Asset Management Strategies:

Fat Tails and Risk Control

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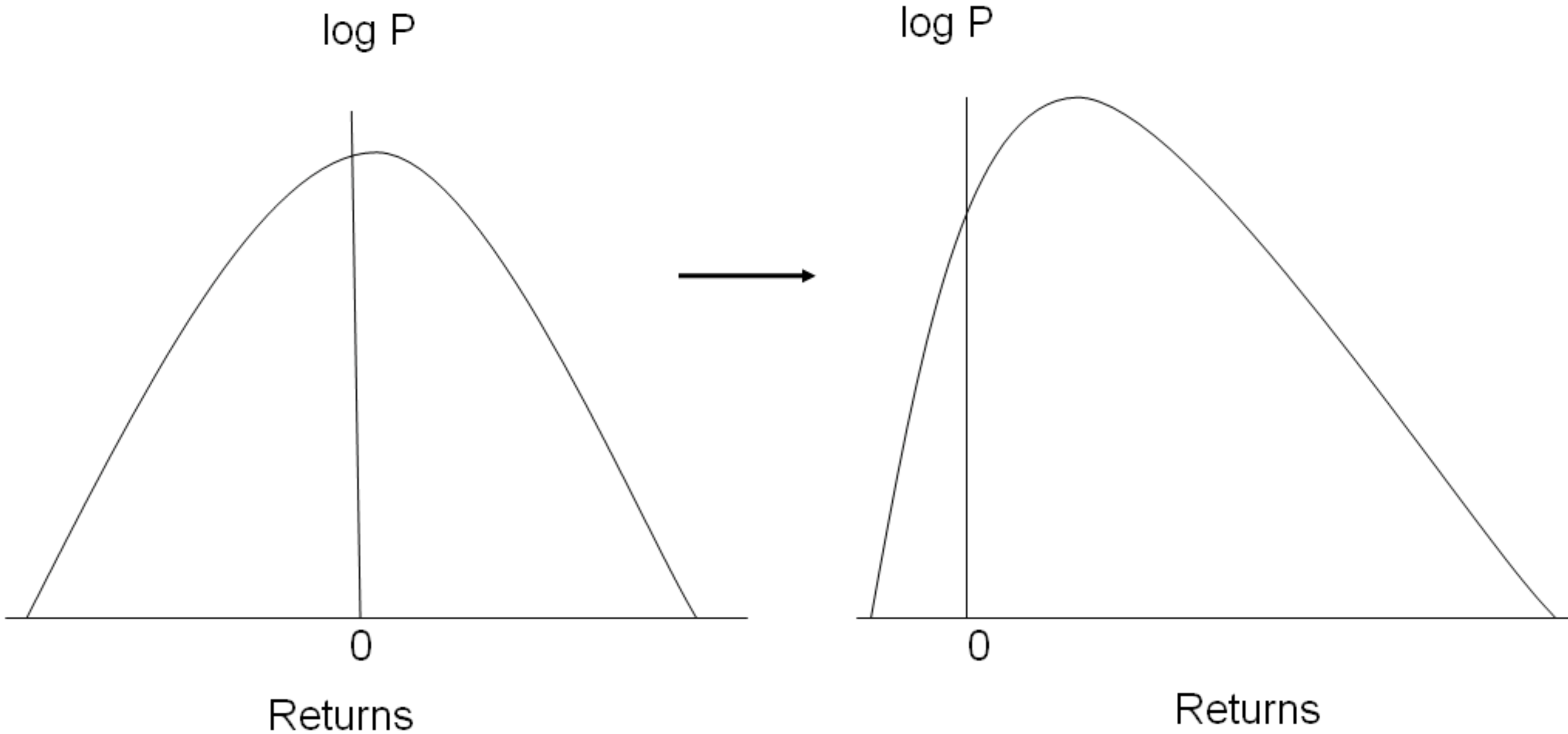
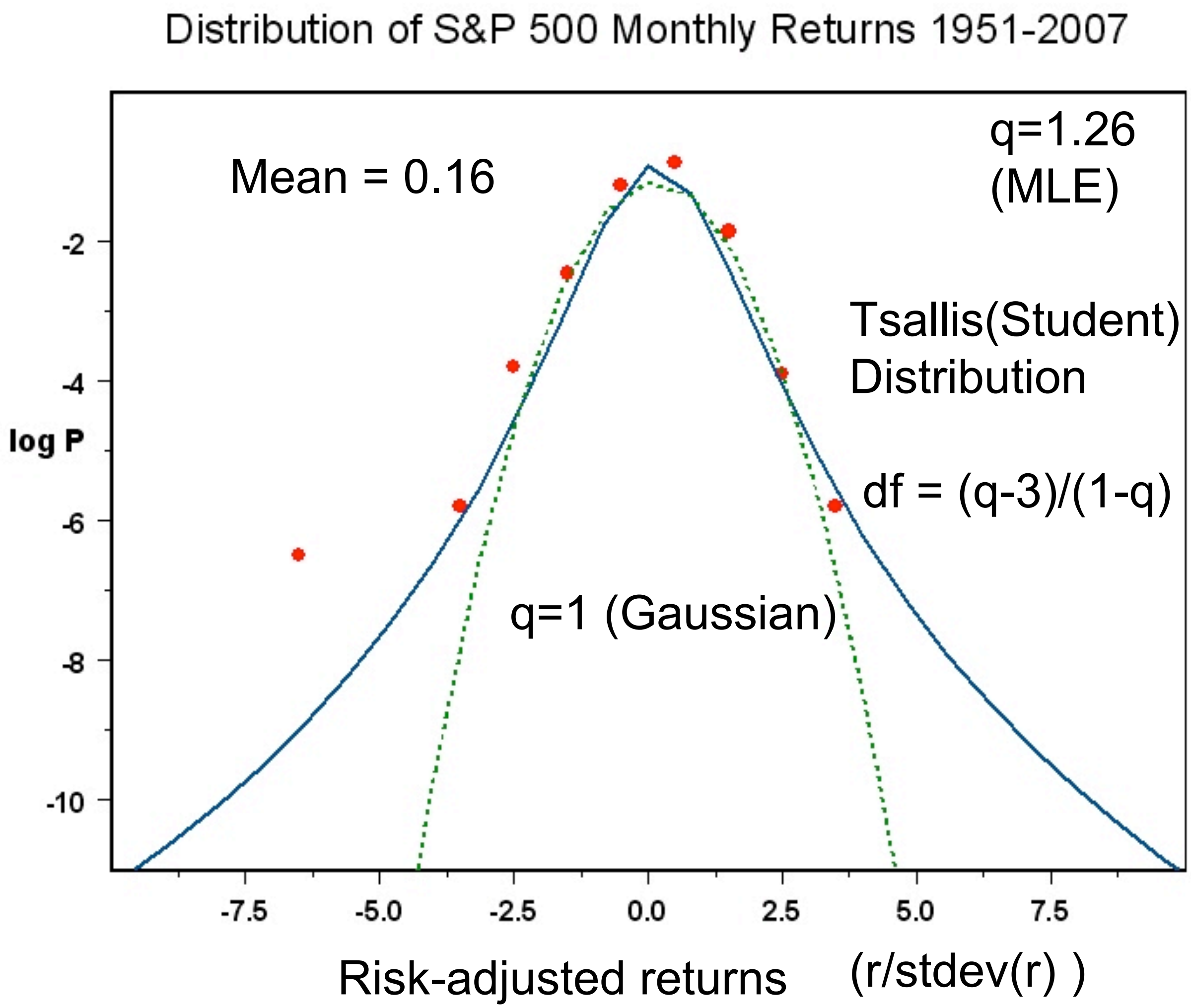
Layout

