

Capital Structure Arbitrage with a Non-Gaussian Asset Model

MS&E444 Investment Practice Project Report

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Abstract

Capital structure arbitrage refers to the practice of exploiting relative pricing inefficiencies in the market by taking long and short positions across different instruments and asset classes of the capital structure of the same firm. This project is a practical investigation into the risk, return and implementation-wise complications of capital structure arbitrage strategies across equity options and credit default swap positions. In specific, we construct and test 4 different strategies, starting from a basic discrepancy threshold strategy to one that looks at convergent momentum. There have been a number of other studies on the profitability of basic capital structure arbitrage strategies (Yu 2005). The key difference to our approach is that we identify mispricings through a model of non-Gaussian asset dynamics based on the Tsallis distribution, and we attempt to identify some general strategy-independent observations by exploring a wider variety of strategies. Our results point toward the need to take a closer look at the dynamics of the spread discrepancies, while demonstrating the difficulties of identifying convergent mispricings.

1. Introduction

Suppose we are observing the stock price and the price of a 3Y European call option of a given firm. Thanks to Black, Scholes, Merton and the numerous developments that have followed in the field of option pricing, we know that if we assume appropriate dynamics for the stock price (e.g. geometric Brownian motion, log Levy), then there is a clearly defined relationship between the price of an option and the price of the underlying stock (the catch is in designating "appropriate" dynamics). Based on this relationship we can keep our options position well hedged by neutralizing first and second degree price, time and volatility sensitivities with positions in the underlying. What about the price of vastly different financial instruments that constitute the capital structure of the same firm? Since both the price (spread) of a 3Y zero coupon bond and the equity of a firm should bear some relation to the financial health of the given company, it seems reasonable to predict there to be a relationship between the price of the two instruments. Suppose that we have established a relationship between the two instruments. This will then enable us to infer the price of one from the other. If we have faith in our model specification and the accuracy of our model parameter estimations, and the theoretical model price differs greatly from the real price observed in the market, our faith implies the belief that the discrepancy will eventually dissipate for the prices to converge. Capital structure arbitrage stems from the question of how one might profit from such situations where relative mispricings have been identified.

Capital structure arbitrage in the traditional sense refers to the practice of exploiting relative mispricings in the market by taking long and short positions across different financial instruments that constitute the capital structure of the same firm. Capital structure arbitrage in recent years has departed from this traditional notion to include trades across instruments contingent on the same firm but not necessarily issued by the firm itself. In particular, the recent surge in interest in the field can be largely attributed to the development of liquid markets for credit derivatives. Traditionally, even with relatively accurate models and correct directional views, investors had difficulties in executing long equity, short debt trades due to liquidity constraints in the debt market. The development of large liquid markets for credit derivatives has facilitated the construction and execution of portfolios that express relative directional views on debt and equity.

The term "capital structure arbitrage" itself is a misnomer in that it is far from our notion of arbitrage in the strict mathematical sense. Discrepancies in theoretical and

market prices may come from a variety of different sources. Model misspecification and errors in parameter estimation are obvious problems but even barring these issues, it may be that there are inherent liquidity differences or unseen systematic risk factors that we are taking on in the heterogeneous instrument markets, that put an intransient wedge between the prices. Even if we do identify genuine mispricings correctly we do not have an accurate gauge on the rate of convergence, and thus we do not know if our position is justified given the capital cost. Since we are applying partial hedges across vastly different instruments, the hedge is not always effective and positions may become extremely risky. Suppose that we have the simple situation where an investor, based on his theoretical model, believes that shareholders have been slower in the uptake of negative information than bondholders. He would then short the stock and long the bond. The life of the strategy lies in convergence. In the ideal situation where both prices inch towards each other he gains on both the position and the hedge. When it was in fact the case that the bondholders were the ones that were overly pessimistic, the hedge still mitigates the losses. However it might be that the model predictions were completely off and the two prices diverge further. In this case losses amount on both the position and the hedge. The hedge not only provides no safeguard but actually exacerbates the situation and can lead to a full loss on the margin. During the course of this project we progressively install measures and refinements to avoid or mitigate the effects of some of the aforementioned difficulties.

The first step towards constructing a capital structure arbitrage strategy is finding the relationship between the two instruments under consideration. In his general empirical overview of the risk and return of basic capital structure arbitrage strategies, Yu (Yu 2005) uses the CreditGrades model based on the Black and Cox first passage, in order to compute the theoretical CDS spreads from equity prices. Hull (Hull, Nelken & White 2003) provides a direct link between CDS spreads and equity option prices by estimating the leverage ratio and asset volatility parameters of the Merton model through the implied volatilities of two equity options. In this project we wish to investigate CDS vs. equity options plays, but in the process of establishing the relationship (computing the theoretical CDS spreads from equity prices) we focus on modeling accurate asset dynamics rather than on the issue of which specific structural model we are to employ. As such we simplify matters by assuming default to be analogous to equity default, occurring when the equity price drops below 95%. However unlike the previous models that are based on geometric Brownian motion, we use the non-Gaussian q-alpha model developed by Lisa Borland et al. at EvA Funds for equity dynamics in order to realistically model the non-Gaussian (positive excess kurtosis and skew) traits observed in the market.

