MSE 444 Final Presentation

Implied Volatility Modeling

M. Günhan Ertosun, Sarves Verma, Wei Wang

Advisors: Prof. Kay Giesecke, Benjamin Ambruster

Outline

- Brief Literature Review
- Data and Methodology
- Results
- Interpretations
- Conclusions

Introduction

- **Four Different Ways to model :**
  - Using a Deterministic Volatility Function (DVF) used by Derman[1], Dupire[2]
  - Using Stochastic Volatility Model such as in Hull-White[3]
  - Using factor based models constructed using time dependent parameters such as Rama Cont. et. al[4] which used O-U process
  - Using empirical statistical techniques to fit data and then use PCA (principal component analysis) to understand the dynamics (as in Roux et.al[5])

[1] E. Derman, I.Kani "Stochastic Implied Trees: Arbitrage Pricing with Stochastic Term and Strike Structure of Volatility", International Journal of Theoretical and Applied Finance, 1998
[2] B. Dupire, " Pricing with a smile", RISK, 1994
[3] J Hull, A White, "The pricing of options on assets with stochastic volatilities", Journal of Finance, 1987
[4] R Cont, J. Fonseca, V Durrleman "Stochastic models of implied volatility surfaces, Economic Notes, 2002.
[5] M.L.Roux, "A long term model of the dynamics of the S&P 500 Implied Volatility Sufeace", working paper ING institutional markets.

Data & Methodology

- Use S&P 500 index options (daily data) from June 2000-June 2001
- Sort Data:
  - All options with less than 15 days of maturity were ignored as they result in high volatility.
  - Data values with call prices less than 10 cents were also ignored.
  - Average value of ask & bid price was taken to represent the call price.
  - All call prices which were less than the theoretical value (calculated using Black-Scholes) were ignored for arbitrage reasons
- Divide the data into:
  - Moneyness Buckets (**New!**)
  - Maturity Buckets (Skiadoupoulos et.al[6])
- Model Implied Volatility by incorporating both maturity & moneyness (**New!**)
- Ultimately, answer the following question:
- Which Principal component is important for different regimes of moneyness & maturity

	Out of Money	At the money	In the Money
Short Term Maturity (8-30 days)	?	?	?
Medium Term Maturity (60-90 days)	?	?	?
Long term Maturity (150-250 days)	?	?	?

G. Skiadopoulos, S.Hodges, L.Clewlow, " The Dynamics of the S&P 500 Implied Volatility Surface", Review of Derivative Research, 1999

Implied Volatility Models

Model 1: $I(m,\tau) = \beta_0 + \varepsilon$

Model 2: $I(m,\tau) = \beta_0 + \beta_1 m + \beta_2 m^2 + \varepsilon$

Model 3: $I(m,\tau) = \beta_0 + \beta_1 m + \beta_2 m^2 + \beta_3 \tau + \beta_4 \tau m + \varepsilon$

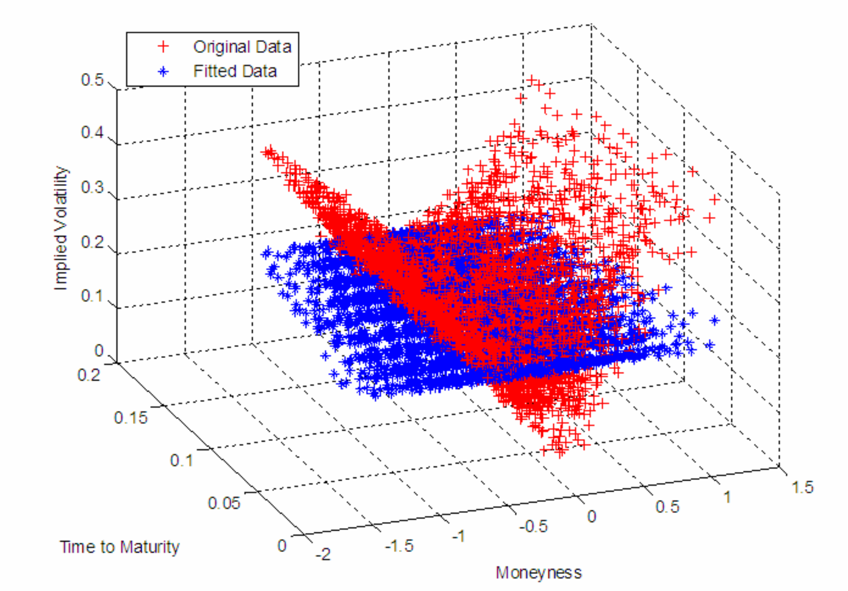
Model 4: $I(m,\tau) = \beta_0 + \beta_1 m + \beta_2 m^2 + \beta_3 \tau + \beta_4 \tau m + \beta_5 \tau^2 + \varepsilon$

