# Modelling Stock Orders Using Hawkes's Self-Exciting Process

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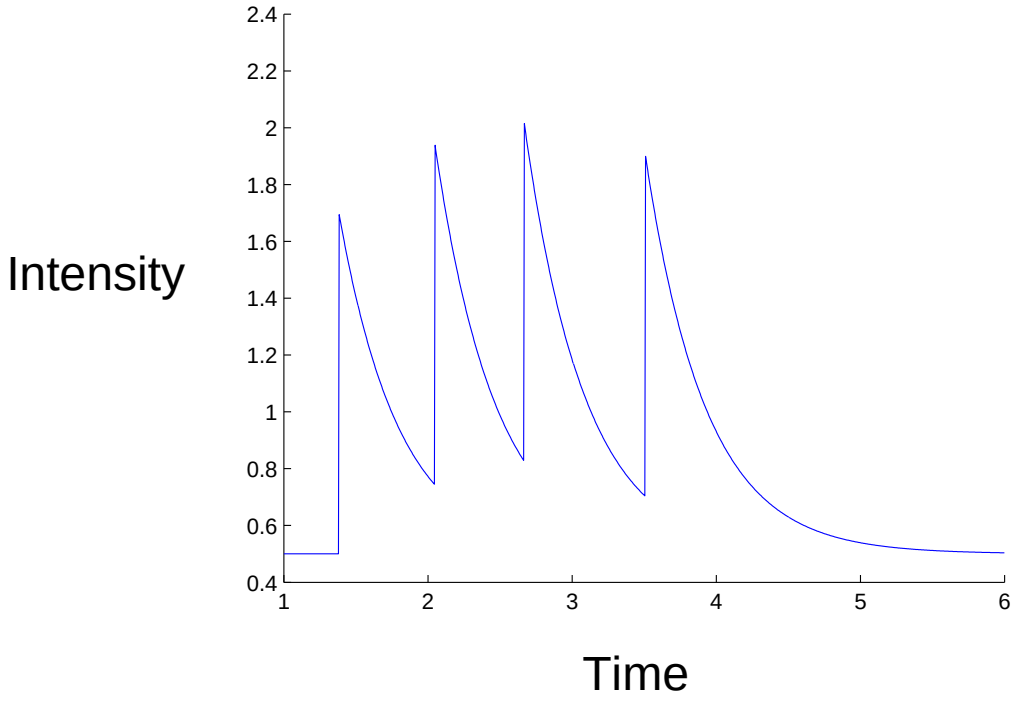
# Hawkes Process

- Univariate: intensity at time $t$ is given by

$$\lambda_t = \mu + \alpha \int_{u<t} e^{-\beta(t-u)} dN_u$$

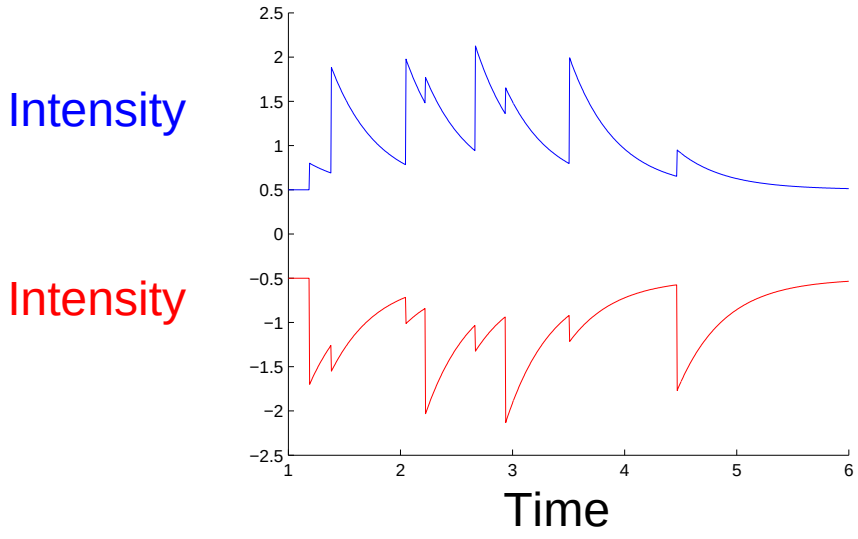


# Hawkes Process

- Bivariate: intensities of processes $1$ and $2$ are given by

$$\lambda_t^1 = \mu_1 + \alpha_{11} \int_{u<t} e^{-\beta_{11}(t-u)} dN_u^1 + \alpha_{12} \int_{u<t} e^{-\beta_{12}(t-u)} dN_u^2$$

$$\lambda_t^2 = \mu_2 + \alpha_{22} \int_{u<t} e^{-\beta_{22}(t-u)} dN_u^2 + \alpha_{21} \int_{u<t} e^{-\beta_{21}(t-u)} dN_u^1$$



