Lecture 4: Bonds

In our analysis of the Social Security system, we assumed that the trust fund would grow at 2.9% in real terms (i.e. adjusted for inflation; its purchasing power would grow 2.9% each year). By studying bonds, we see that this model is a simplification, and we learn about some associated risks. We will also be able to approach questions such as:

- You are a saver and have \$300,000 which you want to invest for 10 years, at which time you will use the future value to buy a house. You have the opportunity to buy any of 5 different bonds. Which should you choose?
- A company needs to borrow \$10 million now to spend on a new production line. Should the company issue commercial paper that must be repaid in 3 months, or bonds that must be repaid in 20 years?

1 How does a Bond Work?

Its features include:

- *face value*, *principal*, or *par* F: this is like an amount that is lent
- *maturity* T is when the principal is repaid
- *coupon payments* are made at equal periods up to and including maturity:
  - the *coupon frequency* m is the number of times per year that coupon payments are made
  - the *coupon rate* c is the percentage of principal that equals the total coupon payments in a year

Let $n = Tm$ be the number of periods until maturity. Then the bond's issuer will pay

- a coupon payment of Fc at periods $1, 2, \ldots, n-1$
- a coupon payment of Fc plus the face value F for a total of $F + Fc$ at maturity, which is period n

So, if you buy a bond, it's like you're lending to the bond's issuer: you pay money up front, and you have the issuer's promise to pay money in the future.

The big difference between a bond and a loan is that a bond is a financial security that can be traded in the marketplace—you can buy a bond and then sell it.

2 Bond Price Formula

When a bond is issued, if its price is par (F), the coupon rate c is the rate at which the investment in the bond grows.

In general, the rate at which the investment in the bond grows is the *yield* or *yield to maturity* y. This is defined to be the number which, when you use it as a discount rate, makes the present value of the bond's future payments equal to its price.

To be precise, we need to use the *bond yield quotation convention.* If the yield (quoted annually) is y for a bond making m coupon payments per year, the corresponding per-period discount rate is (because of the yield quotation convention)

$$r = y/m.$$

Then the price P of a bond with face value F, coupon rate c, m coupon payments per year, next coupon payment in k periods, n coupon payments remaining, and yield y is

$$P = F(1+r)^{-n)} + \frac{Fc}{y}(1-(1+r)^{-n}).$$

You can check that if we plug in c for y, we get $P = F$. The intuition behind this is that it's like lending F and then getting interest Fc at the end of each period, until maturity when you get F back.

EXAMPLE: $F = \$10,000$, $n = 5$, $c = 6\%$, $m = 1$. Then $y = 6\%$ implies

$$F(1+y)^{-n} = \$7473 \quad \text{and} \quad \frac{Fc}{y}(1-(1+y)^{-n}) = \$2527$$

which sum to $P = \$10,000 = F$. But $y = 8\%$ implies

$$F(1+y)^{-n} = \$6806 \quad \text{and} \quad \frac{Fc}{y}(1-(1+y)^{-n}) = \$2527$$

which sum to $P = \$9,201$.

In general, when the yield goes up, the price goes down, and vice versa. INTUITION: if price drops while you get the same promised cashflows, your (smaller) investment must grow faster.

REMARK: All else being equal, a higher yield makes for a more attractive investment opportunity: your investment in a higher-yielding bond is growing faster. Or, you can think that a higher yield is associated with a lower price, and you'd prefer to pay less for a bond.

3 Risks in Bond Investing

1. Inflation risk. Consider the plan of investing $300,000 for 10 years. A bond provides payments that are fixed, measured in dollars. But if inflation makes house prices go up, so the house-purchasing value of a dollar shrinks, the money you have in 10 years after investing in a bond might buy *less* house than $300,000 buys now! If house prices double in 10 years, you might ended up in a better house if you just bought a $300,000 house now.

Next, consider this menu of bonds:

Issuer	Maturity	Yield
U. S. Treasury	5	2.3%
U. S. Treasury	10	3.3%
U. S. Treasury	20	4%
Exxon Mobil	20	5%
Brazil	20	6%

Which is the best choice? Is it always the one with the highest yield?

2. Credit risk: you might not get the money you were promised if the issuer can not pay you, or chooses not to. *Credit ratings agencies* rate the credit risk of bonds. Brazil's bonds might be rated Baa by Moody's, the lowest *investment grade* (i.e. suitable for a bond investment fund to own); Exxon Mobil (which has lots of cash in the bank) and the U. S. Treasury are rated AAA, the highest rating.

What the last 3 bonds in the table show is a risk-reward tradeoff: the bond with the most credit risk has the highest yield.

See `yieldcurve.xls`. The yield curve shows there's no single growth rate for money: the yield depends on how long you're lending to the U. S. Treasury. One reason for this is that longer-maturity bonds have more interest-rate risk.

3. Interest-rate risk is the risk that a bond may drop in price because its yield rises. The yield curve can change over time, and bond prices change with it.

The reason that longer-maturity bonds have more interest-rate risk is illustrated by Luenberger Figure 3.4.

If our goal is to have money to buy a house in 10 years, then we expose ourselves to interest rate risk by investing in a bond of 20-year maturity. In 10 years, its price could have dropped, so that we could lose money by investing in this bond for 10 years.

4. Reinvestment risk is the risk associated with uncertainty about what yield you will get when it comes time to invest payments that you get from owning a bond. In our example of saving to buy a house, there is reinvestment risk because we want to have the money in

10 years. But, even if we buy a bond that matures in 10 years, we don't know exactly how much money we'll have in 10 years. This is because we get coupons before maturity, and we don't know yet at what rate we'll be able to make those coupon payments grow when we reinvest them in the future.

However, it's clear that there's much more reinvestment risk associated with the 5-year bond than the 10-year bond. With the 5-year bond, you'll have to reinvest the principal in 5 years; with the 10-year bond, you'll never have to reinvest the principal because 10 years is your time horizon.

4 Risks in Bond Issuance

Similar issues arise from the bond issuer's perspective, as a borrower.

Suppose our company could pay for the new production line by issuing

- \$10 million of commerical paper with yield 0.2%, maturing in 3 months. (*Commercial paper* is a word for very short-term debt issued by a corporation and paying no coupons.)
- \$10 million of 20-year bonds with yield 5%.

For an issuer, a low yield is good, but financing the new production line by issuing commercial paper creates funding liquidity risk. This is the result of a mismatch between when we have to repay the money and when the investment in a new production line will produce \$10 million to repay our creditors. If we issue commercial paper, we'll have to repay our creditors before we make much money by selling the new product. This means we'll have to borrow money again in 3 months to repay our creditors. This is a risky strategy because perhaps in 3 months, investors will be panicking and will not want to lend us any money, or would demand a very high yield to do so.