➡ Moneyness $m = \dfrac{\log(S_t e^{r\tau_t}/K)}{\sqrt{\tau_t}}$

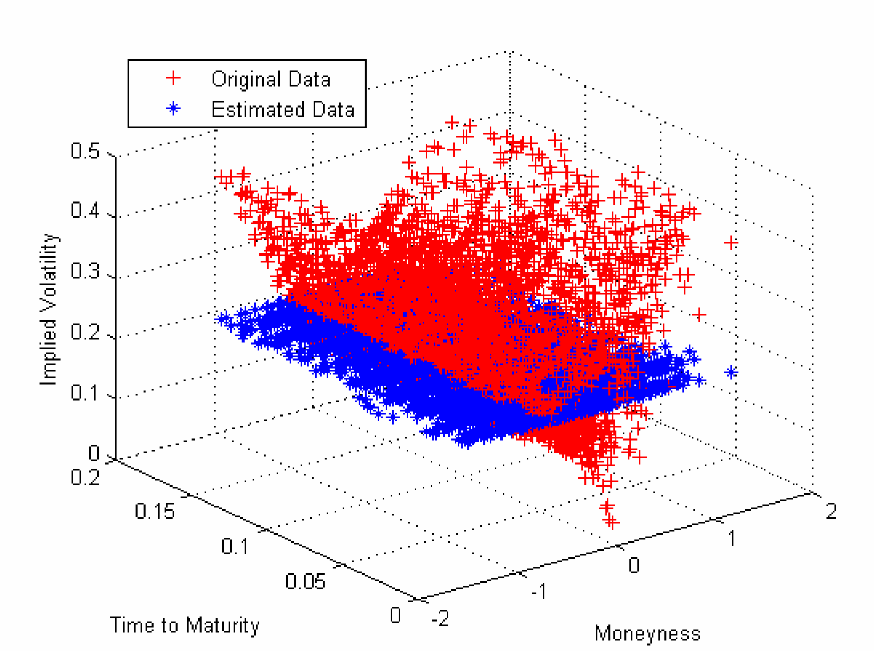
- S&P 500 index options (daily data) from June 2001-June 2002 (ie. Next years') is used to verify our models via out of sample prediction