# Maximum Likelihood Estimation

- Hawkes log-likelihood function can be written explicitly (e.g. univariate case):

$$\log L\left(t_1,\ldots,t_n|\mu,\alpha,\beta\right) =$$

$$-\mu t_n + \sum_{i=1}^{n} \frac{\alpha}{\beta}\left[e^{-\beta(t_n-t_i)}-1\right] + \sum_{i=1}^{n} \log\left(\mu + \alpha \sum_{t_j<t_i} e^{-\beta(t_i-t_j)}\right)$$

- Gradients and Hessians are also available

# Model Description

- Bivariate process:  buy/sell orders for stocks arrive throughout the day
- Use Lee-Ready tick test to classify orders as buy/sell
- Ignore volume and price levels

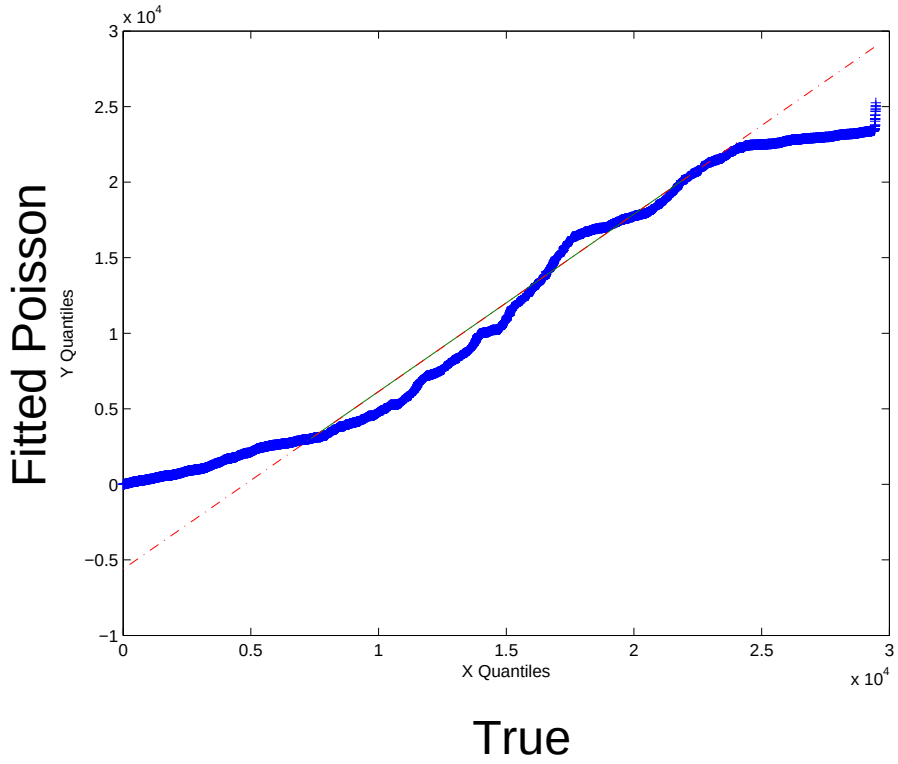
# Results

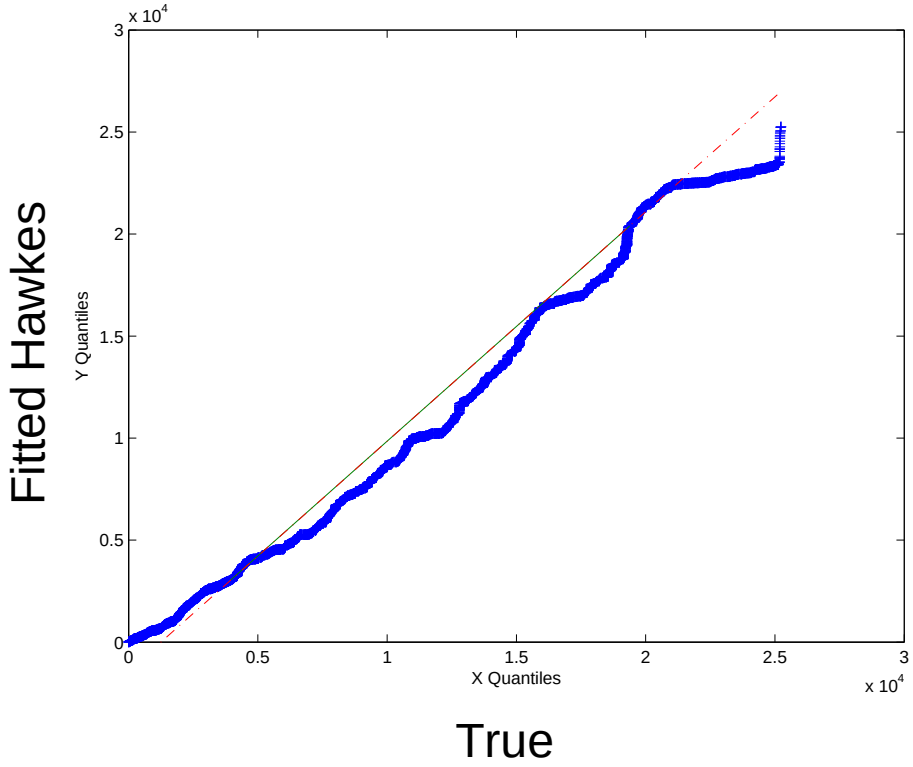
- Most companies (Google, Microsoft) exhibit negligible cross-correlation
- Most parameter estimates have very high decay, e.g. 3 seconds

A typical day's parameter estimates of buy orders for Google stock:

