Exciting Times for Trade Arrivals

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Road Map

- use of a bivariate Hawkes process to model buy and sell trade arrivals
- a more general Hawkes model
  - much, much faster to fit
  - more accurate function for excitation decay
  - higher likelihood
  - more flexible to incorporate additional data sources ($e.g.$ price, volume, market index, option data)
- simple trading strategy showing promising results

Bivariate Hawkes Model

- processes $N_1(t), N_2(t)$ that correspond to buy, sell orders
- intensities at time t, for $i = 1, 2$:

$$\lambda_i(t) = \mu_i + \int_{u<t} h_{i1}(t-u)dN_1(u) + \int_{u<t} h_{i2}(t-u)dN_2(u)$$

- log-likelihood function

$$\mathcal{L} = \int_{u<t} -(\lambda_1(u) + \lambda_2(u))\, du + \log(\lambda_1(u))dN_1(u) + \log(\lambda_2(u))dN_2(u)$$

Traditional Hawkes Model Fit

- basic assumption: exponential decay
- fitting done by maximizing log-likelihood $\mathcal{L}$:

$$\begin{array}{ll} \text{maximize} & \mathcal{L} \\ \text{subject to} & h_{ij}(t) = \alpha_{ij} e^{-\beta_{ij} t} \\ & \text{positivity, stationarity} \end{array}$$

- this optimization program involves typically 10 variables
- but, it is not convex...

  $\Rightarrow$ no efficient way of solving it
  $\Rightarrow$ no optimality certificate

A New Way To Do It...

- idea: consider a time-limited, piecewise linear form of $h_{ij}(t)$
  - if the length of each piece δt is $\leq$ to the time resolution of the data, no information is lost
  - exponentials are also 'time-limited', since they die off after 20-30 secs
- intensities can be written as:

$$\lambda_i(t) = \mu_i + \sum_{k=t-n}^{t-1} w_{i1}(t-k)N_1(k\delta t) + \sum_{k=t-n}^{t-1} w_{i2}(t-k)N_2(k\delta t)$$

where $h_{ij}(t)$ consists of n pieces, with the t-th having value $w_{ij}(t)$

Problem Formulation

the fitting problem can now be cast as a convex optimization program:

$$
\begin{array}{ll}
\text{maximize} & \mathcal{L} \\
\text{subject to} & \mu_i \geq \epsilon \\
& w_{ij} \succeq 0 \\
& z_{ii} \geq \epsilon \\
& \frac{z_{12}^2}{z_{11}} \leq z_{22} \\
& z_{ii} = 1 - \mathbf{1}^T w_{ii} \\
& z_{ij} = \mathbf{1}^T w_{ij} \\
& z_{12} = z_{21} + s \\
& s \geq 0
\end{array}
$$

- constraints correspond to positivity and stationarity

- the piecewise linear form of $h_{ij}(t)$ results in many more variables, $w_{ij}(t)$, typically 200
- still, it can now be readily solved (using a primal-dual interior point solver for instance)
  - much, much faster
  - with an optimality certificate

