Hwk 3 Solutions

1) Suppose you got a job in LA and want to buy a new car. This being LA you narrow your choice to either a Prius (costs \$25k) or a Miata convertible (costs \$20k). The Prius gets roughly 50 mpg (miles per gallon) while the Miata gets 25. Suppose gas is \$2/gal. and you drive 15,000 miles a year. Use a 5% discount rate compounded monthly.

a) Which is the more economical choice assuming either car lasts 10 years? (We're ignoring salvage value, license fees, and repair costs here.)

A: 15,000 miles/yr = 1250 miles/mo. So the Prius uses 1250/50=25gal./mo. while the Miata uses 50gal./mo. Therefore the gas costs are \$50/mo. for the Prius and \$100/mo. for the Miata. The NPV (net present value) of these gas savings is (using Excel) $= PV(5\%/12, 10*12, \$100 - \$50) \approx \$4,714.07$. Since this is less than the \$5k additional cost of the Prius, the more economical choice is to buy the Miata.

b) Will you change your decision if the average gas price is \$3/gal?

A: Since the gas price is 50% greater than in part a, the NPV of the gas savings will also be 50% greater or \$7,071.10. Since this exceeds the \$5k additional cost of the Prius, you should change your decision and get the Prius instead.

c) At \$2/gal what is the more economical choice if you decide to sell the car after 5 years for half its original price?

A: The NPV of the gas savings is $= PV(5\%/12, 5*12, \$100 - \$50) \approx \$2,649.54$. If you get the Prius you can sell it for \$2500 more after 5 years (worth $\$2500(1 + 0.05/12)^{-60} \approx \$1948.01$ today). At \$4597.55 together, these savings don't exceed the additional \$5k cost of the Prius. Thus the Miata is the more economical choice.

2) Let Z_i be iid standard normal random variables. What is $\Pr[\frac{1}{30}\sum_{i=1}^{30} Z_i < 0.1]$?

A: $\Pr[\frac{1}{30}\sum_{i=1}^{30} Z_i < 0.1] = \Pr[\mathcal{N}(0, 30) < 0.1 * 30] \approx 0.71$.

3) Let $X_i \sim \text{Bernoulli}(p)$ be correlated random variables such that $\text{corr}(X_i, X_j) = q$ for any $i \neq j$. Let $Z = \sum_{i=1}^{1000} X_i$.

a) Calculate the mean and standard deviation of Z.

A: $\text{E}[Z] = \sum_{i=1}^{1000} \text{E}[X_i] = 1000p$.

Note that $\text{Var}[X_i] = p(1-p)$. So $\text{Cov}(X_i, X_j) = q\sqrt{\text{Var}[X_i]\,\text{Var}[X_j]} = p(1-p)$ for $i \neq j$. Therefore, $\text{Var}[Z] = \sum_{i=1}^{1000}\sum_{j=1}^{1000} \text{Cov}(X_i, X_j) = 1000\,\text{Var}[\text{Bernoulli}(p)] + (1000^2 - 1000)qp(1-p) = 1000p(1-p)(1+999q)$. So the standard deviation of Z is $\sqrt{1000(1+999q)}\sqrt{p(1-p)}$.

b) What is the standard deviation of Z if $q = 0$? if $q = 1$?

A: If $q = 0$, then the standard deviation is $\sqrt{1000}\sqrt{p(1-p)}$ and if $q = 1$ it is $1000\sqrt{p(1-p)}$.