Model I

In Sample Fit



Out of Sample Prediction

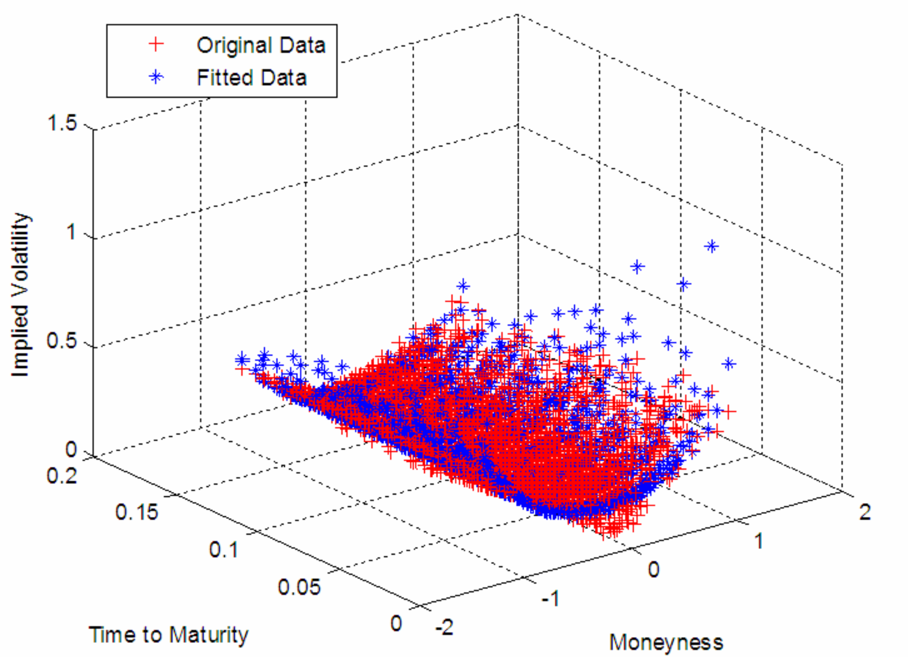


- Black-Scholes like model assuming constant volatility

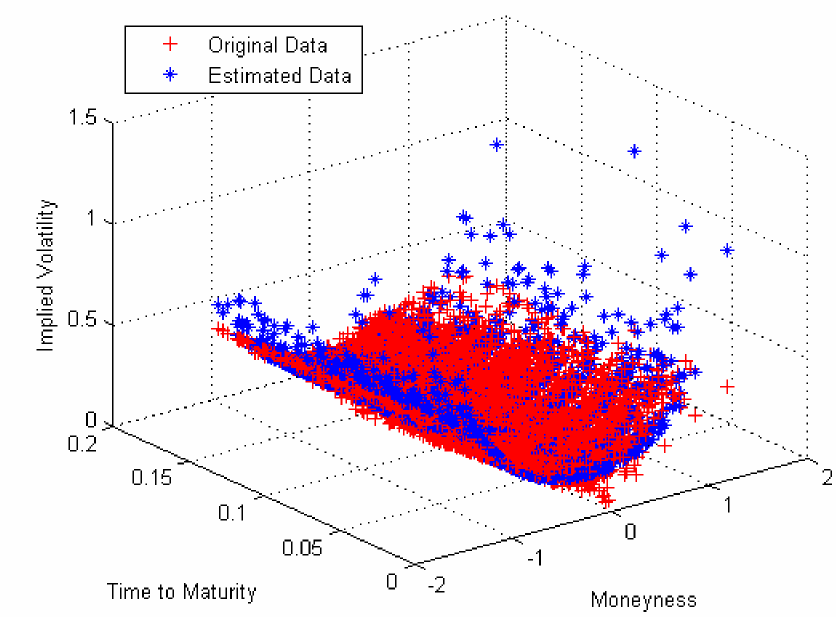
$$\text{Model 1:} \qquad I(m,\tau) = \beta_0 + \varepsilon$$