$$\mu = 0.1389;\ \alpha = 1.2131;\ \beta = 1.9805$$

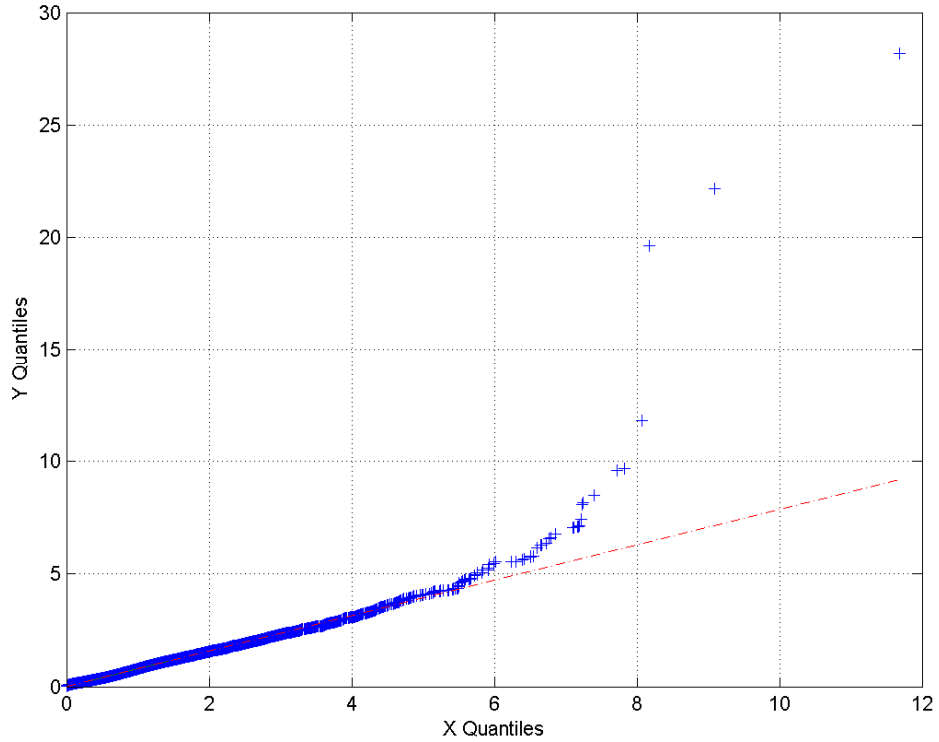
# QQ plots


Fitted Poisson
x 10^4
Y Quantiles
X Quantiles
x 10^4
True


Fitted Hawkes
x 10^4
Y Quantiles
X Quantiles
x 10^4
True

# Classification of Buy/Sell Orders

- Classical Lee-Ready ‘tick test’