- Hedge Fund Returns: Distributional Characteristics
- How to Calculate Robust VaR Numbers
- Portfolio Construction in the presence of fat tails



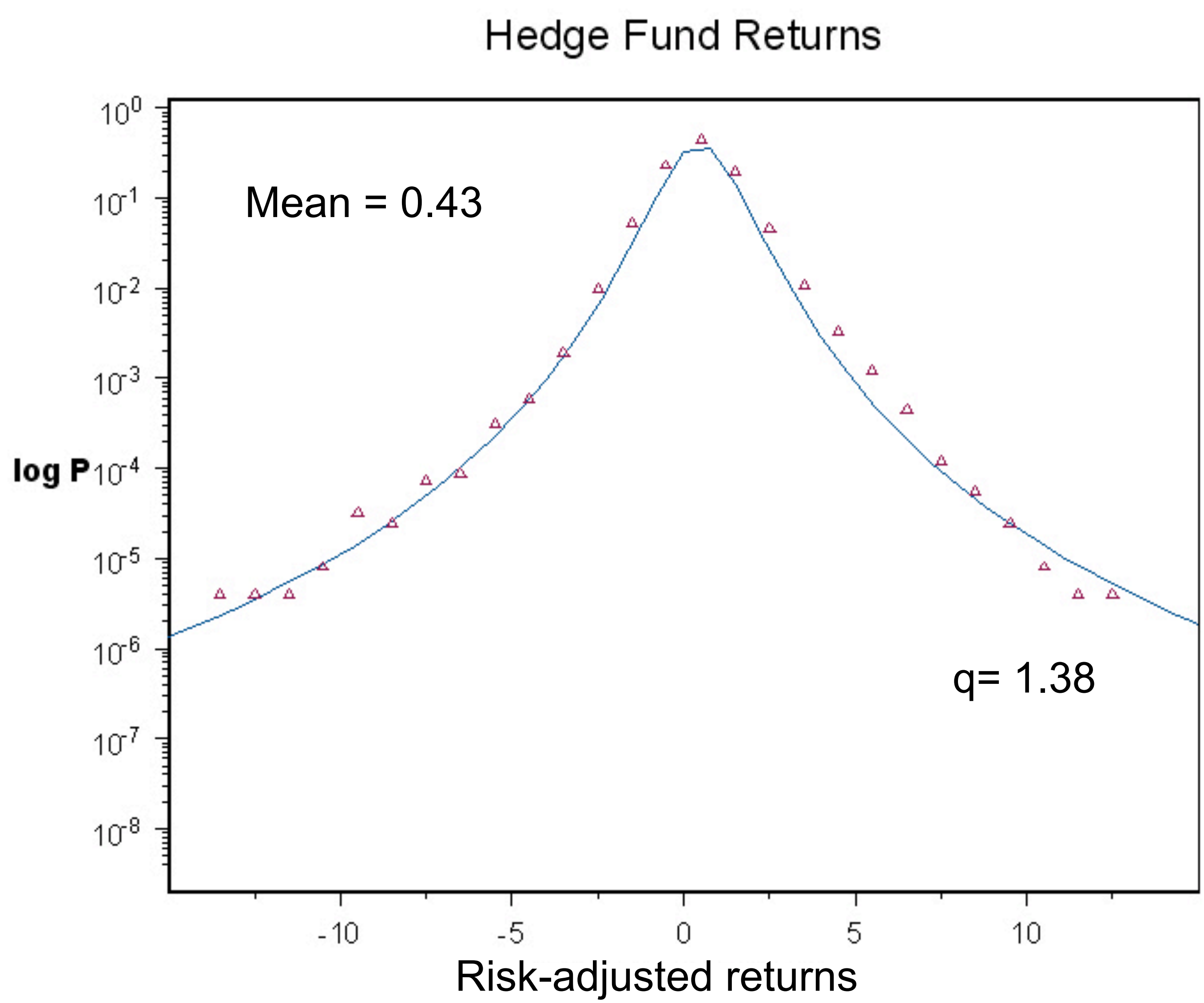
Ideally: A trading strategy transforms underlying asset return distribution favorably