The second section of the paper provides a brief overview of the most salient features of the q-alpha model used in this project for asset dynamics. The third section explains how we use this model to compute the default probabilities from which we calculate the theoretical CDS spreads. The fourth section provides a detailed description of the four strategies that we designed and tested in this project. The first strategy is a basic discrepancy-threshold strategy where trades are put on when the difference between the theoretical and market rates are above a certain threshold and taken off when convergence occurs. In the second and third strategies we rank and trade only the top decile of discrepancies. In the second strategy the portfolios are static within the holding periods while they evolve dynamically in the third. The final strategy uses convergent momentum as the ranking criteria. In the fifth section we provide details of the data set and some illustrative results from our tests, while we conclude in the final section with suggestions for further extensions.

2. The q-alpha Model

While Brownian motion based continuous diffusion models provide tractable means of modeling financial instruments and underlying asset value processes, empirical studies over the years have shown that they do not describe properly the distributions typically observed in the market. In particular, these models are inherently inadequate in modeling jump discontinuities, asymmetry and positive excess kurtosis (fat tails), traits that frequently occur throughout empirical data. One possible remedy that is of recent interest is the use of log-Levy models. However unless we apply strict simplifying restrictions on the characteristics of the underlying Levy process, computations are often intractable with no closed form analytical pricing formulas that involve first hitting probabilities (Kim 2007). Another unattractive feature of Levy and stochastic volatility models is that they revert quickly back to the Gaussian (Borland 2007).

Lisa Borland et. al (Borland and Bouchaud 2006) tackle the issue of skewness and positive excess kurtosis by introducing the non-Gaussian q-alpha model based on the Tsallis distribution, traditionally used in the field of non-extensive thermo-statistics. Capitalizing on the power law characteristic of the distribution, both positive excess kurtosis and skewness are modeled via statistical feedback. The asset dynamics are specified by the following evolution

$$dS = \mu A + \sigma(A_0)^{1-\alpha} A^\alpha d\Omega$$

where

$$d\Omega = (P[\Omega])^{(1-q)/2} dW$$

and dW is a standard Wiener process. α controls the level of skewness while q is the entropy index giving fat-tails. $q=1$, $\alpha=1$ corresponds to the ordinary Gaussian. P is the conditional probability with statistical feedback, and Ω evolves according to the non-linear Fokker-Planck equation

$$\frac{\delta P}{\delta t} = \frac{1}{2} \frac{\delta P^{2-q}}{\delta \Omega^2}.$$

This gives rise to the generalized Black-Scholes PDE (Borland and Bouchaud 2006)

$$\frac{\delta f}{\delta t} + rA \frac{\delta f}{\delta A} + \frac{1}{2} \frac{\delta^2 f}{\delta S^2} \sigma^2 (A_0)^{2(1-\alpha)} A^{2\alpha} p^{1-q} = rf$$

from which we are able to obtain analytical pricing expressions on contingent claims. The empirical work in their paper shows that this model closely reproduces market distributions. This is the model that we adopt for equity price dynamics in this project.

3. Computing the Default Probabilities and Theoretical CDS Spreads

As mentioned previously, we adopt the simplifying assumption that default occurs when the stock price of a given firm declines by more than 95%. Ideally we would run Monte-Carlo simulations to determine the first passage probabilities of the stock process. This was the route first attempted, however the feedback loops proved to be excessively time consuming, prompting us to devise an alternative scheme. Our essential observation was that we have a closed form options pricing formula through which we can obtain the entire option surface. This surface already implies the probability distribution of the stock process under q-alpha dynamics. We can thus obtain the default probabilities via numerically differentiating the price of either deep ITM calls or deep OTM puts.

Note that

$$\mathbf{E}[(X-c)^+] = \int_c^\infty (X-c)q(x)dx.$$

Hence

$$-\frac{\delta}{\delta c} \mathbf{E}[(X-c)^+] = -\frac{\delta}{\delta c} \int_c^\infty (X-c)q(x)dx = -(X-c)q(x)|_c^\infty + \int_c^\infty q(x)dx = \mathbf{Q}[X > c]$$

The call price C is the discounted expected value of the payoff $(S_T - K)^+$ where S_T is

the stock price at the exercise date T , and K is the strike. Therefore

$$C = e^{-rT} \mathbf{E}^{\mathbf{Q}}[(S_T - K)^+] \Rightarrow \mathbf{Q}[S_T < K] = 1 + \frac{\delta C}{\delta K} e^{rT}.$$

Assuming that default is an absorbing state and thus ignoring the possibility of recovery after going under the default barrier, we obtain the default probabilities via numerically differentiating the option prices.

This enables us to build the theoretical CDS curve by finding the CDS rate values for each maturity that minimize the norm of the difference between the default probability curve implied by the options market and the default probability curve that results from putting the theoretical CDS curve through the stripper function. We sequentially solve for each CDS rate while preserving the rates that have already been established - all the previously solved CDS rates remain unaltered during the search for the latest entry in the theoretical CDS curve.

4. Trading Strategies

4.1 Data

We used CDS and options data spanning a 2 year period leading into 2003, on the 100 most liquid single names. The data was provided to us by EvA. Although we had 5 different maturities for the CDS rates at our disposal, we principally carried out the tests with the 3Y. The P/L results showed very little variation when tested with the other maturities.

