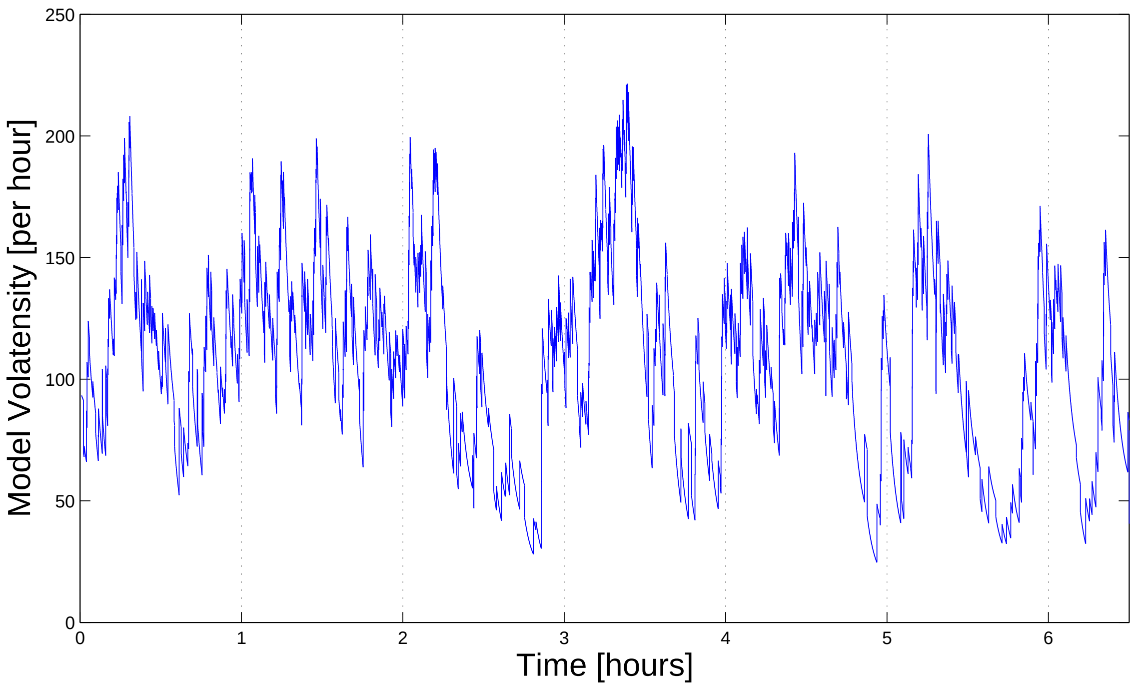
A model for intraday volatility

Yongyang Cai, Baeho Kim, Matthew Leduc,
Kamil Szczegot, Yang Yixiao, Manuel Zamfir



Volatility of stock returns

Let y be the return on a stock. Then the instantaneous volatility is

$$\begin{aligned}
\sigma_t^2 &= \lim_{\Delta t \to 0} \frac{1}{\Delta t} E\big\{(y(t+\Delta t) - y(t))^2 \big| \mathcal{F}_t\big\} \\
&= \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_0^\infty P\big\{(y(t+\Delta t) - y(t))^2 \geqslant \alpha \big| \mathcal{F}_t\big\}\, d\alpha \\
&\approx \lim_{\Delta t \to 0} \frac{1}{\Delta t} \sum_{k=1}^{K} P\{|y(t+\Delta t) - y(t)| \geqslant k\,\varepsilon | \mathcal{F}_t\}\, C(k,\varepsilon) \\
&= \sum_{k=1}^{K} \lambda_t^{(k\varepsilon)}\, C(k,\varepsilon)
\end{aligned}$$

assuming the changes in returns are multiples of some small $\varepsilon > 0$. Here