Tsallis distribution: Fits well to daily returns also with q = 1.4.

Used for non-Gaussian option pricing [Borland 2002, Borland &Bouchaud 2004].

Lipper TASS Database: 2883 Funds, 1300 Funds of Funds, Monthly Returns



- Hedge Fund managers do shift the mean from 0.16 to 0.43

- Tails are much fatter, monthly returns well-fit by q=1.4

- The “ideal” distribution (small left tail) is not achieved, but also no significant negative skew

- $q = 1.3 – 1.5$ fits well to hedge fund monthly returns

- How can we use this for

  - Risk Control
  - Portfolio Construction

VaR

- 5% VaR: You have a 5% chance of getting returns less than VaR (per $)
- Common calculation methods:

  - Assume a distribution (eg. Gaussian)

  - Use the past N days historical price changes

  - Use MC simulations of future returns

VaR

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- Common calculation methods:

  - Assume a distribution (eg. Gaussian)

    Can't be good! Fat tails!

  - Use the past N days historical price changes

    Simple. Can we do better?

  - Use MC simulations of future returns

    Very compute-intensive!

Robust Calculation of VaR

An experiment:

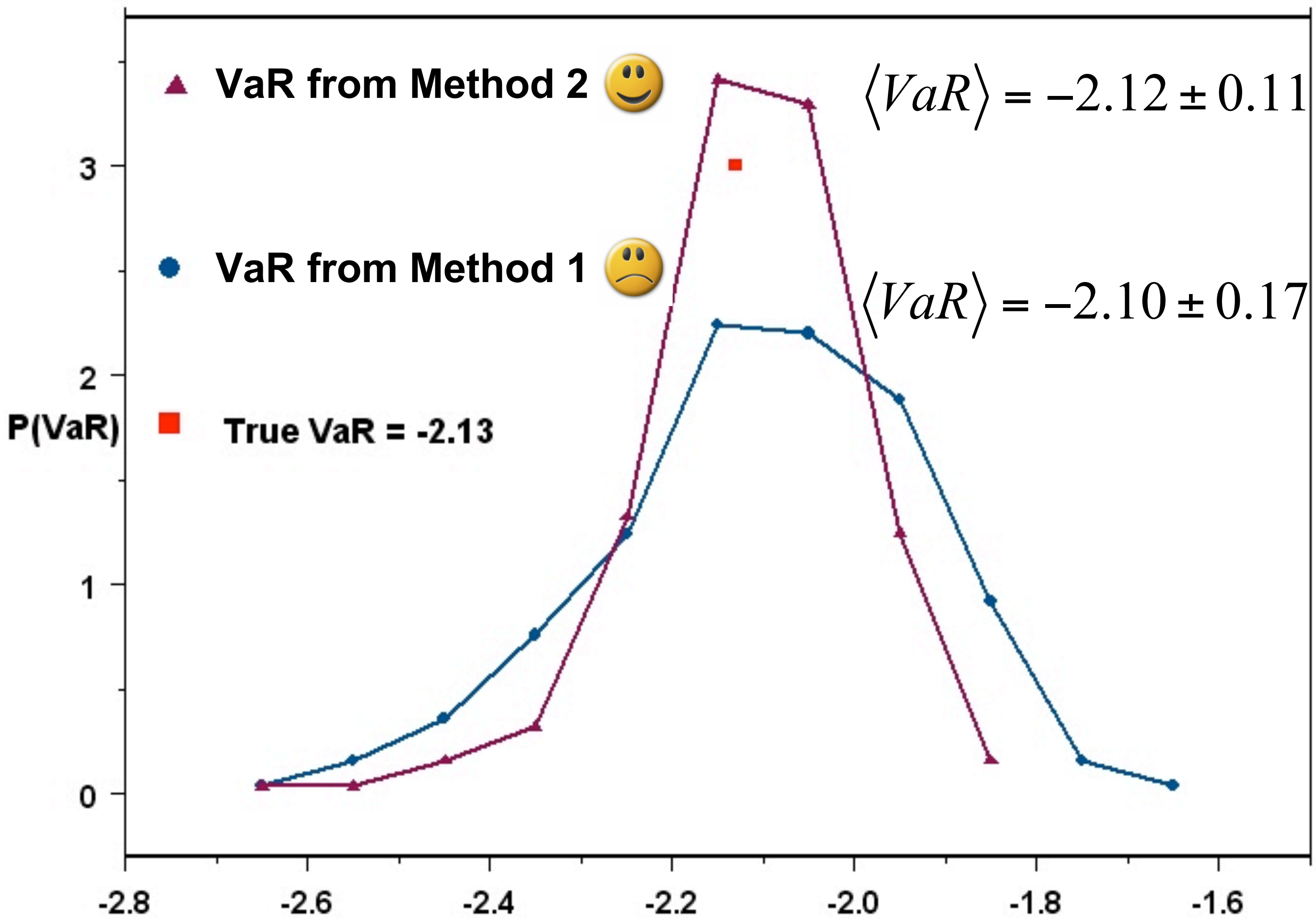
- Simulate 500 returns drawn from q = 1.4 Tsallis distribution. Repeat 250 times.