- ~30000 data points

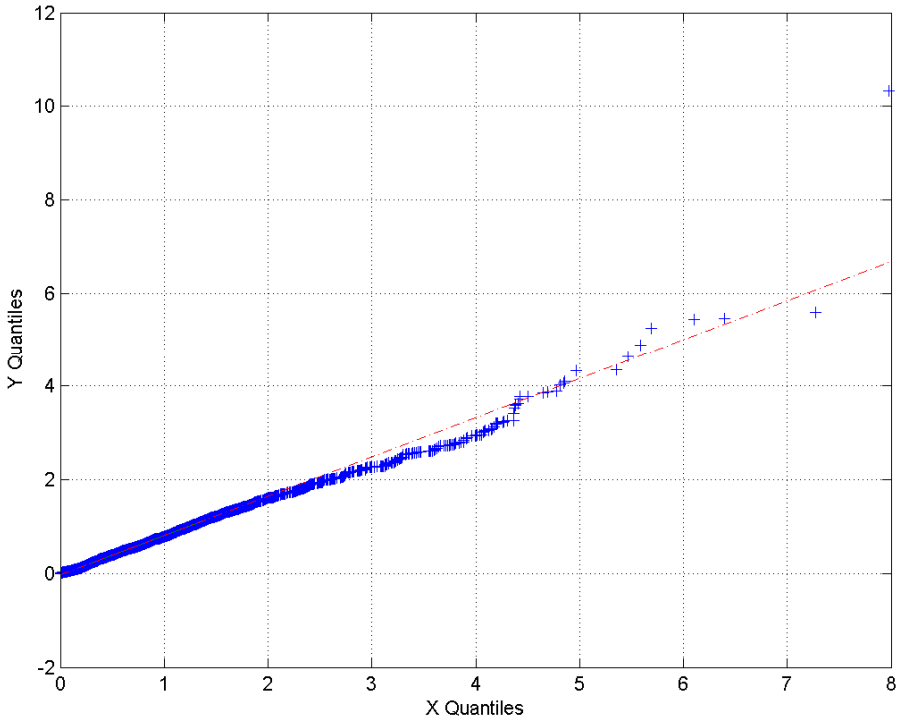


# Data Thinning

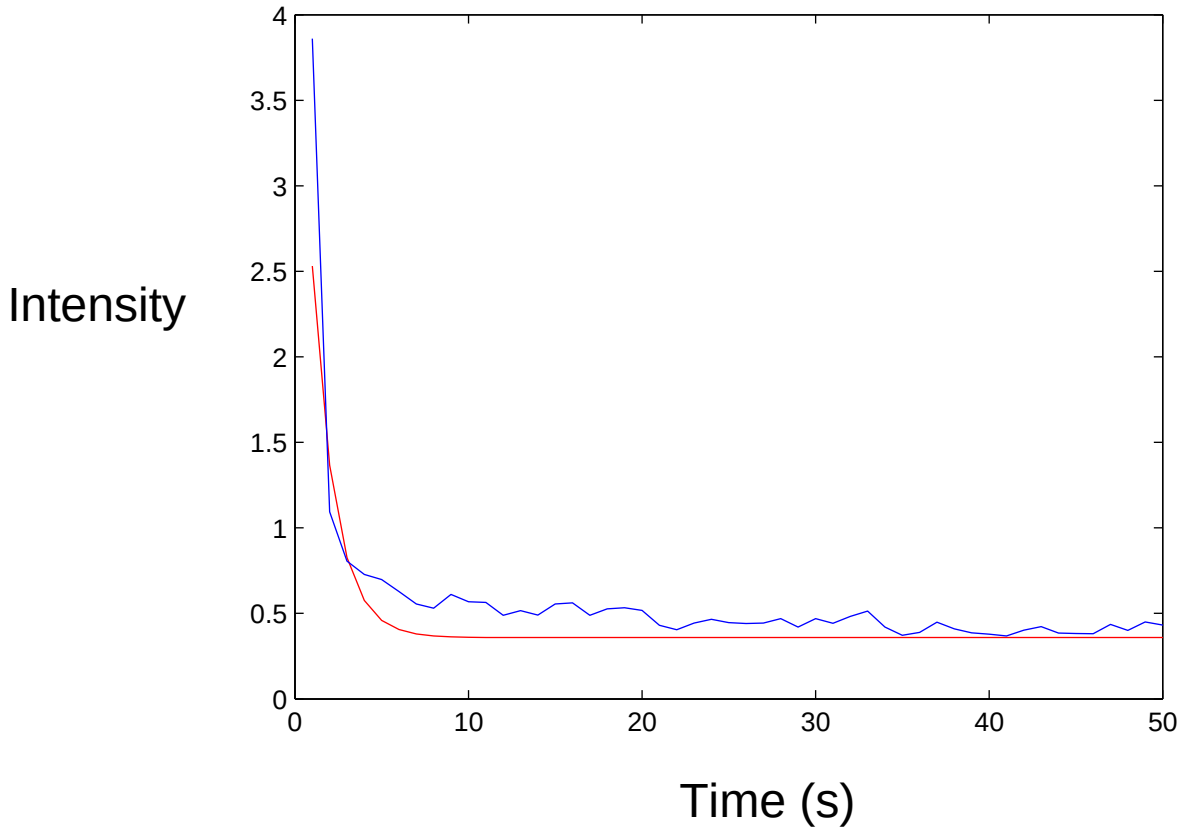
- Dropped orders with

$$Price_{previous} = Price_{current}$$

- ~10000 data points



# Conditional intensity following a buy



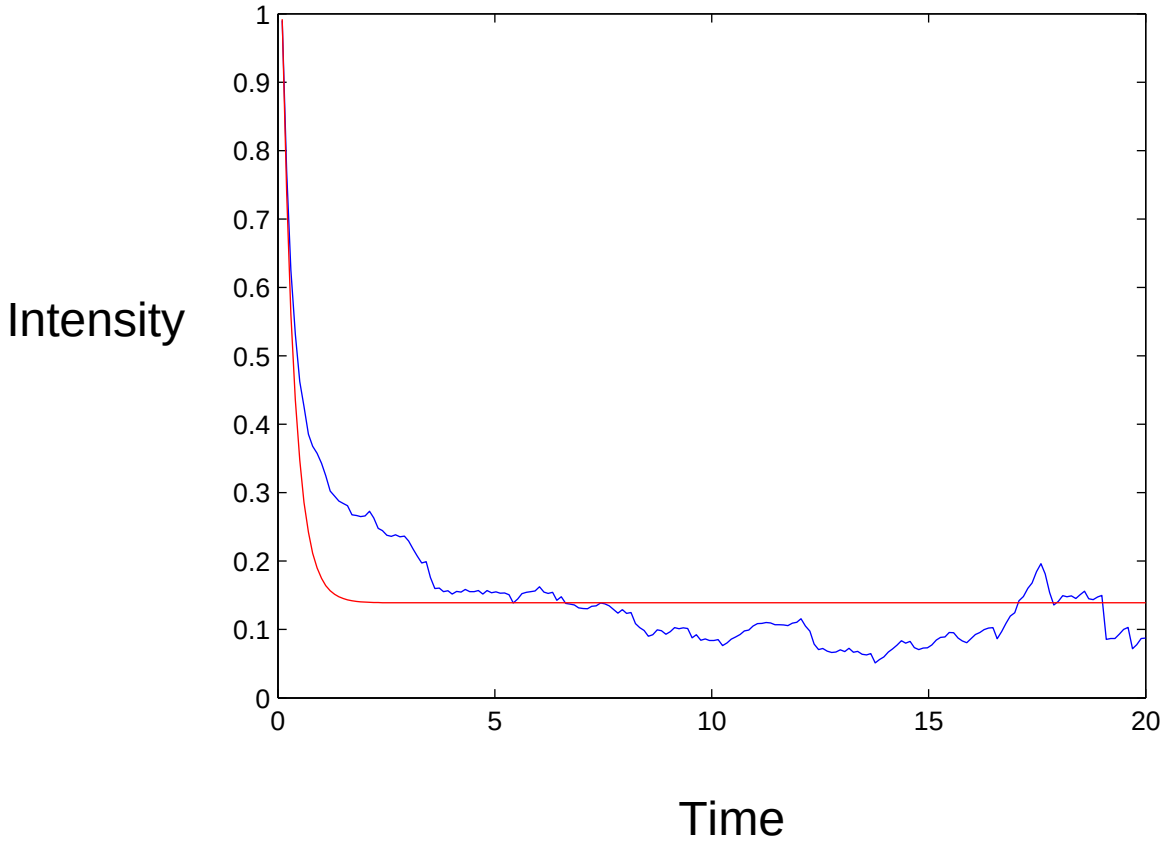
# “Most Recent” Conditional Intensity

- Suppose the most recent event occurred at time $t$. What is the intensity at time $t+s$?
- Want this to be

$$\lambda_{t+s} : (\text{most recent event at } t) = \mu + \alpha \int_{u<t} e^{-\beta(t-u)} dN_u$$

- In particular, it should decay to $\mu$!