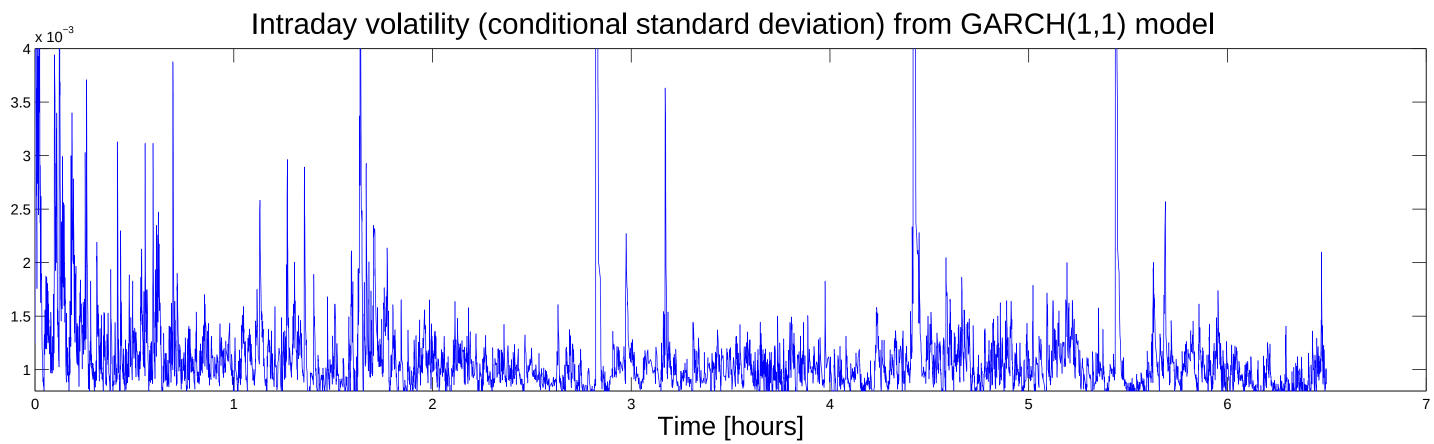
$$\lambda_t^{(\Delta y)} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P\{N(t+\Delta t) - N(t) > 0 \,|\, \mathcal{F}_t\}$$

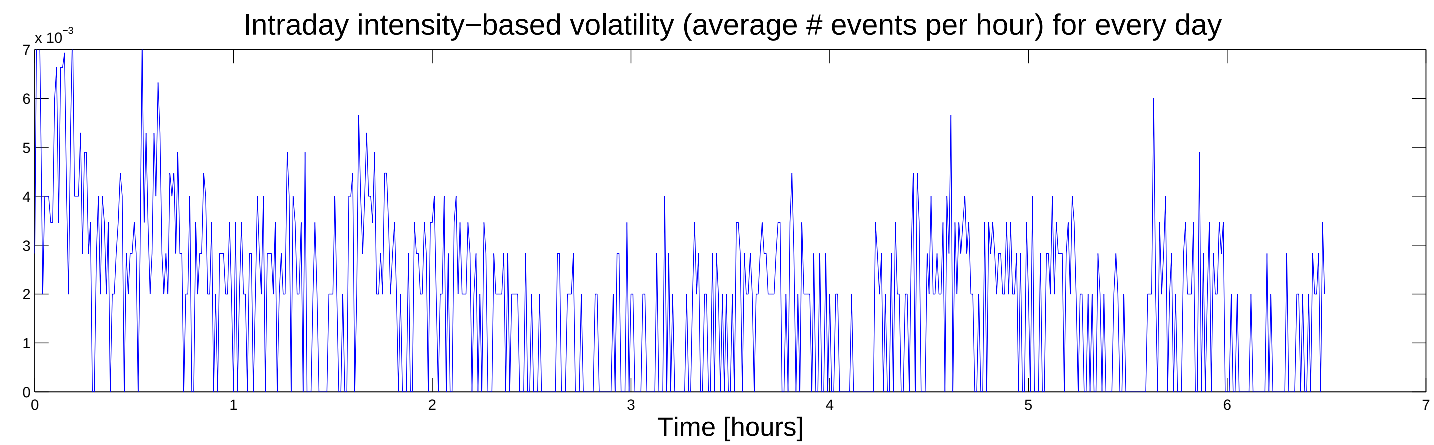
is the intensity of the counting process N, which counts the number of jumps of y exceeding some fixed threshold Δy.

