01/21/09

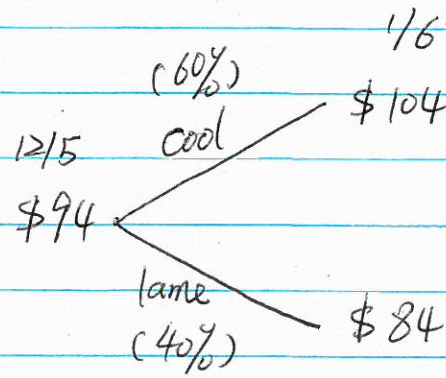
Simulation

Decision Trees

1) Calculate the NPV of your outcomes
2) Combine to get E[NPV] for decision.
3) Choose action with highest E[NPV]

ex. Stock Option

Dec. 5, AAPC $94



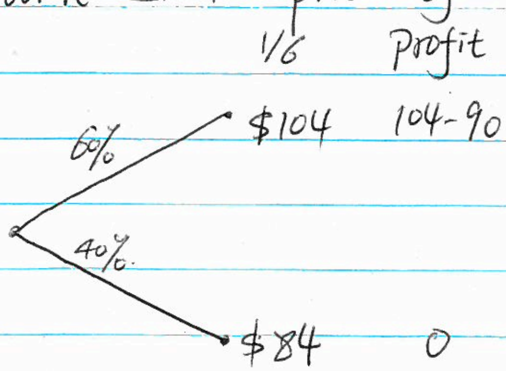
Question: To buy or not to ?

$$E[\text{buy}] = 60\%(104-94) + 40\%(84-94) = 2$$

$$E[\text{NOT buy}] = 0$$

So you should by.

Option with strike price of $90

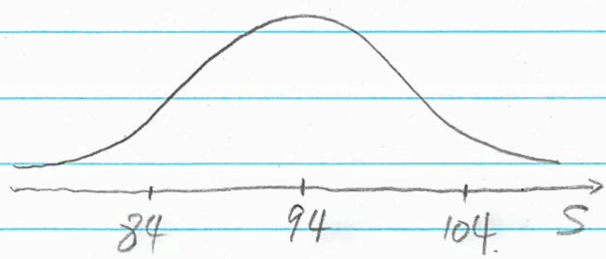


Value of the option is $\begin{cases} S-90 & \text{if } S>90 \\ 0 & \text{if } S<90 \end{cases}$, S is stock price.

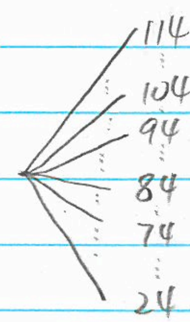
$E[\text{Value}] = 0.6 \times 14 = 8.4$

Now let us assume the stock price is normally distributed, i.e.

$S \sim N(94, 10^2)$



Then we have infinite scenarios on Jan 6th.



We can simulate S from its distribution.

for example: we get

S	option value
89	0
92	2
97	7
95	5
⋮	

The average of all option values.

$\approx E[\max\{S-90, 0\}]$

The standard deviation

$\approx \sigma[\text{option value}]$

Generate random values from Normal distribution in Excel.

NORMINV (RAND(), 94, 10)

For every value you get from this function it is a realized stock price in the future (1/6).

Assume you have got $S_1, \cdots, S_n$ from $N(\mu, \sigma)$.

That means you have n scenarios in the future.

For $\forall\, i=1, \cdots, n$, calculate the option payoff $S_i - K$,

where K is the strike price.

The average of $(S_1-K, \cdots, S_n-K)$

Then you can calculate the sample average of option payoff and the sample standard deviation of option payoff. based on $\begin{pmatrix} S_1-K \\ \vdots \\ S_n-K \end{pmatrix}$

- Another way to generate random numbers in Excel.

  DATA → DATA ANALYSIS → random number generation.

- Simulate stock price via trinominal tree

$$S = S_0 + X_1 + X_2 + \cdots + X_n \qquad (X_i,\ i=1,\cdots n \text{ are random variables})$$

$$X_i = \begin{cases} +1 & \text{with probability } p \\ 0 & \quad " \quad\quad " \quad\quad q \\ -1 & \quad " \quad\quad " \quad\quad 1-p-q. \end{cases}$$

ex. $S = 94 + X_1 + X_2 + \cdots + X_{20}$

$$X_i = \begin{cases} +1 & 35\% \\ 0 & 35\% \\ -1 & 30\%. \end{cases}$$

Generate Normal and Bernoulli random numbers:

$N(\mu, \sigma^2)$: NORMINV(RAND(), μ, σ).

Bernoulli(p): IF(RAND()<p, 1, 0).