Math Handout 1
Summation Notation and Geometric Series

Let a_1, a_2, a_3, ... be a sequence of numbers. Then

$$\sum_{j=3}^{8} a_j = a_3 + a_4 + a_5 + a_6 + a_7 + a_8. \quad (1)$$

For example, if $a_i = (i+2)^2$, then

$$\begin{aligned}\sum_{k=3}^{6}(1 - a_{k-1}) &= (1 - a_{3-1}) + (1 - a_{4-1}) + (1 - a_{5-1}) + (1 - a_{6-1}) \\ &= (1 - a_2) + (1 - a_3) + (1 - a_4) + (1 - a_6) \\ &= (1 - (2+2)^2) + (1 - (3+2)^2) + (1 - (4+2)^2) + (1 - (6+2)^2) \quad (2)\end{aligned}$$

We can also have infinite sums. For example,

$$\sum_{i=0}^{\infty} a_j = a_0 + a_1 + a_2 + \cdots \quad (3)$$

For geometric sums we have special formulas. Suppose $a_i = b^i$ where b is some constant. Then

$$\begin{aligned}\sum_{i=j}^{k} a_i =& a_j + a_{j+1} + \cdots + a_k \\ =& \sum_{i=j}^{k} b^i = \frac{b^j - b^{k+1}}{1-b}.\end{aligned}$$

We have formulas for the values of infinite geometric sums when $-1 < b < 1$

$$\sum_{i=j}^{\infty} b^i = \frac{b^j}{1-b}. \quad (4)$$