4.2 Preliminaries

The core program reads in the raw CDS and options data and cleans, sorts, merges the data to create the two matlab data files: cdsData.m and optionsData.m. If those two data files are present then the program will not recreate them. Next the program processes the market data in 3 stages to create tradeData.m. tradeData is the matrix with all the results including theoretical CDS rates, trading/hedging decisions, and running P&L calculations. This matrix encapsulates all the current information accumulated. The 3 stages to create the matrix are:

buildTradeData
makeTradeDecisions
calcPL

buildTradeData reads in the raw data, unless the cdsData and optionsData refined data files are present. It then populates the initial data into the tradeData matrix, including building the theoretical CDS curves. The process is very time consuming so if the process is interrupted and restarted later, the buildTradeData program will resume from where it left off.

makeTradeDecisions evaluates the market vs theoretical CDS rates each day and decides to go long / short / or take no position, which results in a +1, -1 or 0 in the 'active' field of the tradeData matrix. At the time of a new trade, the date and traded spread are recorded. The current simple trade rules stipulate that when a stop loss occurs a new trade will not be entered into for 'n' days.

calcPL first populates the tradeData matrix with the number of options to use as the partial hedge for the CDS position. The options hedge is only computed on the day a new trade is made and then the number is copied for all rows for which the trade is active. To calculate how many options are needed to hedge the CDS trade we need to know the sensitivity of the theoretical CDS rates to a 1 point change in volatility – which we call $dCDSdV$. With the simulation method this would mean calling MCSurvivorSimulation twice, both times with a high number of simulations but we avoid the excessive time load by the options implied probability scheme mentioned in the previous section. After the option hedge data is populated into tradeData then calcPL computes the dailyPL based on 1) movements in the actual CDS rates, 2) the vega P&L on the options position, 3) the gamma P&L on the options position. The positions are delta neutral.

In implementing the strategies we were forced to make a number of simplifying assumptions. The most noticeable one is that the options and CDS positions are 'constant maturity' and 'constant at-the-money' (Yu 2005, also makes these assumptions in his hedging scheme). In a real trading situation, if the 1 year theoretical and market CDS rates are out of line then the trader will go long the 1 year CDS and short the 1 year at-the-money options. In a few weeks the spot is likely to be quite different from the strike, which will change the vega of the options position, and the maturity of both legs of the trade will not be 1 year anymore. We make the simplifying assumption that the options are always at the money and that both legs of the trade are a constant 1 year maturity. In theory it's possible maintain the trade at a near constant maturity,

constant at-the-money state by liquidating the trade at the end of each day and then repurchasing 1 year options that are currently at-the-money and transacting in a new 1 year CDS. That approach would incur significant bid-offer costs, so by ignoring these bid-offer costs with the constant maturity, constant at-the-money assumption, we are in effect demonstrating an upper bound on the profitability of the strategy. The options delta rebalancing is done once a day, at the close of the trading day and the options vega and gamma are estimated from closed form Black Scholes. The trading rules are implemented on a close of business basis - ie there is no intra day stop loss possibility. We use the interpolated 1 year interest rate for all calculations. Interest rates are a second order effect in the phenomena we'll be testing, so this simplification should not have a great impact on our results.

The dCDSdV, the sensitivity of CDS rates to a 1 point change in the volatility level, is computed at the time of the trade and then the number of options used to hedge the CDS is held constant. This will cause some second order distortions. In addition we tested the strategies with real implied q values but for most cases there were no noticeable differences with using $q = 1.2$ which was suggested in the Borland (Borland and Bouchaud 2006) paper. In some cases the implied q values were completely out of line, returning illogical values for the other program chains, so we hard-coded the value as $q = 1.2$.

We have 2 years of daily market data on 100 companies. We've given each company an arbitrary code number, based on alphabetic sorting. The market data is in two matrices, cdsData & optionsData:

cdsDatacolumns:

companyCode, date, cds1, cds2, cds3, cds4, cds5

The date field is an integer which is excel's internal date number

optionsDatacolumns:

companyCode, date, optionsMaturityDate, impliedVol, q, alpha, interestRate

There are other extraneous data columns. impliedVol is the implied volatility to be used in the Tsallis distribution model.

We create a third matrix, tradingData

tradingDatacolumns:

- 1 - Company number
- 2 - Date
- 3 - CDS1
- 4 - CDS2
- 5 - CDS3
- 6 - CDS4
- 7 - CDS5
- 8 - spot price
- 9 - sigma
- 10 - alpha
- 11 - q
- 12 - r
- 13 - volatility data maturity date
- 14 - dv01_1
- 15 - dv01_2
- 16 - dv01_3
- 17 - dv01_4
- 18 - dv01_5
- 19 - theoCDS1
- 20 - theoCDS2
- 21 - theoCDS3
- 22 - theoCDS4
- 23 - theoCDS5
- 24 - dCDSdV1
- 25 - dCDSdV2
- 26 - dCDSdV3
- 27 - dCDSdV4
- 28 - dCDSdV5
- 29 - optionGamma
- 30 - optionVega
- 31 - active
- 32 - tradedSpread
- 33 - stoppedOut
- 34 - optionPosition
- 35 - dailyPL

The sigma/alpha/q/r parameters are as close as possible to the 1 year from the optionsData matrix. First option data between 9 months and 15 months is searched. If none is found then maturities greater than 15 months is searched. If none is found then data between 6 months and 9 months is searched. If no data with maturity greater than 6 months is found then 0's are put into these fields. The date of the data's maturity is recorded in column 13.