- For each sample:

Method 1: Estimate 5%-ile from 500 day generated data → 250 values of VaR.

Method 2: Fit Tsallis distribution of index q to 500 day generated data. Then calculate 5%-ile of that fitted distribution → 250 values of VaR.

Robust Caculation of VaR (Example 5%)



VaR from 250 runs each of length 500

- Fitting Tsallis distribution to data and then calculating VaR → More robust estimate
- Using q=1.4 is a better prior than the Gaussian distribution
- Better than unconditional VaR using historical data (recent history might be anomalous)

Portfolio construction in the presence of fat-tails

- Single strategy case:

How to calculate optimal holdings?

One strategy is:

Maximize expected long-run profit based on log-utility function (Kelly criterion)

$$\left\langle \log(1+hx) \right\rangle_{P(x|\mu,\sigma)}$$

h = holding (position size)

μ = expected return

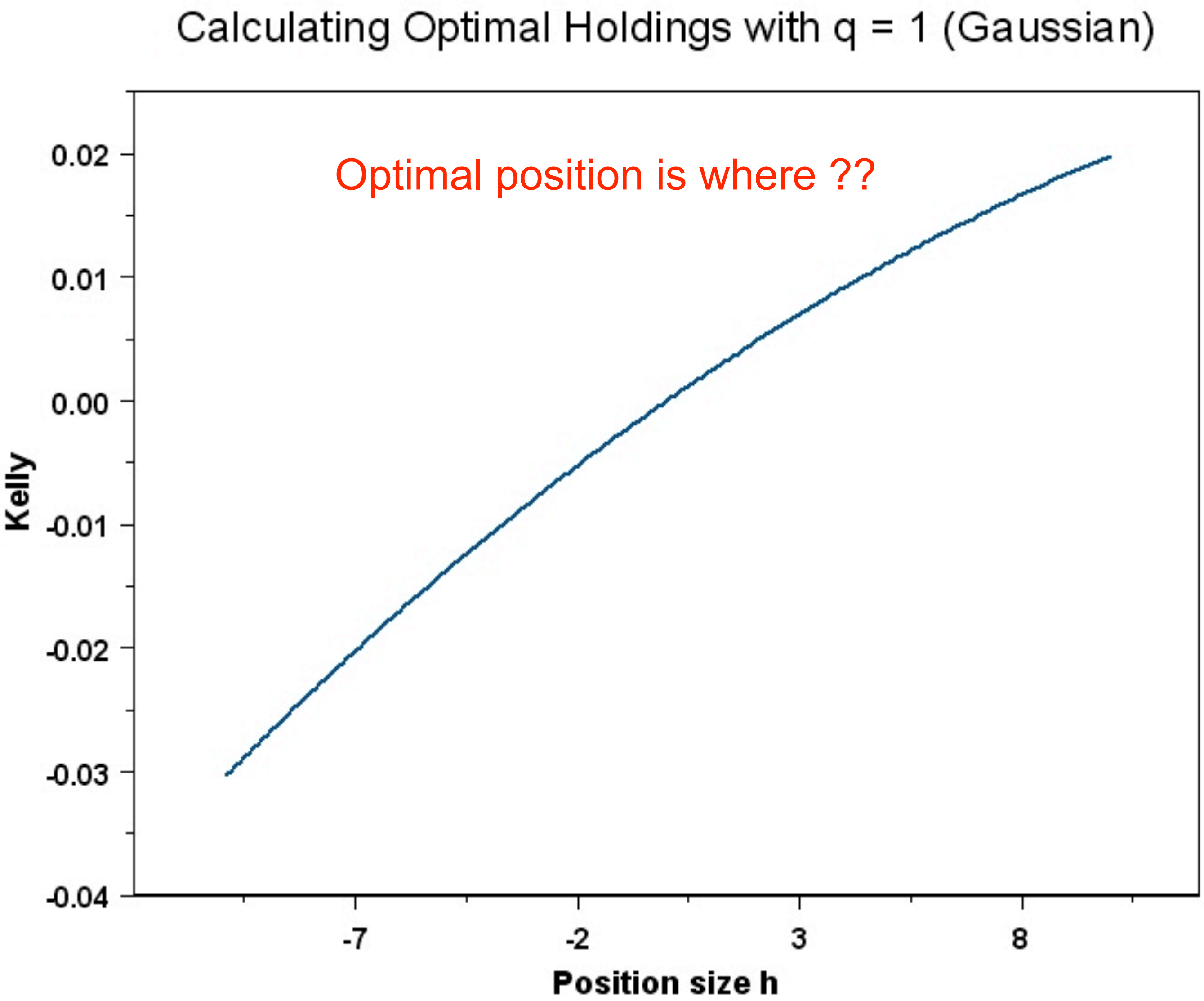
σ = standard deviation (volatility) q-Kelly criterion

$$\int \log(1+hx)\,N\left(1+(q-1)\frac{(x-\mu)^2}{\sigma^2}\right)^{\frac{1}{1-q}} dx$$