# “Most Recent” Conditional Intensity



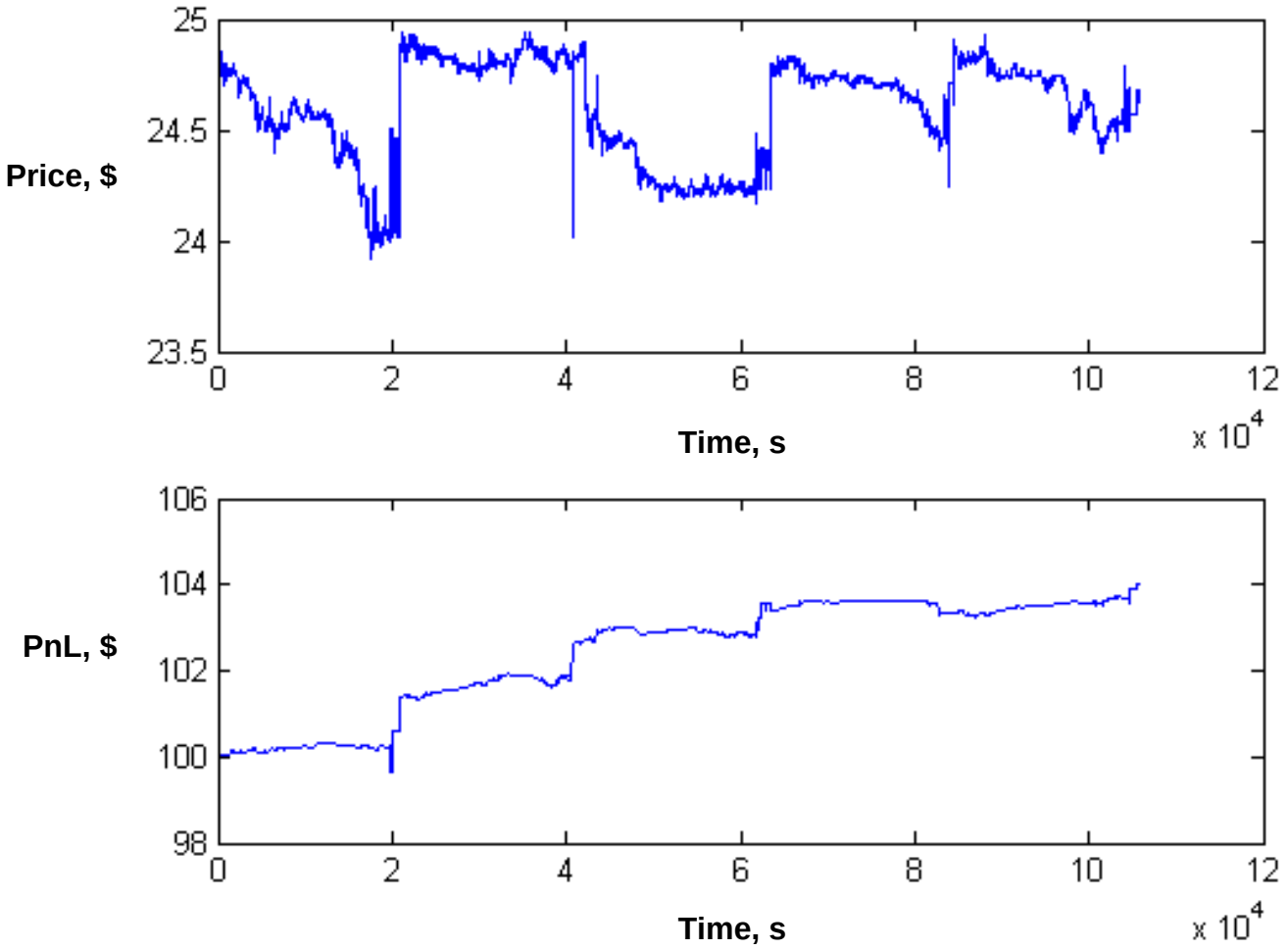
# Trading Strategy

- Performed across 3 counters (Dell, Yahoo, Oracle) from Nov 1$^{st}$ – 7$^{th}$ , 2006
- MLE parameters recalibrated 6 times a day, using 8000 second blocks
- Ignore Bid-Offer spread
- Ignore transaction costs
- No restrictions on short selling