The theoretical CDS rates are calculated as described in the previous section. The dv01 data fields give the p/l of an 'n' year maturity cds swap position when the 'n' year cds rate moves by 1 basis point. The option gamma and vega are computed using the standard Black Scholes model which uses sigma. We make the assumption that the strike = spot, and maturity = 1 year. The 'active' data field holds values -1, 0, 1 where -1 = short CDS trade is active, 0 = no trade is active, 1 = long CDS trade is active. The options position on the data of the trade is calculated as:

$$\text{position} = (\text{dCDSdV} * \text{dv01}_n * \$10\text{M}) / \text{optionsVega}$$

The 'n' in the expression above is an input parameter because we could in theory test different maturity trading strategies. stoppedOut records the day a stop loss occurred.

4.3 Strategy 1 : Basic Threshold Strategy

Since the other strategies are variations of this basic strategy, despite its simplicity, we will devote some detail to the technical and algorithmic aspects. The basic scheme is extremely simple. Each day we calculate the theoretical CDS rates for all the companies. We have a trade activate threshold α , take profit level β , convergence bound ϵ , and stop-loss level of γ , specified as input parameters. Let

$$\text{spread} = \text{theoretical CDS rate} - \text{market CDS rate}$$

When no trade is currently active, if the $\text{spread} > \alpha$ we go long CDS and short the appropriate options hedge. If $\text{spread} < -\alpha$ we activate a short CDS position and long the appropriate options hedge. When a long CDS trade is currently active, if $\text{spread} > \gamma$ our stop-loss policy is invoked by which we close out the position. If $\text{spread} < \beta$ we take profits by closing out the position. When a short CDS trade is currently active, if $\text{spread} < -\gamma$ our stop-loss policy is invoked by which we close out the position. If $\text{spread} > -\beta$ we take profits by closing out the position. Note that if we do not wish to use either of these stop-loss or take-profits policies we can nullify them by specifying an extremely high value for their thresholds. However, as we will see, the stop-loss policy is critical in avoiding huge losses in divergence-divergence situations where we lose on both the position and the hedge. α, β, γ can all be specified in terms of

absolute values or relative to the current spreads. All single CDS trades are made on a US\$10M notional, and the options hedge is put in place to delta neutralize the portfolio. Hence we can either hedge with calls or puts and each day there will be a gamma rebalancing P/L. Convergence is defined to occur when the two rates come within some small ϵ neighborhood ($|theoretical - market| < \epsilon$) and checked daily to execute daily convergence close-outs.

4.4 Strategy 2: Rank and Hold

Strategy 2 is a refinement of the previous strategy. In strategy 1 we take on any trade that exhibits a wide enough discrepancy between theoretical and market CDS rates. As we will see in the results in the next section, by doing so we take on too many bad trades that never converge, and often even diverge further. The motivation for the second strategy is that perhaps the most egregious discrepancies are the ones that have a genuine mispricing component more likely to dissipate over time. We accept that there may be some other factors at play that put a systematic wedge between the prices. Hence we do not always aim at full convergence, but try to capitalize on convergent movement whenever possible. The strategy works as follows. First we define a rebalancing or holding period. At the beginning of the period we rank the companies according to the absolute value of the difference between the theoretical and market CDS rates. Then we form a portfolio that consists of the top 10% of the companies in terms of these differences. Each day, we check for convergence. If a company's theoretical and market rates converge, then we close out its position and pocket the profit. At the end of the holding period, we close out the whole portfolio (this allows us to both close out positions that appear to be non-convergent, and pocket from any partial convergence that may have occurred in between the periods). At the same time, we again form a new portfolio that consists of the top 10% of discrepancies. However, in this new portfolio, we do not hold any companies that have just been closed out due to non-convergence in the immediate previous portfolio even though those companies may have very large differences between the market and theoretical rates.

As a demonstration, suppose that in the beginning, we form a portfolio consisting of IBM, Microsoft, HP and Dell with a holding period of 30 days. Each day, we check for convergence. On day 5, Microsoft converges and we close out Microsoft. Between day 5 and day 30, no companies converge and the portfolio remains unchanged. On day 30, the holding period ends and on that day IBM and Dell finally converge, but HP still has not converged. We close out IBM and Dell due to convergence and kick out HP because we perceive it as having no hope of convergence in the immediate future.

We then form a new portfolio. We look at companies that have the largest differences between the market and theoretical rates. Suppose Microsoft and HP once again have large differences. We are not going to include HP since it is in the previous portfolio and has failed to converge previously. The new portfolio will consist of other companies with large differences in the spreads including Microsoft.

4.5 Strategy 3: Rank and Hold with Active Holding Periods

This is a variation of strategy 2. In the previous strategy the portfolio remains static in between the holding periods, hence we are not able to capture and capitalize on newly evolving spread dynamics and corresponding inefficiencies in the market in between the (possibly long) holding periods. Here we wish to allow the portfolio to dynamically evolve even in between the holding periods by having distinct holding and rebalancing periods. This will allow us to capture opportunities in between the periods and also have a smoothing effect by compounding a greater number of period-overlapping portfolios. The holding period is an integer multiple of the rebalancing period. For each rebalancing interval we continue to form new portfolios by appending the new top decile items to the existing portfolio. Concurrently we also close out the positions from the previous holding periods. Thus at any time we may have a portfolio of items with different dates to expiration which we have to keep track of separately.