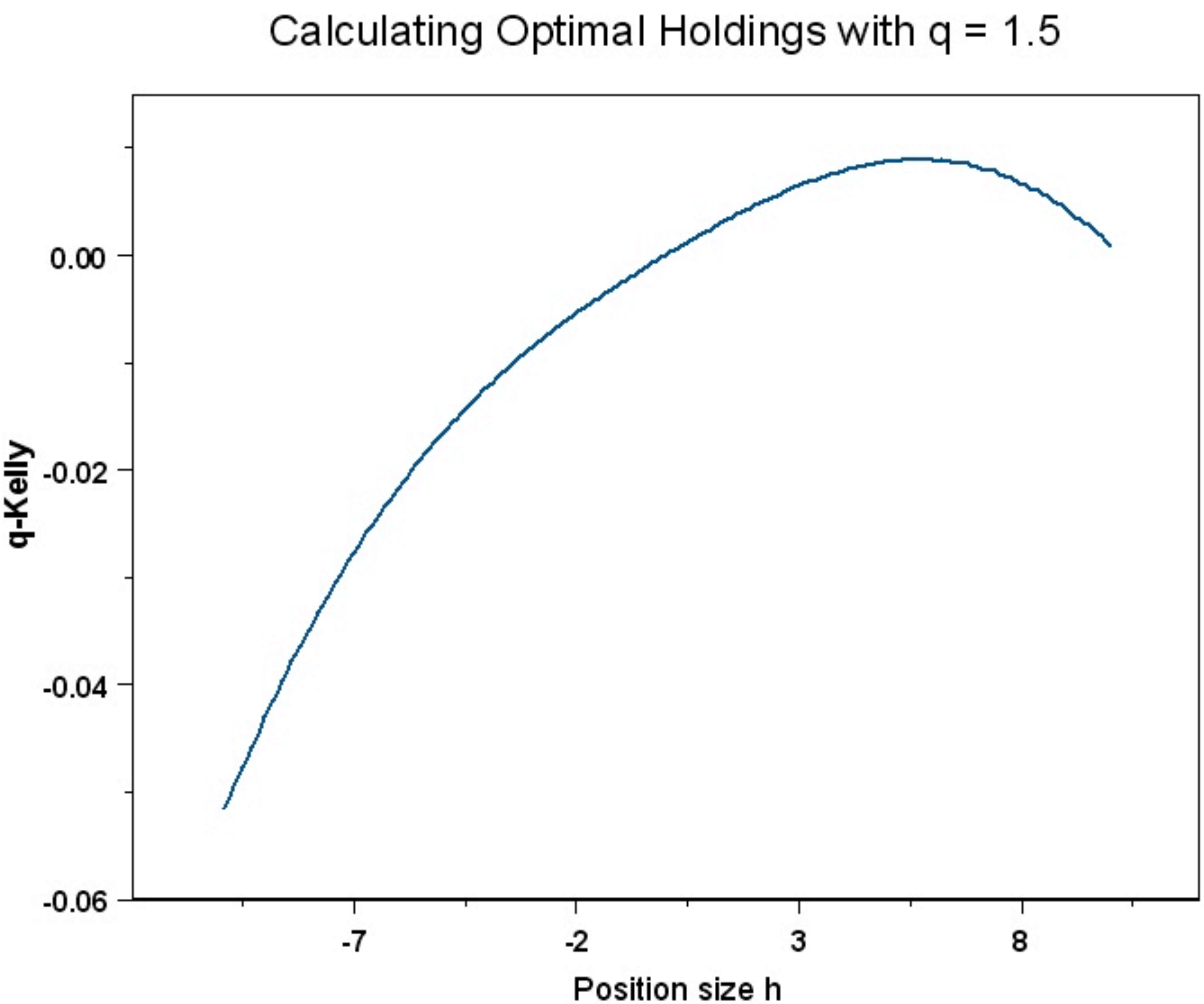
Gaussian, q=1 — Not good –
any slightly positive expected return implies an extremely large position because there is no tail risk

Tsallis, q = 1.5 — Good –
large position sizes are penalized by the tail risk

Example: Daily expected return $\mu = 25$ bp and $\sigma = 1$ %



Example: Daily expected return $\mu = 25$ bp and $\sigma = 1$ %



These portfolios might be optimal, but some investors might not like the high leverage

i) Might not be log-utility maximizers
ii) Might be irrational

- One more ingredient:

- Prospect Theory

(Tversky and Kahneman, Nobel Prize 2002)