Model II

In Sample Fit



Out of Sample Prediction

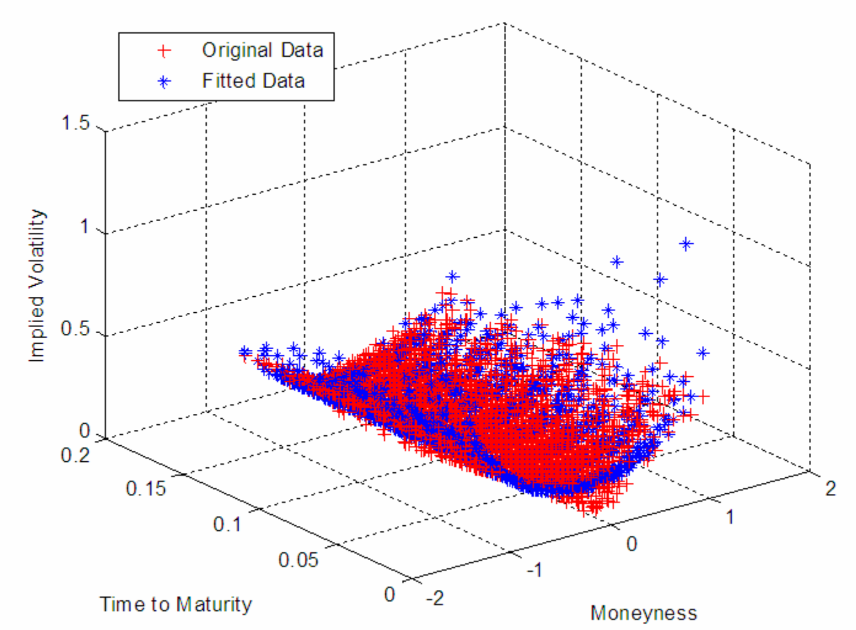


- Model accounting for slope & curvature of moneyness

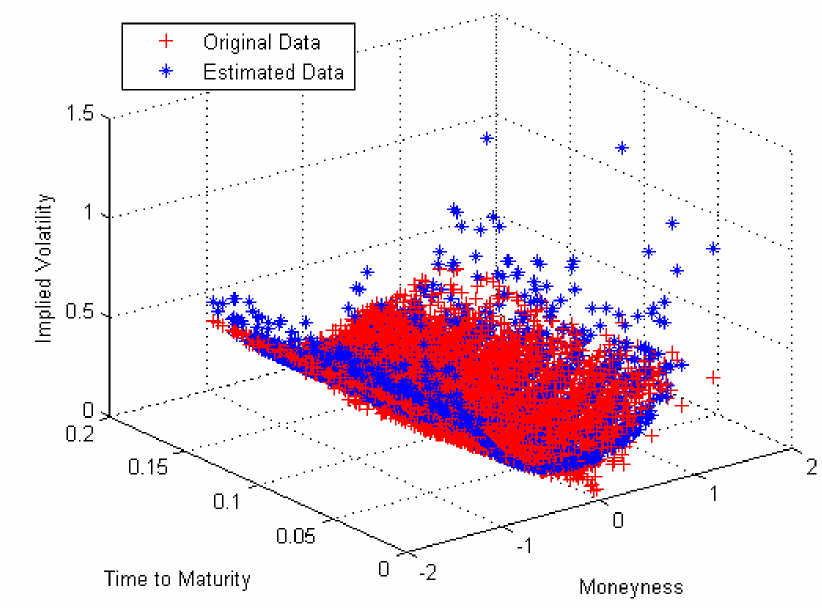
Model 2: $I(m, \tau) = \beta_0 + \beta_1 m + \beta_2 m^2 + \varepsilon$