As an illustration we again form a portfolio consisting of IBM, Microsoft, HP and Dell with a holding period of 30 days, but now we have a rebalancing period of 15 days. We still check for convergence daily. Suppose again that Microsoft converges on day 5, we would then close out Microsoft. On day 15, we rebalance the portfolio. If Intel and Yahoo have a large difference between theoretical and market rates at that time, we would add Intel and Yahoo to the portfolio and the portfolio will consist of 5 companies, namely IBM, HP, Dell, Intel and Yahoo. Between day 15 and day 30, no companies converge and the portfolio remains unchanged. On day 30, IBM and Intel converge. We then close out IBM and Intel. We also kick out Dell and HP since their holding periods have expired on day 30. Yahoo remains in the portfolio. On day 30, we also need to rebalance the portfolio and we will add companies (excluding Dell and HP) that have large differences in the theoretical and market rates into the portfolio. If convergence does not occur Yahoo will be killed at day 45.

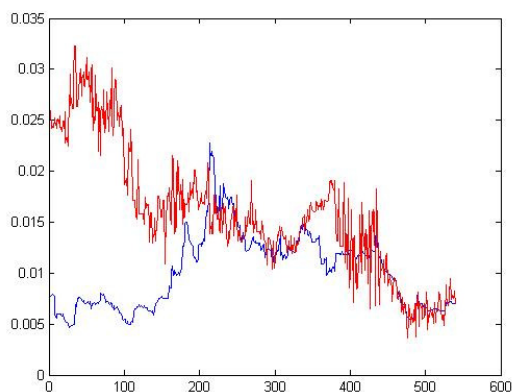
4.6 Strategy 4: Capture the Momentum

This strategy is a variation of strategy 3. It is the most complex strategy that we

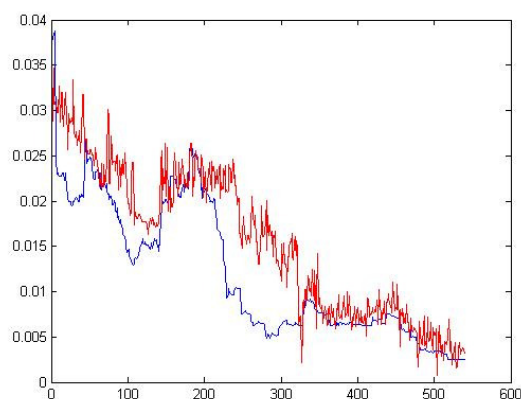
have tested, but thanks to the common infrastructure in place from the previous strategy the description is simple. Up to now, we have not considered any dynamics, only using the magnitude of the discrepancies as the selection criteria. But as we will see in the next section, large discrepancies often become even wider. Here we use a different ranking criterion - we look at convergent momentum. The basic structure of the strategy is the same as 3, but at each interval we have a lookback/formation moving window for which we compute the rate of decrease of the absolute difference of the spreads (rate of convergence) and rank accordingly.

5. Results

Before testing specific strategies we first plotted the real vs. theoretical CDS rates to get an idea on what was happening in terms of their relative movement. As expected some were converging. The following are plots for Eastman Kodak (left) and Halliburton (right). The vertical and horizontal axes represent the CDS spread and time (in days) respectively. The market rates are given in blue, while the red plots are the ones generated by our theoretical model and algorithm explained in previous sections.

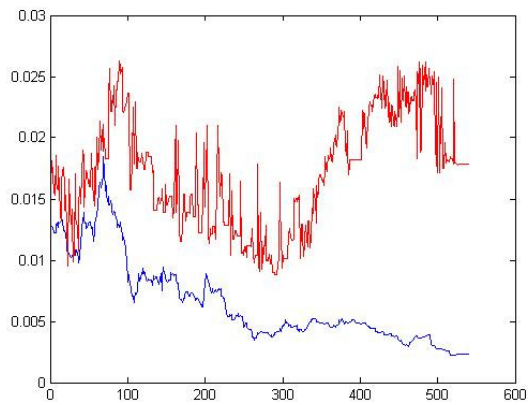


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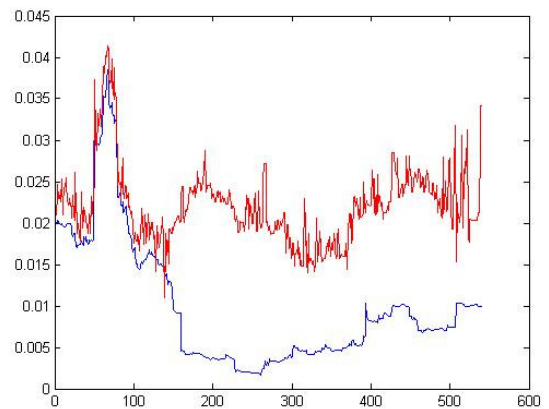


<Halliburton>

Some spreads diverged.

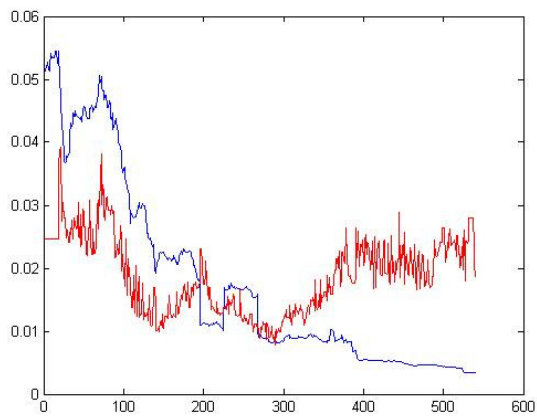


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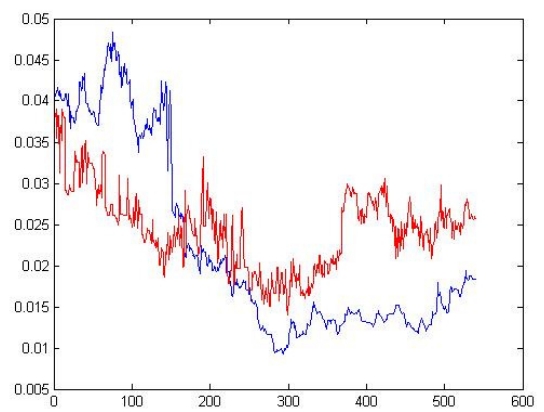