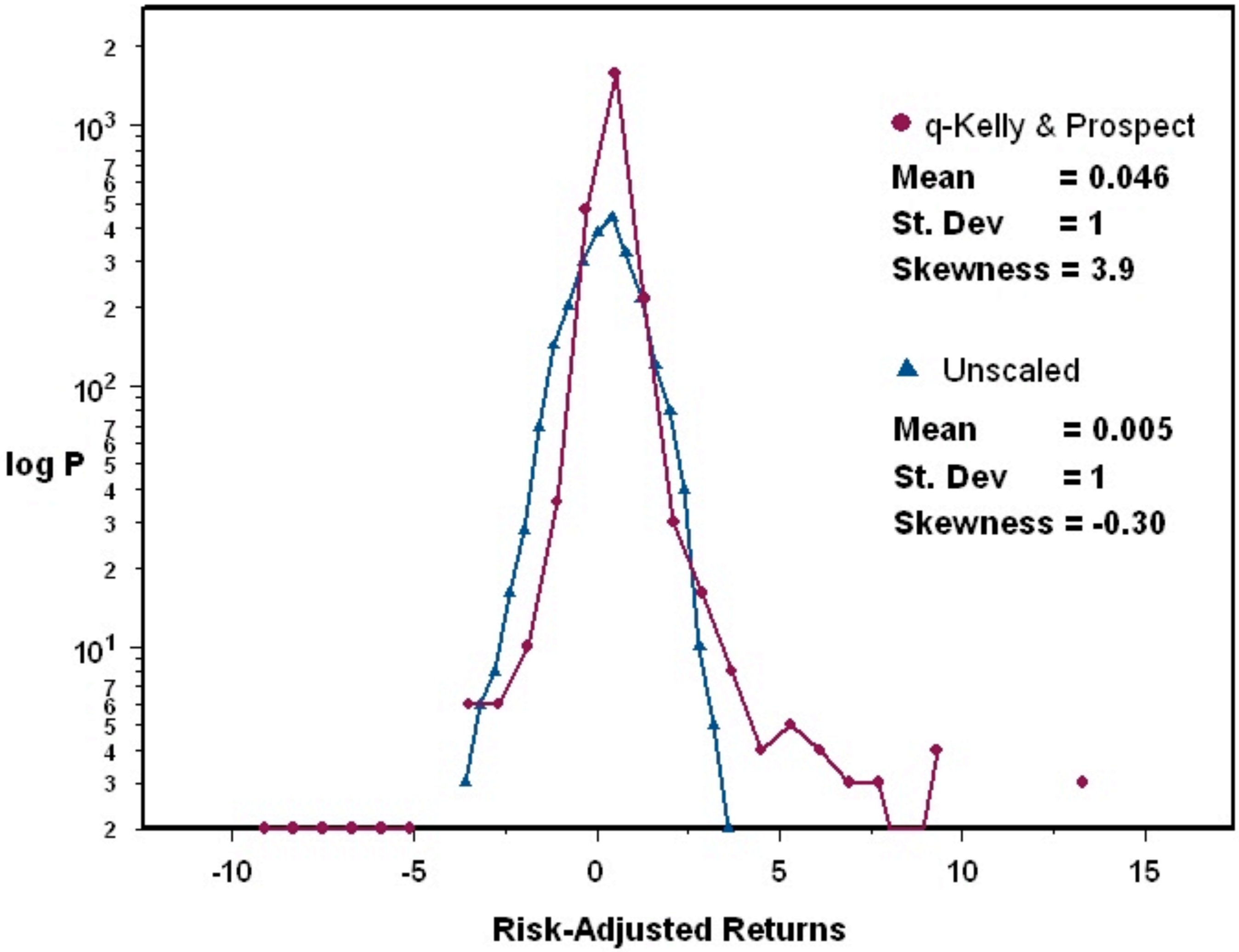
$$\left\langle \log(1+hx) \right\rangle_{P(x|\mu,\sigma)^a}$$

$$a \leq 1$$

Gives even more weight to the tails – incorporates subjective investor fear, not just actual probability of losses

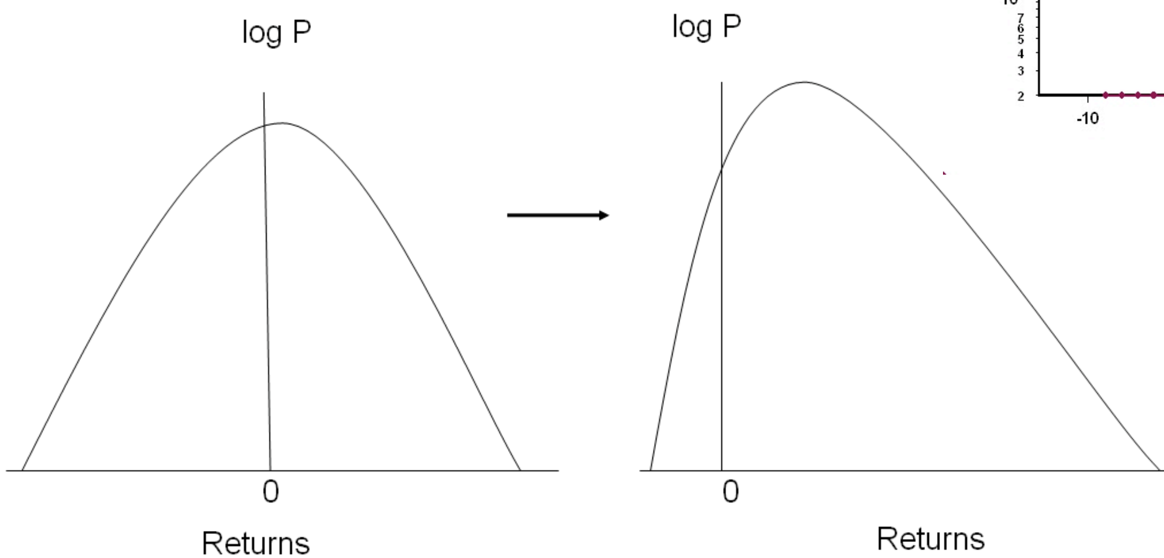
Results using q-Kelly & Prospect Theory   q=1.5, a = 0.8:

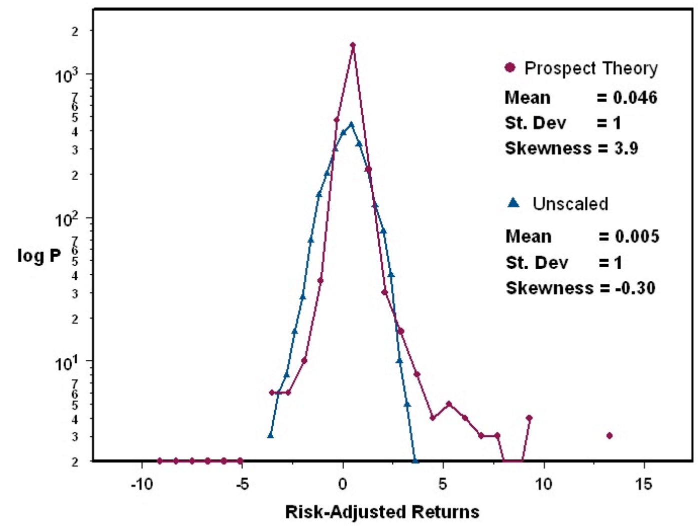
A real trading strategy: returns with and without scaling



Same predictive signal but better risk control → superior returns

Remember our cartoon!