Model III

In Sample Fit



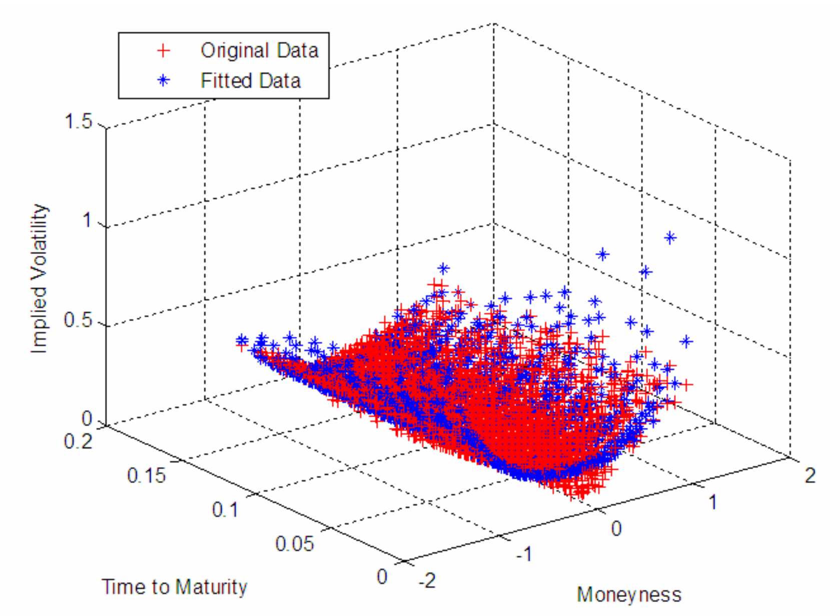
Out of Sample Prediction

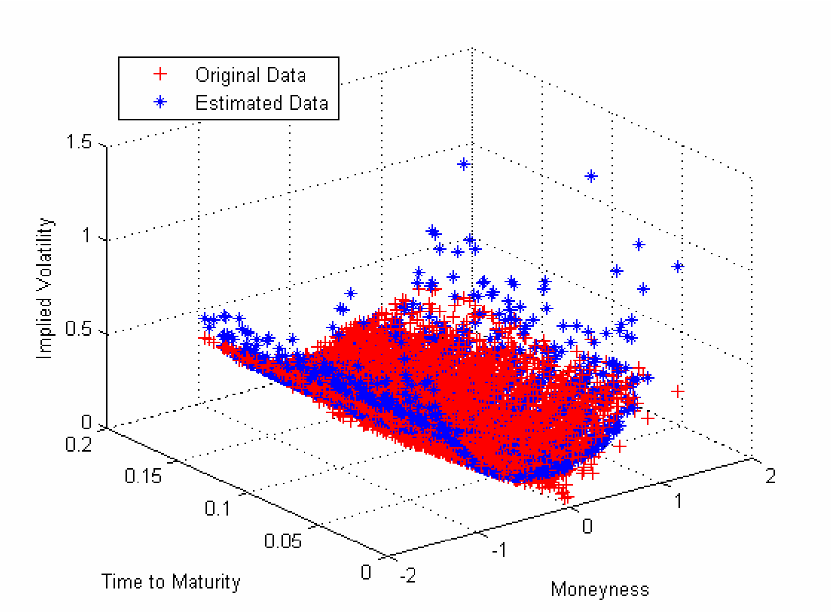


- This model takes in account, the slope contribution of maturity as well as mixed contribution from maturity & moneyness

$$\text{Model 3:} \qquad I(m,\tau) = \beta_0 + \beta_1 m + \beta_2 m^2 + \beta_3 \tau + \beta_4 \tau m + \varepsilon$$

Model IV





- This model takes in account, the slope contribution of maturity as well as mixed contribution from maturity & moneyness

Model 4: $$I(m,\tau) = \beta_0 + \beta_1 m + \beta_2 m^2 + \beta_3 \tau + \beta_4 \tau m + \beta_5 \tau^2 + \varepsilon$$

Comparison of In Sample Vs Out of Sample Prediction

	β_0	β_1	β_2	β_3	β_4	β_5	RMSE (In Sample) (Fitting)	RMSE (Out of Sample) Prediction
Model I	-1.4876						0.3033	0.3362
Model II	-1.6352	0.2702	0.8836				0.1805	0.2001
Model III	-1.6244	0.2504	0.8779	-0.1208	0.2565		0.1802	0.1999
Model IV	-1.6108	0.2538	0.8783	-0.5613	0.2202	2.5269	0.1801	0.1998

PCA (Principal Component Analysis)

PCA on Moneyness Bucket

Moneyness of Call Option (in %)	1st PC (in %)	2nd PC (in %)	3rd PC (in %)	Total explained Variance by 1st three PCs (in %)
m<-1	51.561	39.379	9.0596	100
-1<m<-0.5	50.548	26.729	11.646	88.923
-0.5<m<0	45.248	23.932	18.656	87.836
0<m<0.5	50.017	19.536	16.346	85.899
0.5<m<1	37.732	24.999	22.1	84.831
m>1	62.871	23.417	10.996	97.284

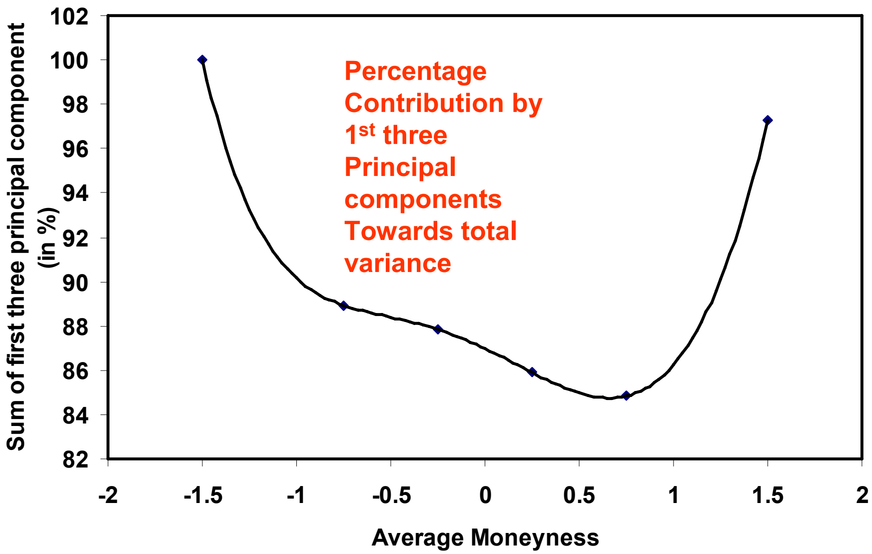
- Moneyness= $m = \ln(\frac{S_t e^{r\tau}}{K}) / \sqrt{\tau}$