For convenience we call $\lambda_t^{(\Delta y)}$ the **volatensity**.

Volatensity is a proxy for volatility

This can be seen, for example, by comparing it with the GARCH(1,1) fitted to the return series[1], which is a very common representation of volatility.





1. This and following based on TAQ data for IBM stock on 1/05/2007

Our model for volatensity

$$\lambda_t = \underbrace{\lambda_\infty(N_t)}_{\text{ACD part}} + \underbrace{\int_0^t g(t-s)\, dN_s}_{\text{Hawkes part}}$$

where

$$\lambda_\infty(n)^{-1} = C + \sum_{i=1}^{m} \alpha_i \,(T_{n-i} - T_{n-i-1}) + \sum_{j=1}^{q} \beta_j \,\lambda_\infty(n-j)^{-1},$$

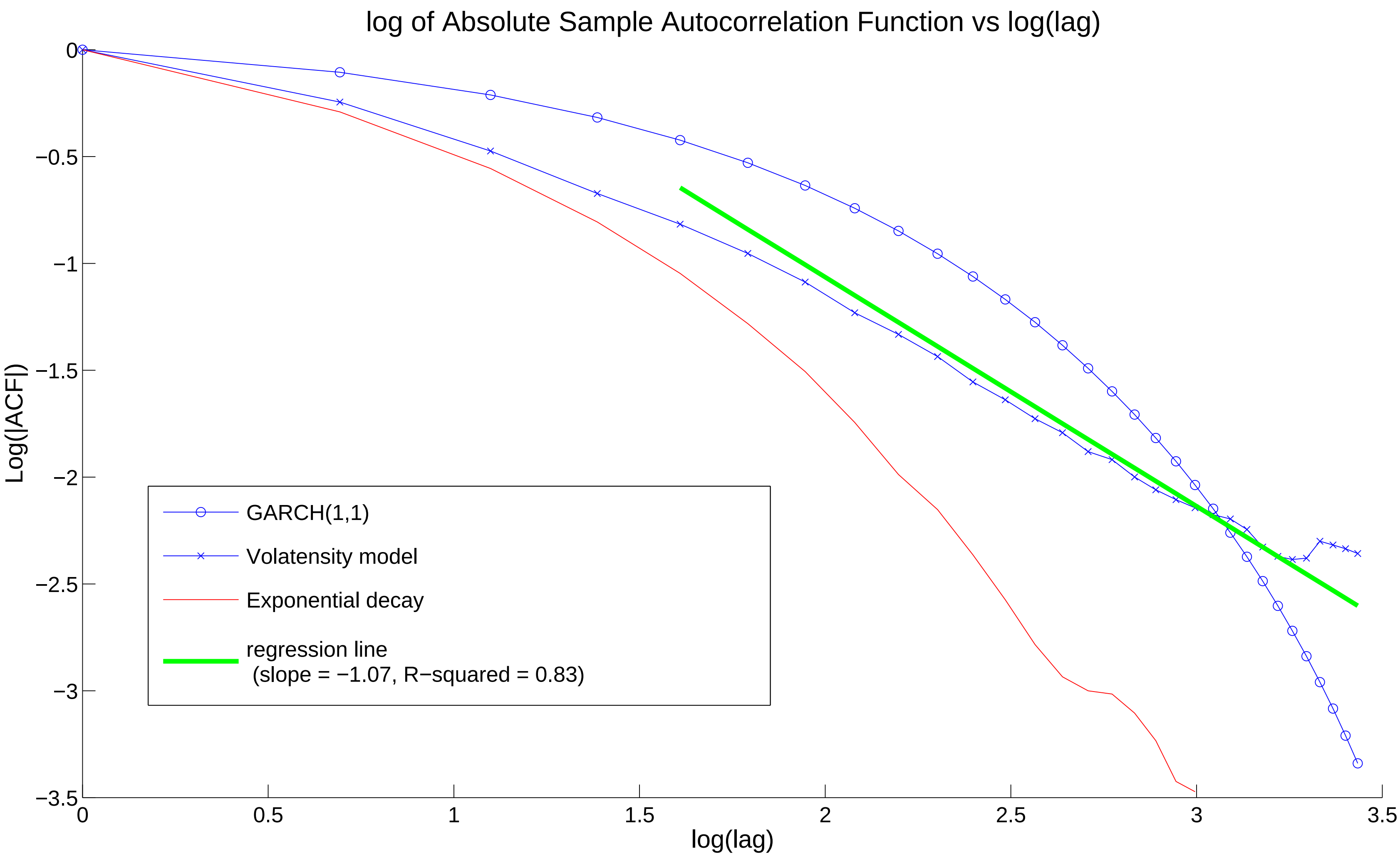
and T_i is the time of the ith jump.