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Some discrepancies converged and then reopened

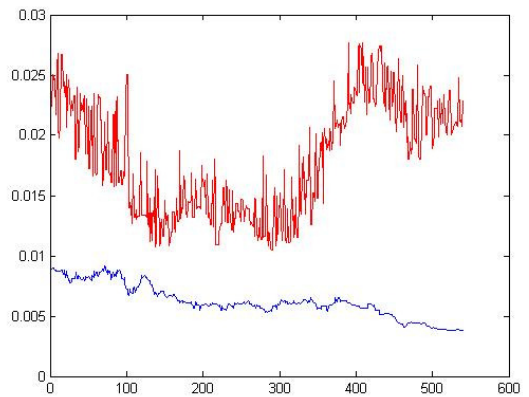
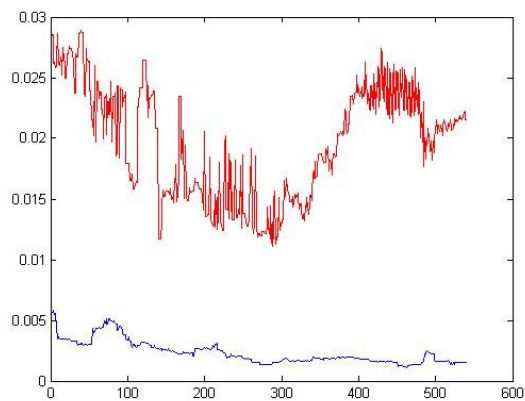


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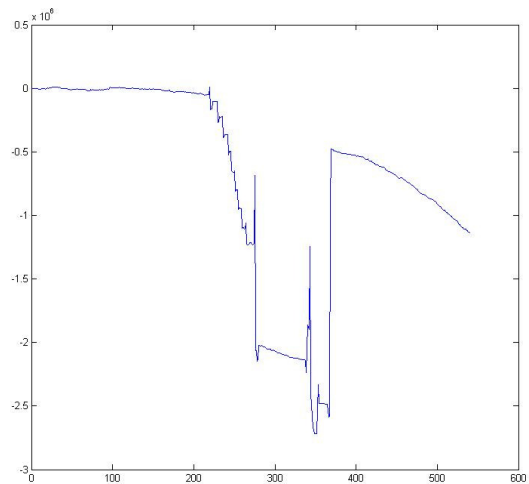
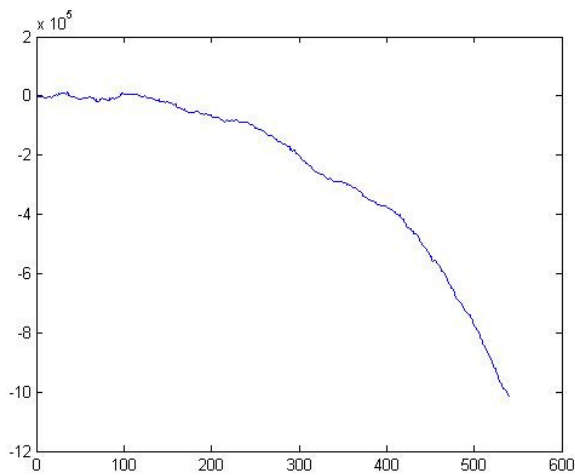
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In many cases the discrepancies appeared to be persistent.



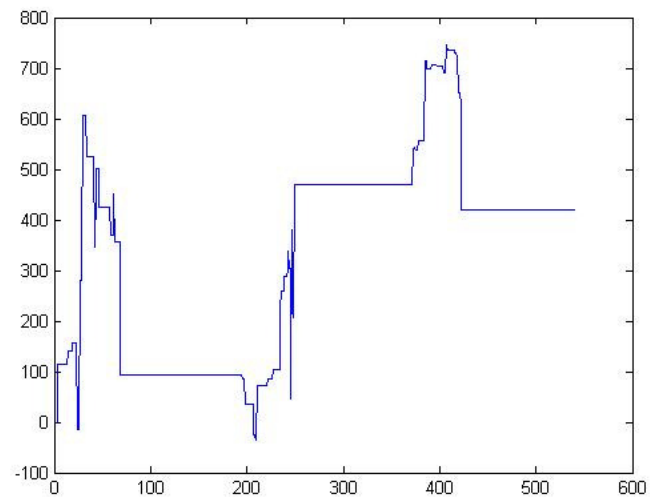
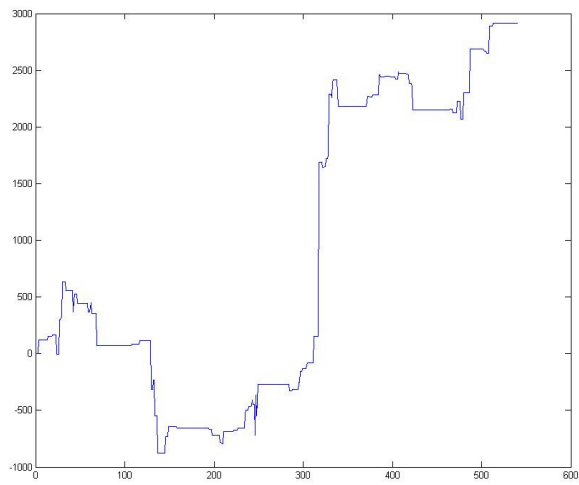
(American Electric Power / International Paper)