# Heuristic

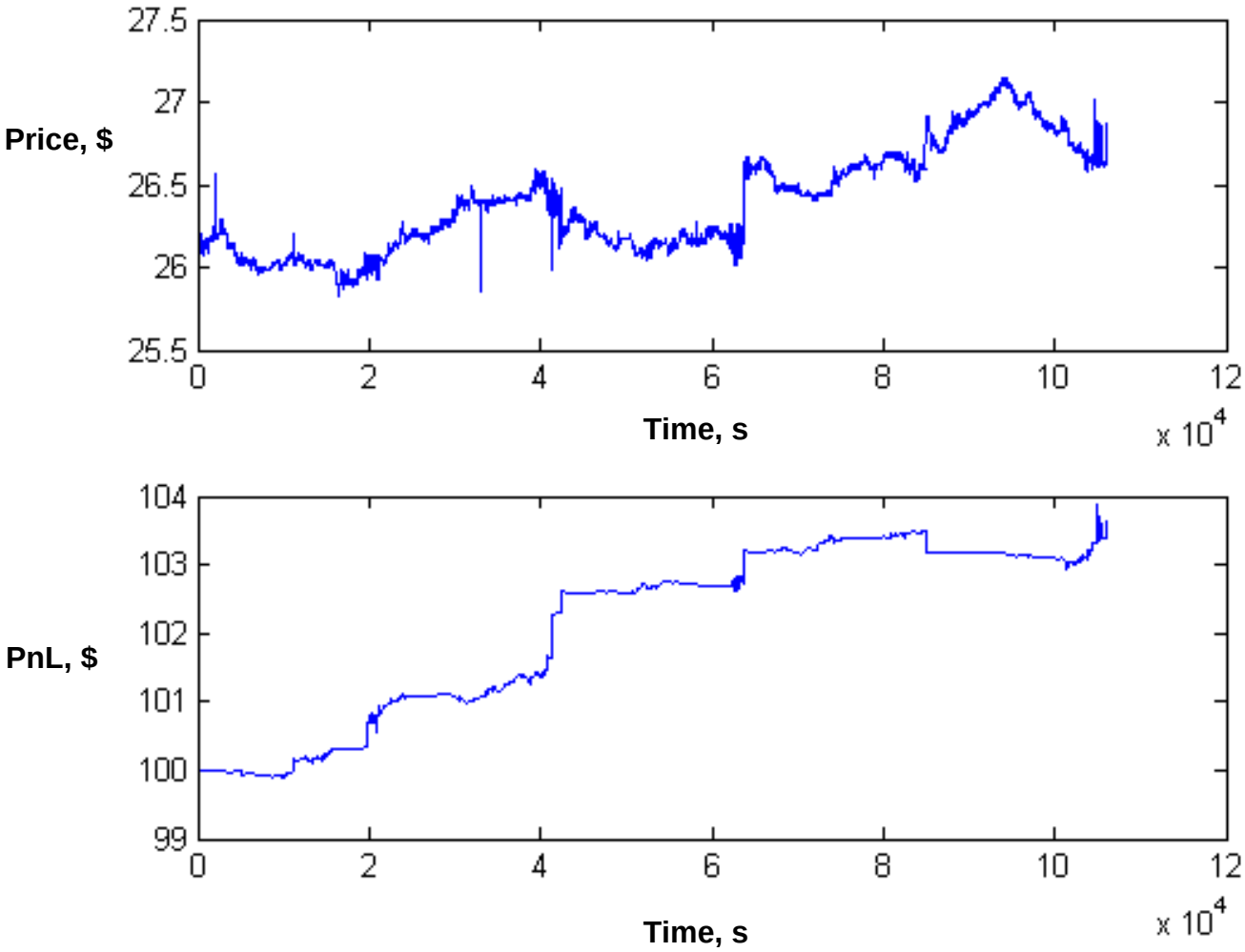
- Begin with $100
  - If buy/sell intensity ratio > 5, purchase stock and exit position 10 seconds later
  - If sell/buy intensity ratio > 5, short sell stock and cover position 10 seconds later