We take g to be equal to

$$g_{\text{general}}(x) = \frac{1}{\sum_{l=0}^{L} c_l\, x^l} \qquad \text{or} \qquad g_{\text{special}}(x) = \frac{a}{(b+x)^c}$$

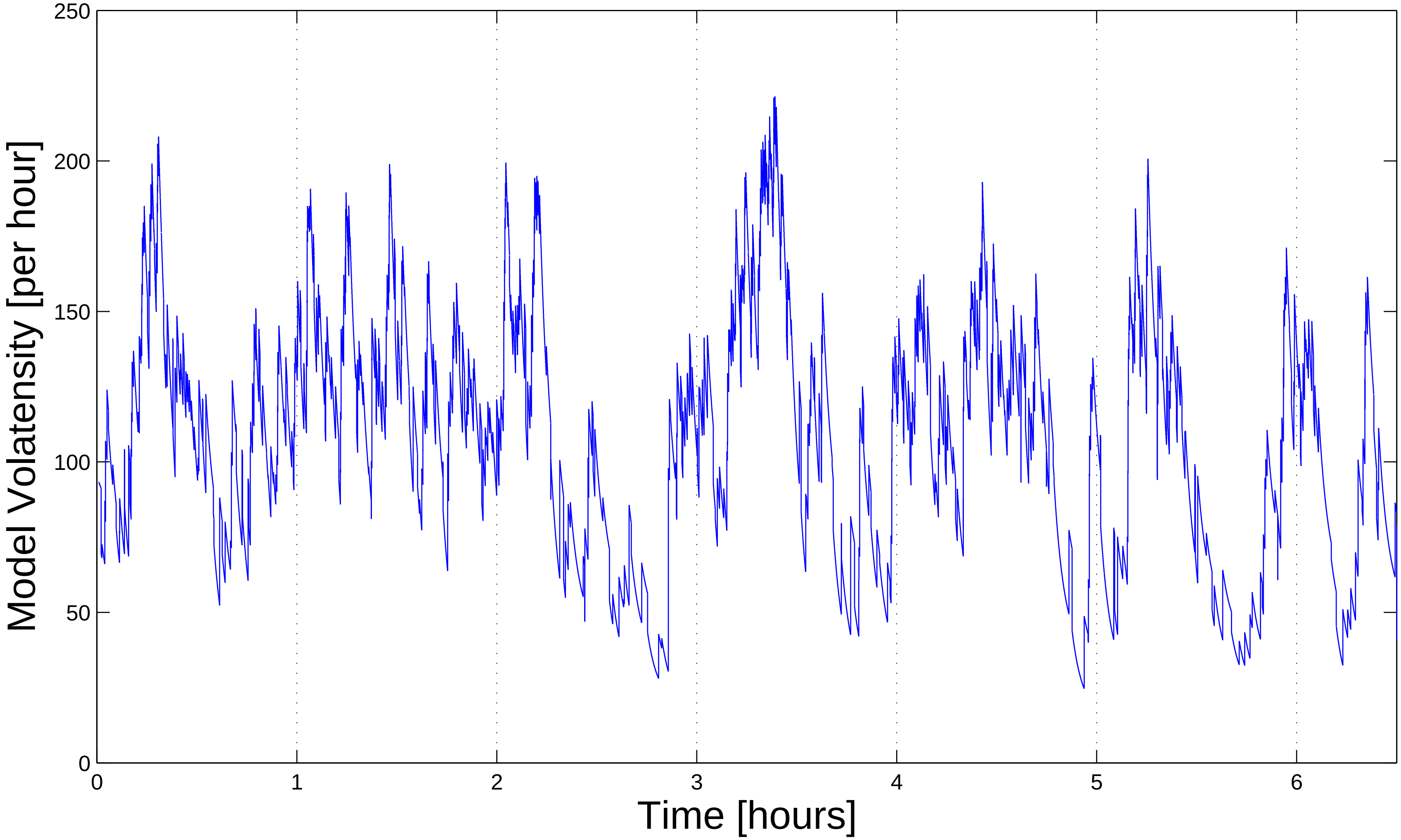
Reproducing stylized features of volatensity

