Hwk 1: Math Review
Solution

Let a_1, a_2, a_3, ... be a sequence of numbers and assume that $a_1 = 1$.
1) If $a_{n+1} = 1.03a_n$ for all $n \geq 1$, then what is $\sum_{i=1}^{30} a_i$?
A: Clearly $a_n = 1.03^{n-1}$ for $n \geq 2$. Applying the formula for a geometric series,

$$\sum_{i=1}^{30} a_i = \sum_{i=1}^{30} 1.03^{i-1} = \sum_{i=0}^{29} 1.03^i = \frac{1 - 1.03^{30}}{1 - 1.03} = 47.58\ldots \tag{1}$$

2) If $a_{n+1} = \frac{a_n}{1.03}$ for all $n \geq 1$, then what is $\sum_{i=1}^{\infty} a_i$?
A: Clearly $a_n = (1/1.03)^{i-1}$. Applying the formula for an infinite geometric series,

$$\sum_{i=1}^{\infty} a_i = \sum_{i=1}^{\infty}(1/1.03)^{i-1} = \sum_{i=0}^{\infty}(1/1.03)^i = \frac{1}{1 - \frac{1}{1.03}} = 34.33\ldots \tag{2}$$

3) If $a_{n+1} = 1.03a_n - 0.05$ for all $n \geq 1$, then what is a_{30}?
A: If we calculate the first few terms of the sequence, $a_2 = 1.03a_1 - 0.05$, $a_3 = 1.03a_2 - 0.05 = 1.03^2 a_1 - 1.03(0.05) - 0.05$, etc., then it is not hard to see that

$$\begin{aligned} a_n &= 1.03^{n-1}a_1 - 0.05 - 0.05(1.03) - 0.05(1.03^2) - \cdots - 0.05(1.03^{n-2}) \\ &= 1.03^{n-1}a_1 - 0.05\sum_{i=0}^{n-2} 1.03^i. \end{aligned} \tag{3}$$

Now applying the rule for a geometric sum,

$$a_{30} = 1.03^{29} - 0.05\sum_{i=0}^{28} 1.03^i = 1.03^{29} - 0.05\frac{1 - 1.03^{29}}{1 - 1.03} = 0.10\ldots \tag{4}$$