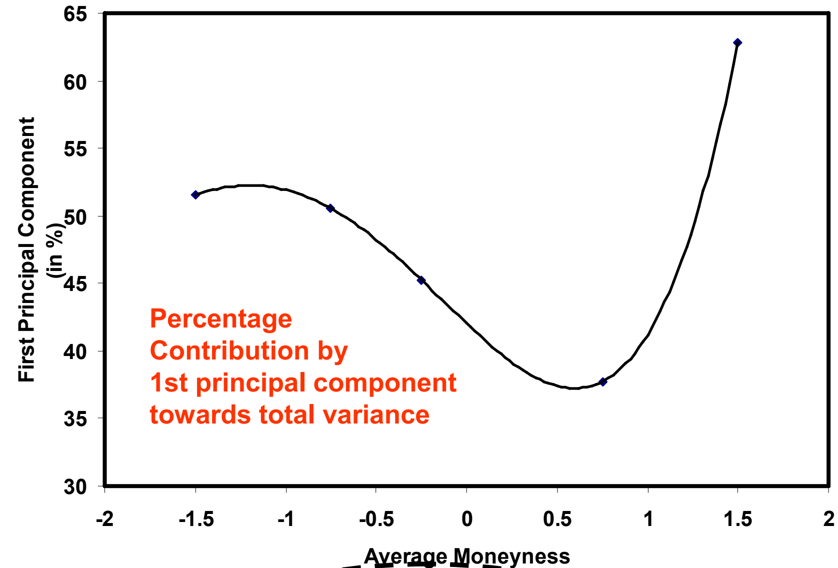
PCA on Maturity Bucket

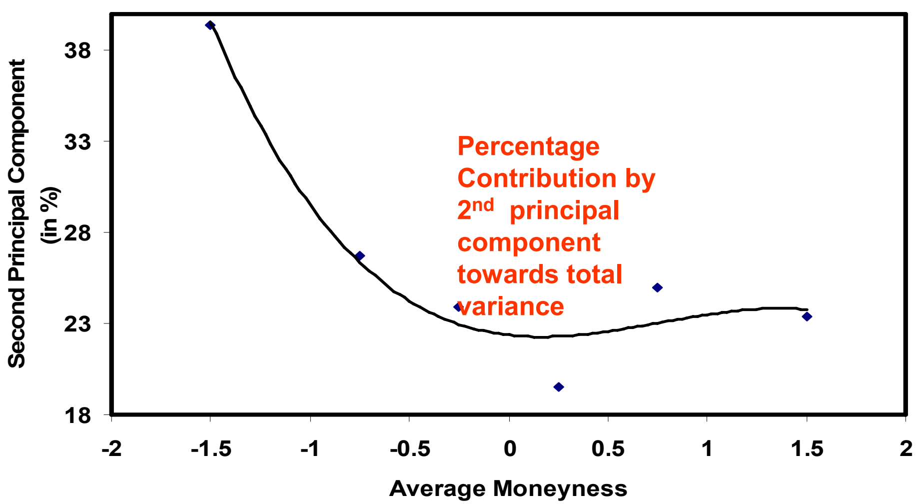
Maturity of Call Option	1st PC (in %)	2nd PC (in %)	3rd PC (in %)	Total explained Variance by 1st three PCs (in %)
15-30	56.929	21.359	12.072	90.41
30-60	69.426	15.266	10.496	95.188
60-90	88.71	5.41	2.79	96.92
90-150	81.419	10.712	7.2489	98.83
150-250	77.38	15.55	4.58	97.5