Slowly decaying autocorrelation:



Reproducing stylized features of volatensity

Volatensity bursts:



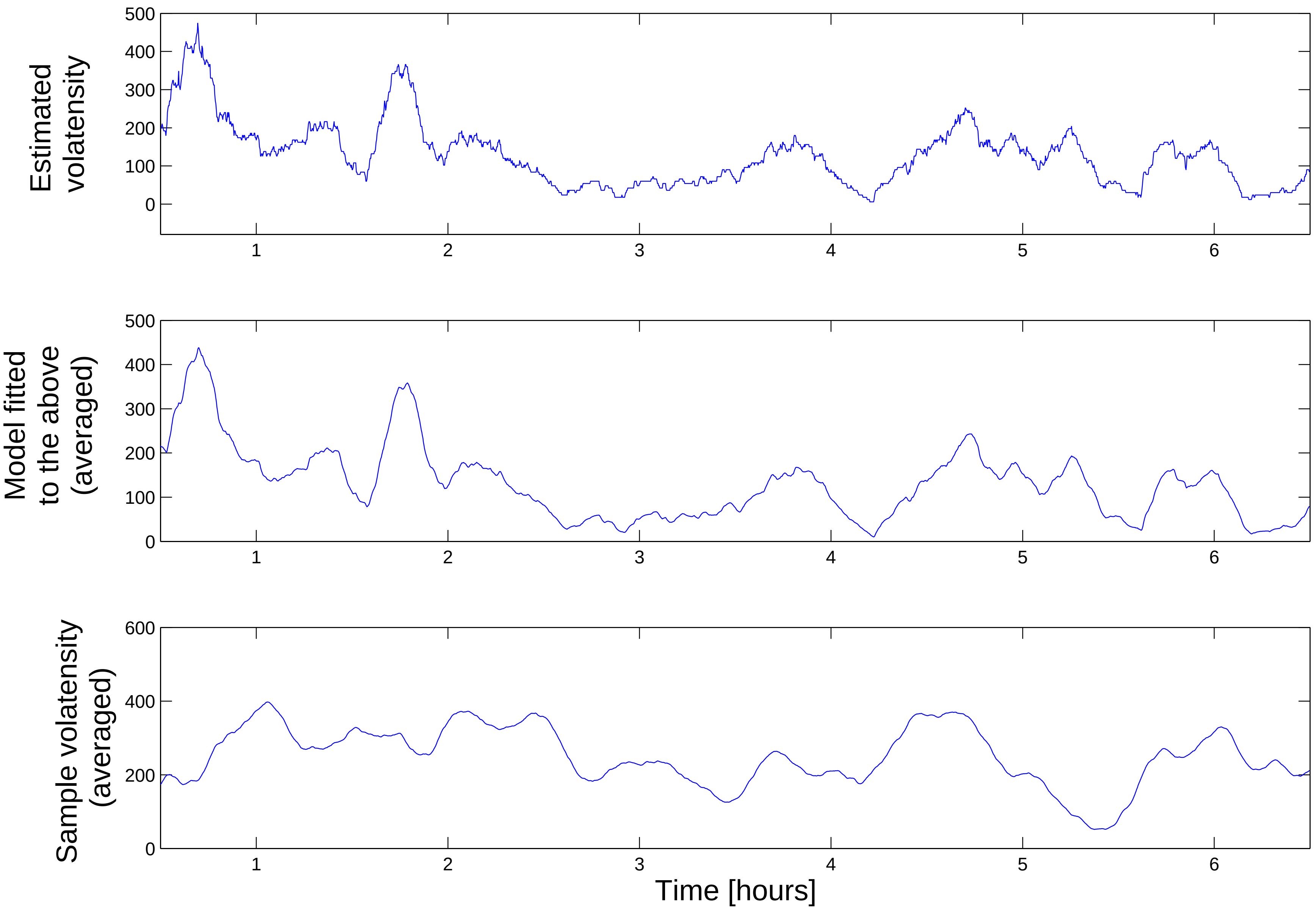
Why do we need the two terms?