# Dell (DELL)



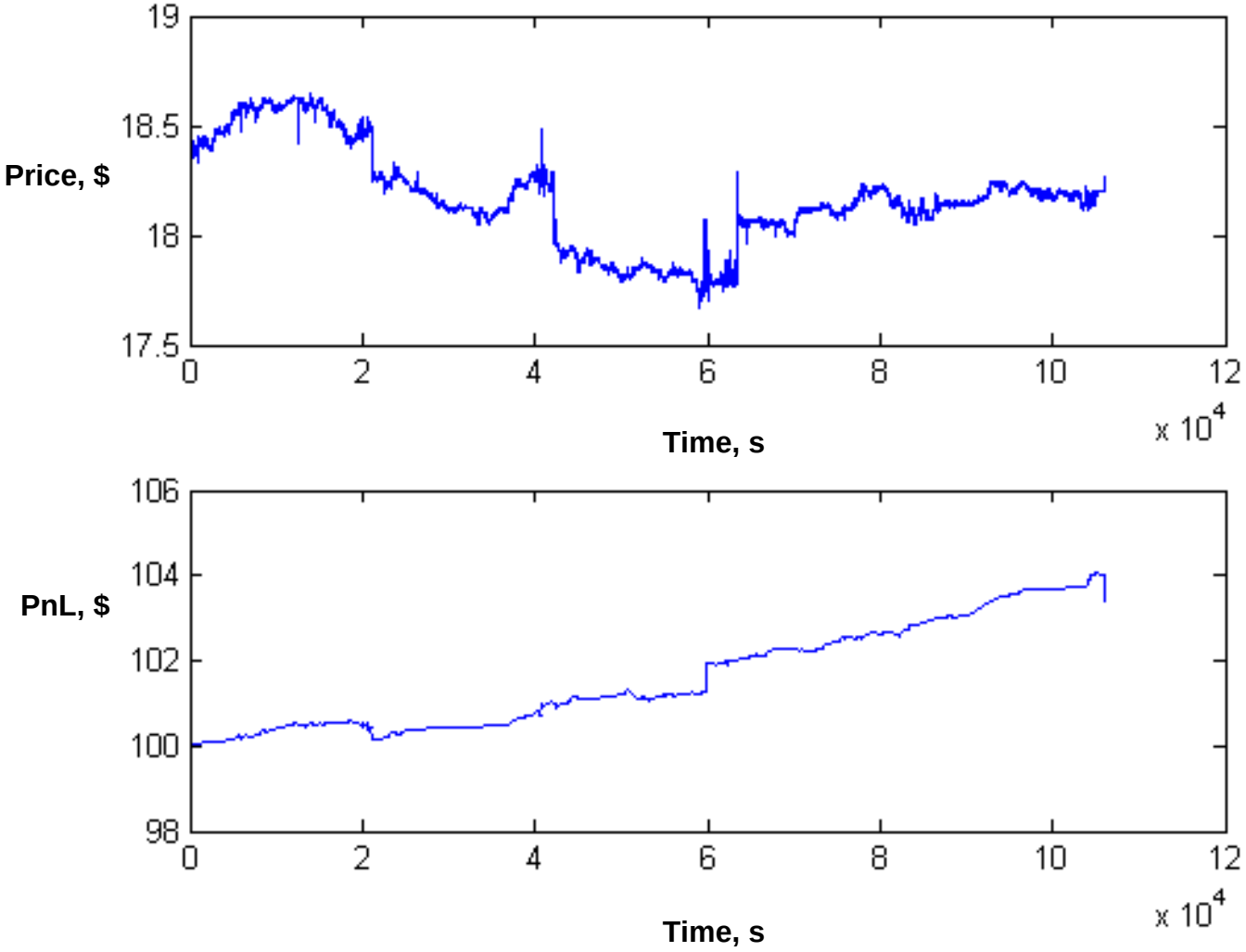

$104.03

# Yahoo (YHOO)




$103.97

# Oracle (ORCL)




$103.71

# Challenges & Further Work

- Challenges
  - MLE algorithm computationally intensive
- Further Work
  - Incorporate volume into model
  - Cross counter trading strategies