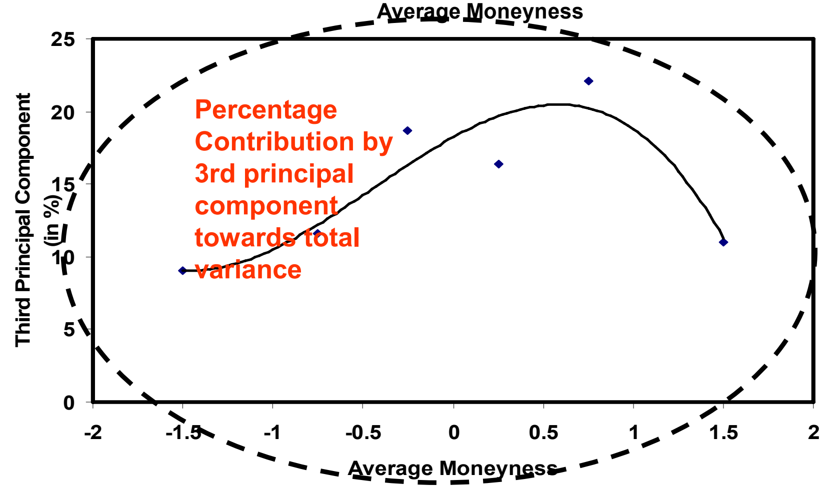
- For short term maturities: All three PCs important.
- For long term maturities: Only the first PC most important

PCA Analysis on Moneyness Bucket



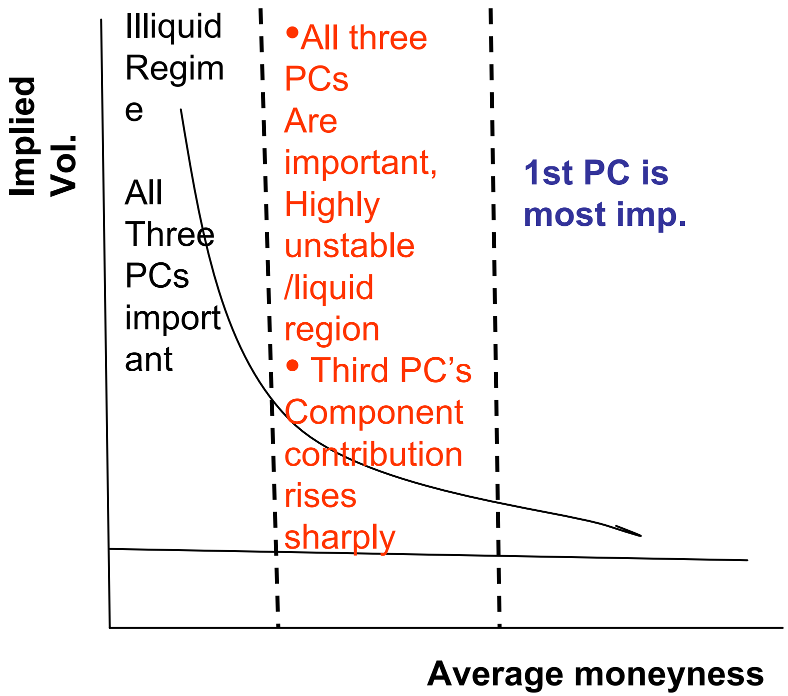






Novel Way of Option Hedging

Summary of PCA & Model



Model Constructed

$$Log(I) = \eta_0 + \varepsilon$$

$$Log(I) = \eta_0 + \eta_1 m + \eta_2 m^2 + \varepsilon$$

$$Log(I) = \eta_0 + \eta_1 m + \eta_2 m^2 + \varepsilon$$

$$Log(I) = \eta_0 + \eta_1 m + \eta_2 m^2 + \eta_3 \tau + \eta_4 \tau m + \varepsilon$$

- Incorporates both Maturity & Moneyness
- R^2 & RMS taken to check for accuracy
- The model fitting is sensitive to data sampling

Observations:

- At the money regime most sensitive; hence 1st three principal components not sufficient
- 'In the money' Regime, 1st PC most important
- 'Out of Money' Regime, All three PCs Important
- Note both out of money & In the money options are illiquid

Conclusions

- Developed an Implied Volatility model on S&P 500 Index options (from June 2000-June 2001)
- The model incorporated slope and curvature of moneyness and maturity
  - Incorporating maturity (slope and curvature) does not improve the model appreciably
- Out of sample prediction shows good matching with our model
  - The coefficients change with time, however, for a shorter to medium horizon they are pretty constants
- PCA analysis was done on moneyness & maturity (see Clewlow 1999) buckets
  - We observed that the three components (corresponding to moneyness buckets) are significant enough & have shapes confirming our intuitional understanding
- The shapes of different principal components are important to develop hedging strategy