	ACD part	Hawkes part
self-exciting	•	•
slow autocorrelation decay	•	
power-law decay of intensity between jumps		•
suitability for intraday modelling	•	•
ease of simulation	•	•
feasibility of calibration	•	•

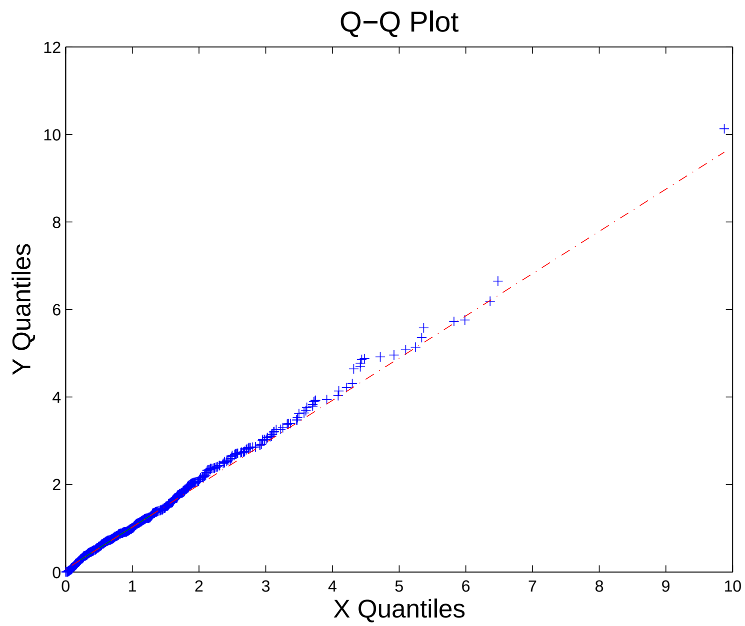
Calibrating the intraday volatensity model