These examples illuminate the fact that there is unlikely to be a simple methodology by which we will be able to weed out the "bad" trades. As expected, in the simple indiscriminate threshold strategy (strategy 1), we end up with a cumulative loss for all 16 input parameter combinations tested. In particular the following are graphs for trade activate threshold, stop loss bound, kickout period, and convergence threshold values of (.01, .02, 90, .0025), and (.02, .05, 30, .005) respectively.

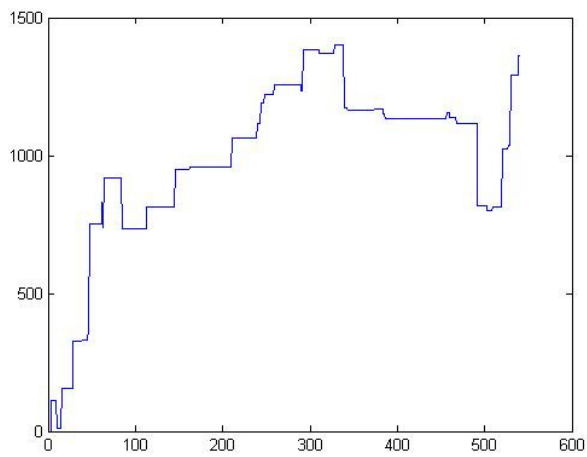


The horizontal axis is in days and the vertical axis is the cumulative P/L in raw US\$. Losses were between US\$1M to US\$2.6M. This strategy is indiscriminate in activating trades as long as wide enough differences are observed. As we have seen from the spread plots, this results in taking on too many bad trades that never converge, or to make matters worse, further diverge. Stop loss becomes the dominant trade and we steadily accumulate losses as time progresses. As an illustration, for the run with parameter values of (.01, .02, 90, .0025), examining the internal counters of the program showed that 448 stop-loss trades were made as opposed to 57 convergence close-outs.

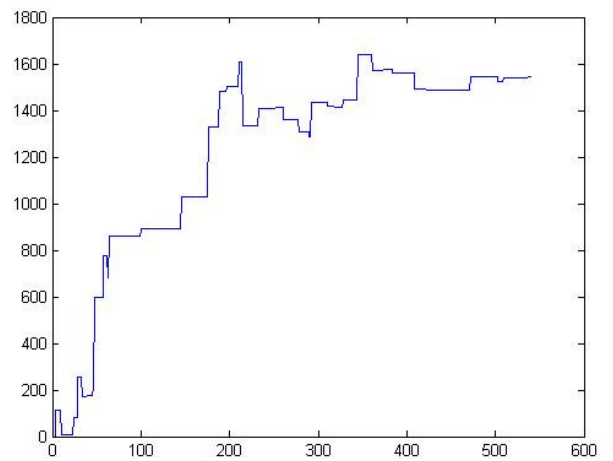
The following are cumulative P/L results for the second strategy with a holding period of 30 (left) and 60 (right) days respectively. The flat regions indicate that frequently no convergence-close outs are made in between holding periods. The jumps typically occur when positions are cleared at the end of holding periods. The jump for instance at day 330 for the first combination, occurred when we cleared out 7 positions, 6 of which had significant convergences (but not complete convergences). All 16 different holding periods tested had cumulative profits of \$800~\$3000, showing that we were successful in eliminating some of the egregiously bad trades.



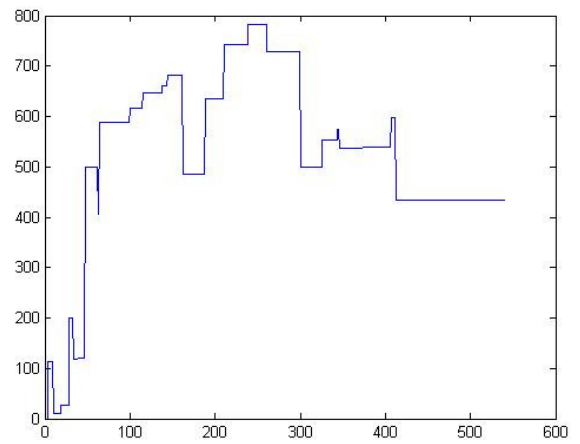
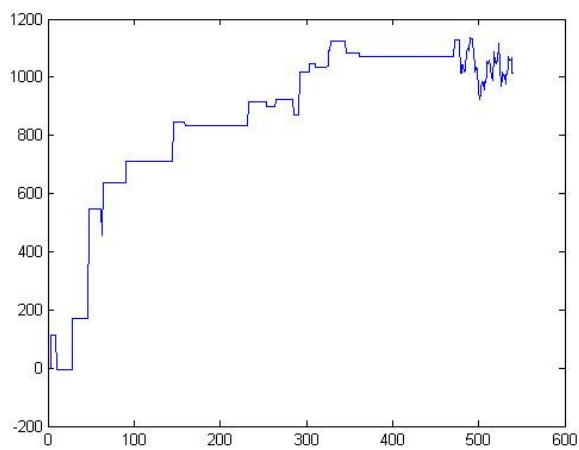
For the third strategy, (rebalancing, holding)



