

# Notes 6: Mortgages

With the traditional 30-year fixed-rate mortgage, you borrow the *principal*  $P_0$  now to help pay for a house, and then you make 360 monthly payments  $C$ . The idea is that the present value of the “annuity” of 360 monthly payments should have present value equal to  $P_0$ :

$$P_0 = \frac{C}{r} \left( 1 - (1 + r)^{-360} \right),$$

where  $r$  is the discount rate per month. It is derived from the mortgage rate you are quoted  $a$  in annual terms with the same interest rate quotation convention seen previously in discussing compound interest:  $r = a/12$ .

**Example 1:** The mortgage rate is  $a = 5.25\%$ , so  $r = 0.4375\%$  per month. Then, if someone borrowed  $P_0 = \$310,000$ , his mortgage payment is

$$C = \frac{P_0 r}{1 - (1 + r)^{-360}} = \$1,711.83.$$

This can be computed in Excel with `pmt(rate/12,years*12,-principal)`.

**Example 2:** The mortgage rate is now  $a = 5.1\%$ , and you can afford to make a monthly mortgage payment of \$1,000. Wow much can you borrow? It is

$$P_0 = \frac{\$1,000}{0.00425} (1 - 1.00425^{-360}) = \$184,179.$$

This can be computed in Excel with `pv(rate/12,years*12,-1000)`.

There are 15-year mortgages as well as 30-year mortgages. Often, the 15-year mortgage has a lower rate. The reason is that mortgage rates are related to the yield curve for U. S. Treasury bonds, which often has a higher yield for maturity 30 years than 15 years.

**Example 3:** The rate  $a$  is 5.1% for a 30-year mortgage but only 4.6% for a 15-year mortgage. However, the 15-year mortgage requires a *higher* payment! Per \$100,000 borrowed, the required payment for the 15-year mortgage is

$$\frac{\$100,000 \times 0.003833}{1 - (1.003833)^{-180}} \approx \$770 > \$543 \approx \frac{\$100,000 \times 0.00425}{1 - (1.00425)^{-360}},$$

which is the required payment for the 30-year mortgage. The reason is that you have a smaller number of payments in 15 years than in 30, so the required payment is larger, even though the interest rate is lower. In this example, although you pay less interest with the shorter-term mortgage, the monthly payment might be unaffordably high.

Mortgages are actually somewhat complicated.

# 1 Amortization, Prepayment and Refinancing

At month  $i$ , your payment of  $C$  goes partly towards making the required interest payment of  $P_{i-1}r$  that you owe because your *loan balance* or *principal balance* was  $P_{i-1}$  at the beginning of the month. The rest of your payment,  $C - P_{i-1}r$ , goes towards reducing your principal balance, so that  $P_i = P_{i-1}(1+r) - C$ . This month-by-month reduction is called *amortization*. See the `mortgage.xls` spreadsheet for an illustration.

A complicating aspect of mortgages is that you do not necessarily have to pay  $C$  every month for 360 months. You are allowed to pay extra principal. That is, you can make a payment  $D_i > C$  at month  $i$  and then your principal balance would be  $P_i = P_{i-1}(1+r) - D_i$ ; the extra amount  $D_i - C$  reduces your principal balance faster than scheduled. This can allow you to finish paying off your mortgage sooner. For example, in `mortgage.xls`, if you make a payment of \$2,000 every month, you will pay off the mortgage during the 22nd year.

Also, if you sell the house, you *must* pay off the remaining principal balance and the mortgage is terminated. Why is that? It's because a mortgage is an example of secured financing. If the borrower does not make the required mortgage payments, the bank can seize the house, which is called *foreclosure*. If you sell the house, there's no collateral any more, so the mortgage must be terminated.

You also have the right to prepay the mortgage in this way even if you don't sell the house. Sometimes people do this and get a new mortgage: this *refinancing* of the mortgage can be advantageous if lower mortgage rates are available.

**Example 1 revisited:** After 16 years of making scheduled payments, the principal balance on this 30-year mortgage with rate 5.25% has decreased to \$203,355.88. The rate for new 30-year mortgages is now 5.1%. What would the monthly mortgage payment be for a new 30-year mortgage? The answer, from `pmt(5.1%/12,30*12,-$203,355.88)`, is \$1,104.12, much less than \$1,711.83. However, refinancing requires a lot of paperwork and expensive fees  $f$  to the bank, lawyers, appraiser, city, etc. Is it a good idea to refinance? The answer depends on many things, especially what your future prepayment behavior will be.

# 2 Down Payment and Secured Financing

Mortgage rates depend on many factors, such as the length of the loan, your personal credit rating, and how much of your own money you spend on the property vs. how much you borrow. What you spend of your own money is called a *down payment*. The larger the percentage of the property value in the down payment, the more security for the lender. With a big down payment, the loan is relatively small. Then even if the value of the property drops and the borrower stops making payments, the lender may not lose money after seizing the property and selling it. You will get the best terms if you have at least 20% of the property's value as a down payment; otherwise you may need to pay extra for "mortgage insurance" (which protects the lender, not you).

### 3 Exotic Mortgages

The mortgage industry has been creative about finding ways to enable people to buy more expensive houses, given the monthly payments they can afford. One way is the *interest-only mortgage*. The interest-only mortgage eliminates the need to make principal payments: if you choose, you can pay just the interest due. In Example 1, the fixed monthly payment was \$1,711.83, but at most \$1,356.25 of that was interest (in the first month). So, with an interest-only mortgage at about the same rate, the borrower can afford to get bigger loan and buy a more expensive house (Example 1). The downside, of course, is that if you only make payments equal to the interest, the principal balance never decreases and you never reduce your debt. Eventually you would need another mortgage.

We saw that a 15-year mortgage could have a lower interest rate than a 30-year mortgage, yet higher payments due to the need to pay off the principal in a shorter amount of time. To avoid this problem while taking advantage of the fact that interest rates for short-term loans are often lower than those for long-term loans, the mortgage industry invented the 30-year *adjustable rate mortgage* (ARM). An ARM can be extremely complicated, but the essence of it is that the borrower's interest rate is fixed at a low level (say, 1.5%) for the first 2 or 3 years, and then it can reset to a new level that depends on what short-term interest rates are at that time in the future.

**Example 1 revisited:** Suppose this home-buyer got an interest-only ARM with a rate of 1.5%. If \$1,711.83 is the payment that he can afford, then the largest possible principal  $P_0$  for the loan satisfies

$$\$1,711.83 = P_0 \frac{1.5\%}{12},$$

which is  $P_0 = \$1.37$  million!

ARMs are involved in the financial crisis. Many ARMs were issued in 2003–2006, and in recent years they have been resetting, often to much higher rates than the initial rates. The consequence is that many borrowers' required mortgage payments doubled or tripled, and then they were unable to make the required payments. At the same time, the housing market collapsed and they were unable to sell their houses for enough to pay off the principal balance. Consequently, many of them have entered foreclosure.