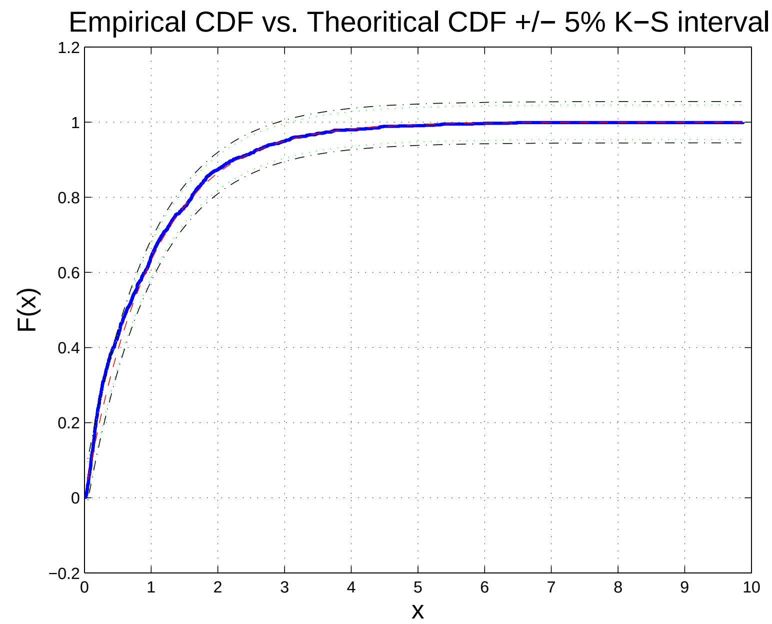
1. Cleaning and converting the data:
   - deleting trades outside the standard trading hours,
   - averaging the trade price in single time points,
   - determining the counting process based on a fixed threshold.
2. Estimating model parameters using a two-step procedure:
   - least-squares fitting of the intensity curve to obtain a starting point,
   - MLE estimation using the starting point.
3. Statistical goodness-of-fit tests for time-changed interarrival times, which should be independent exponentials (Meyer 1971):
   - mean/variance,
   - Ljung-Box test,
   - Q-Q plot,
   - Kolmogorov-Smirnov Test.

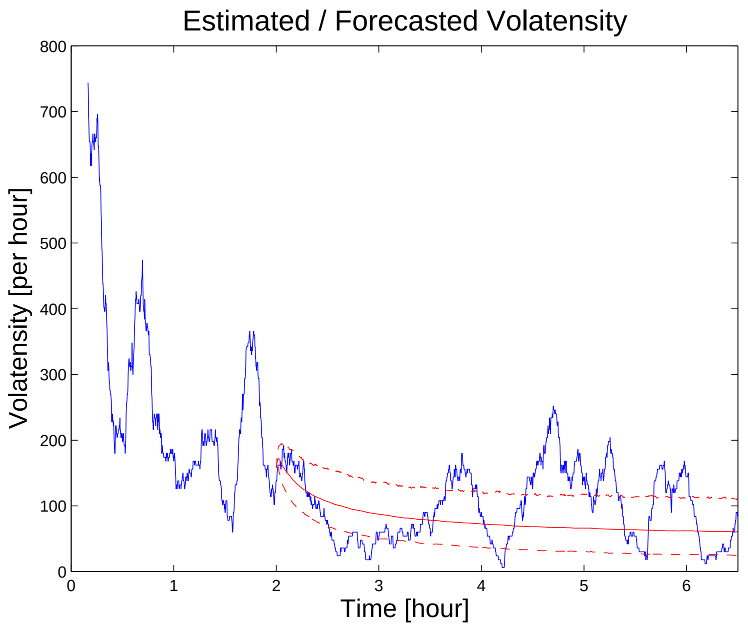
Historical vs. fitted volatensity



Statistical tests & forecasting







For time-changed interarrival times:

- sample mean = 1.00
- sample variance = 1.15
- Ljung-Box test (with lags 10–80) accepts the null hypothesis that the model fit is adequate

Conclusions and future work

- We have succeeded in incorporating in our model most of the stylized features of volatility: clustering, slow decay of autocorellation, and power-law decay after a jump.
- This is an intraday model, but it can be extended to a multiday one.
- One way to achieve this is using the so-called ‘seasonality functions’ to take into account the fact that trading patterns exhibit similarity across different days.
- Another way is to regard the parameters of our model as a realization of a daily time-series and then fitting a model to it (e.g. ARMA).
- Both of those would allow us to make a forecast several days into the future and thus develop a trading strategy.