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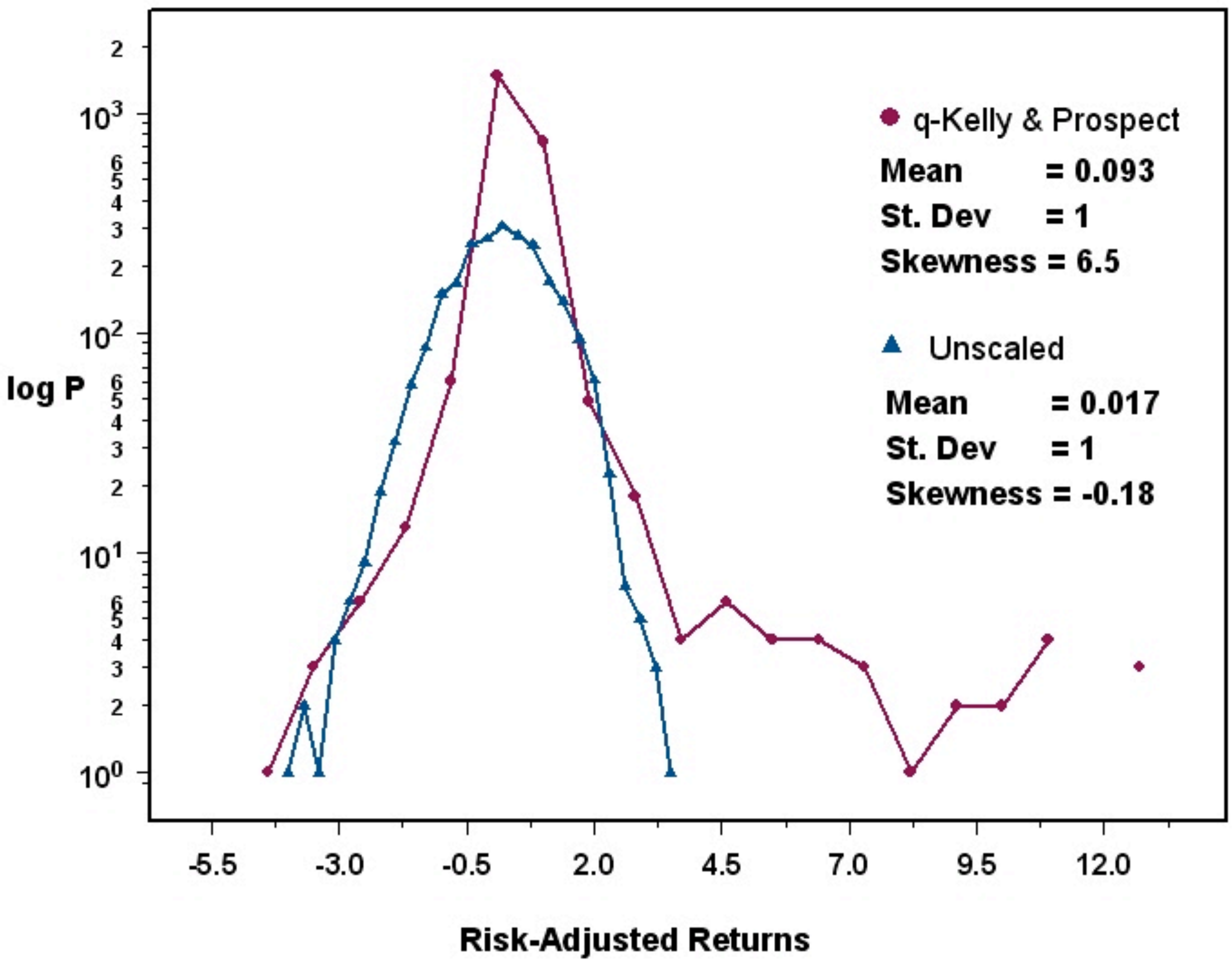
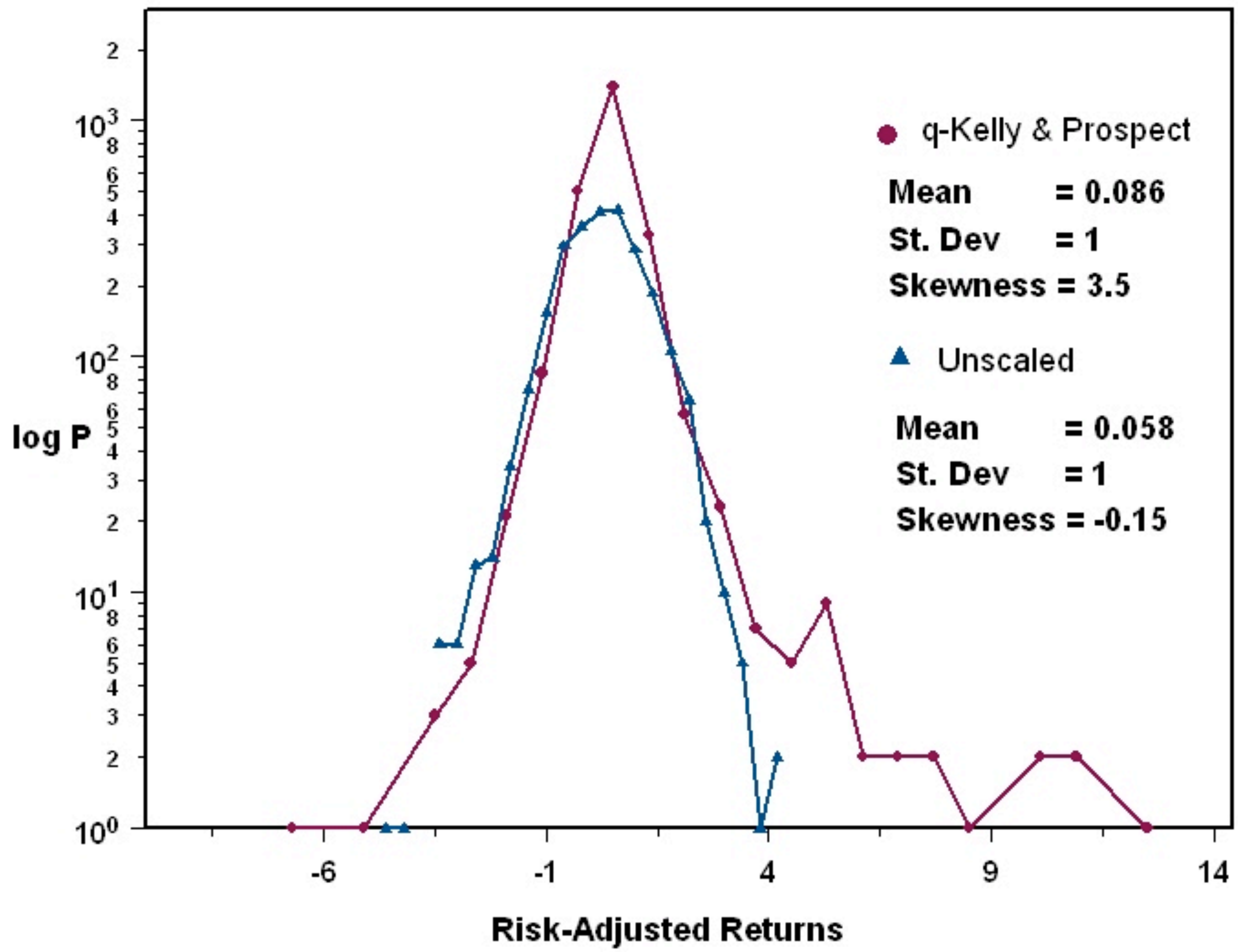


- Multi-strategy case:

Results using q-Kelly & Prospect Theory q=1.5, a = 0.8:

Applied to a multi-strategy portfolio of real trading strategies



- Multi-strategy case:

- Combined strategies in a naive approximation

- Used q-Kelly & Prospect Theory to get leverage rule for whole portfolio

Work still to be done:

- Use q-Kelly & Prospect Theory directly on the multivariate distribution

- Incorporate asymmetry between profit seeking and loss aversion.

Conclusions

- Hedge fund monthly returns distributed according to Tsallis distribution with q = 1.4
- This is quite stable across strategy types
- Using q=1.4, more robust VaR numbers can be calculated
- By taking tail risk into account, optimal position sizes can be found that – at least for the strategies studied here – produce more desirable return distributions