y])] = \mathrm{E}[XY] - \mathrm{E}[X]\mathrm{E}[Y]$$

and the correlation between X and Y is

$$\rho(X, Y) = \frac{\mathrm{Cov}[X, Y]}{\sigma[X]\sigma[Y]}.$$

A linear combination of X and Y, where v and w are constants, has mean and variance

$$\mathrm{E}[vX + wY] = v\mathrm{E}[X] + w\mathrm{E}[Y] \quad \text{and} \quad \mathrm{Var}[vX + wY] = v^2\mathrm{Var}[X] + 2vw\mathrm{Cov}[X, Y] + w^2\mathrm{Var}[Y].$$

If $w_1, \ldots, w_m$ are constants and $X_1, \ldots, X_m$ are random variables, then the linear combination $\sum_{j=1}^{m} w_j X_j$ has mean and variance

$$\mathrm{E}\left[\sum_{j=1}^{m} w_j X_j\right] = \sum_{j=1}^{m} w_j \mathrm{E}[X_j] \quad \text{and} \quad \mathrm{Var}\left[\sum_{j=1}^{m} w_j X_j\right] = \sum_{j=1}^{m} w_j^2 \mathrm{Var}[X_j] + 2\sum_{j=1}^{m}\sum_{k<j} \mathrm{Cov}[X_j, X_k].$$

4 Bonds

A coupon payment of a bond with face value F, coupon rate c and m coupon payments per year is

$$Fc/m.$$

If the yield (quoted annually) is y for a bond making m coupon payments per year, the corresponding per-period discount rate is (because of the yield quotation convention)

$$r = y/m.$$

The price of a bond with face value F, coupon rate c, m coupon payments per year, next coupon payment in 1 period, n coupon payments remaining, and yield y is

$$F(1 + r)^{-n} + \frac{F(c/m)}{r}(1 - (1 + r)^{-n}).$$

5 Capital Asset Pricing Model

Let r_f be the risk-free rate and R_M be the return of the market portfolio. Also let $r_m = \mathrm{E}[R_M]$ be the expected return of the market portfolio and $\sigma_M = \sigma[R_M]$ be the standard deviation of the market portfolio's return.

The beta of an asset whose return is R equals the covariance of its returns with the market portfolio's return R_M, divided by the variance of the market portfolio's return:

$$\beta = \mathrm{Cov}[R_M, X]/\sigma_M^2.$$

Capital Asset Pricing Model: the expected return of this asset is

$$r = r_f + \beta(r_m - r_f).$$

This equation is also known as the Security Market Line.

Capital Market Line:

$$r = r_f + \frac{r_m - r_f}{\sigma_M}\sigma$$

where r is the expected return and σ is the standard deviation of a portfolio that lies on this line.

6 Weighted Average Cost of Capital

A company's (before-tax) WACC is

$$r_d \frac{D}{V} + r_e \frac{E}{V}$$

where

- r_d is the required return on debt,
- D is the value of the company's debt,
- r_e is the required return on equity,
- E is the company's market capitalization, and
- $V = D + E$ is the company's total market value.