(15,45)



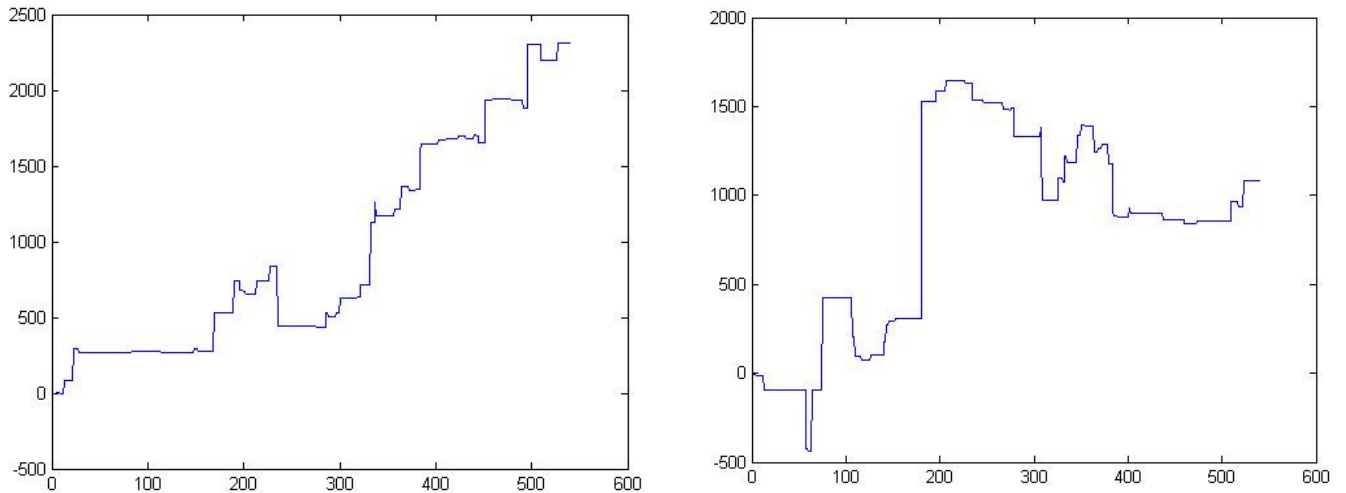
(10,120)



(50, 150)

(40, 160)

The cumulative profit ranges were once again all positive for the 20 combinations tested but they were not significantly different from those of the previous strategy. Finally for the fourth strategy that looks at moving windows of convergent momentum we have,



where the left is with 15 day rebalancing, 30 day lookback, and a 60 day holding period. The right graph is with 15 day rebalancing, 60 day lookback, and a 90 day holding period. Once again all fall within a cumulative profit range of \$1000~\$3000.

While it appears as though we almost always make "something out of nothing" for strategies 2~4, since each position is initiated at zero value and we end up with cumulative profits for all tests, the numbers are deceiving. In reality we need to assume that the arbitrageur has a certain level of initial capital deposited in a margin account to finance the options hedge, to which the profits are added and losses deducted. A full rate of return and Sharpe ratio analysis is impossible at this point since we do not have accurate figures on the margin requirements. However even a very basic analysis reveals that none of these strategies are likely to be "worthwhile". On a \$10M notional, if we consider a 500bp margin analogous to typical IR swap requirements, the annualized returns from the previous strategies range from 0.1~0.8%. If we take into account the fact that, on average, we will have around 7 active trades at any given instance, the most optimistic scenario leads to an annualized rate of return of 0.14% which is clearly lower than the risk free rate. This is even before transaction costs that will inevitably be significant due to the frequency of trades and the "implied slippage" due to our simplifying assumptions in implementing the hedge.

6. Conclusion and Directions for Future Work

In this project we used a model for equity default with non-Gaussian stock dynamics to compute the equity implied theoretical CDS spreads from which we constructed and tested 4 different capital structure arbitrage strategies. Starting from a basic indiscriminate threshold strategy, we applied further refinements to identify trades that are more likely to converge. The results confirmed the anticipated difficulties in constructing capital structure arbitrage strategies from both an implementation-wise and strategic standpoint. From an implementation standpoint, from computing the options hedge position to backing out our implied default probabilities from the option price surface, we are in a sense being too taxing on our model. It may be rewarding to fine tune the model separately for different estimation subtasks, or alternatively to use different estimators for the subtasks instead of relying on faith in coherence across the board. From a strategic standpoint, the purpose of testing relatively simple models was to see if we could come up with robust, systematic, algorithmic ways in which we could identify and profit from market inefficiencies in the aggregate, in an automated fashion, instead of having to rely on some latent "trend" variables constituting the instincts of traders. However our results show that it may be necessary to take a more in depth look at the dynamics of the spread evolution. One strategy would be to perhaps look at the volatility differences relative to the current leverage cycle of the company and to capture cheap volatility in either market by daily gamma trading. With more interesting strategies, and a clearly defined margin requirement we could compare the Sharpe ratios with hedge fund industry benchmarks on fixed income arbitrage strategies. Another promising avenue for further work would be to link the front-end of the strategies to more complex traditional statistical arbitrage strategies, and check the constrained mean and time averaged variance to see if they still constitute statistical arbitrage.

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