Regression Interpretation

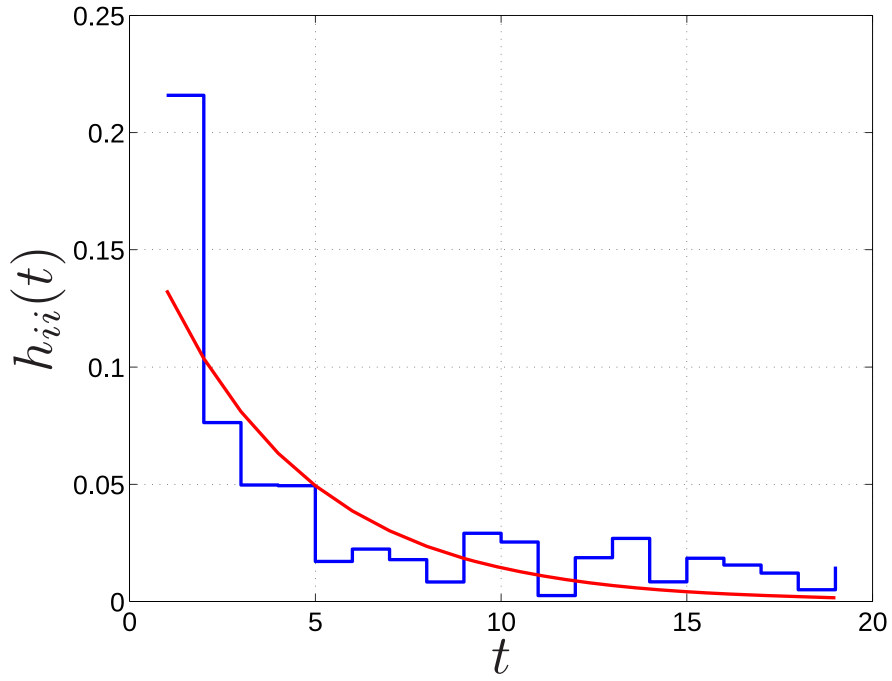
- this model corresponds to a linear Poisson regression with constraints — with features being past data

- other information can easily be used as features: price, volume, market index, option data

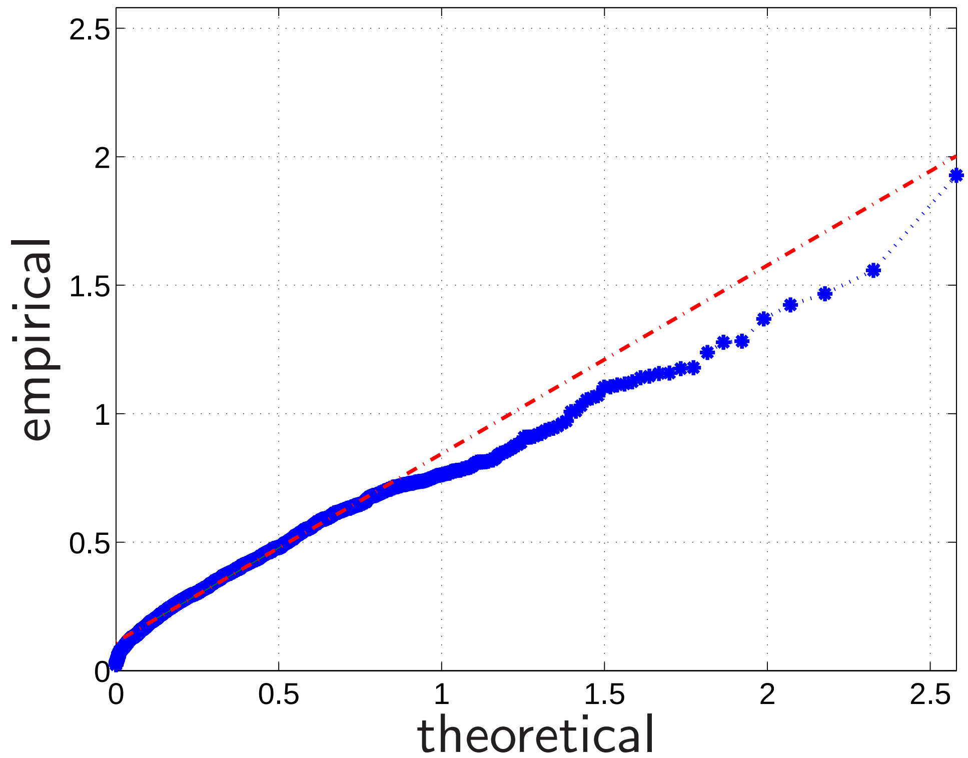
Comparison with Hewlett (2006)

- fitting time drops from 1 hour to 30 sec!!!

- mle objective value is also 33% higher!



QQ plot



(data from YHOO 07/03/06)

Strategy Simulation

- data from NYSE TAQ
- market assumptions
  - discrete time (1-second resolution for TAQ)
  - we execute a trade at worst possible price over next five seconds
  - trades do not affect the market
  - no short selling constraints

Use of Hawkes Model

- types:
  - exponential
  - piecewise linear

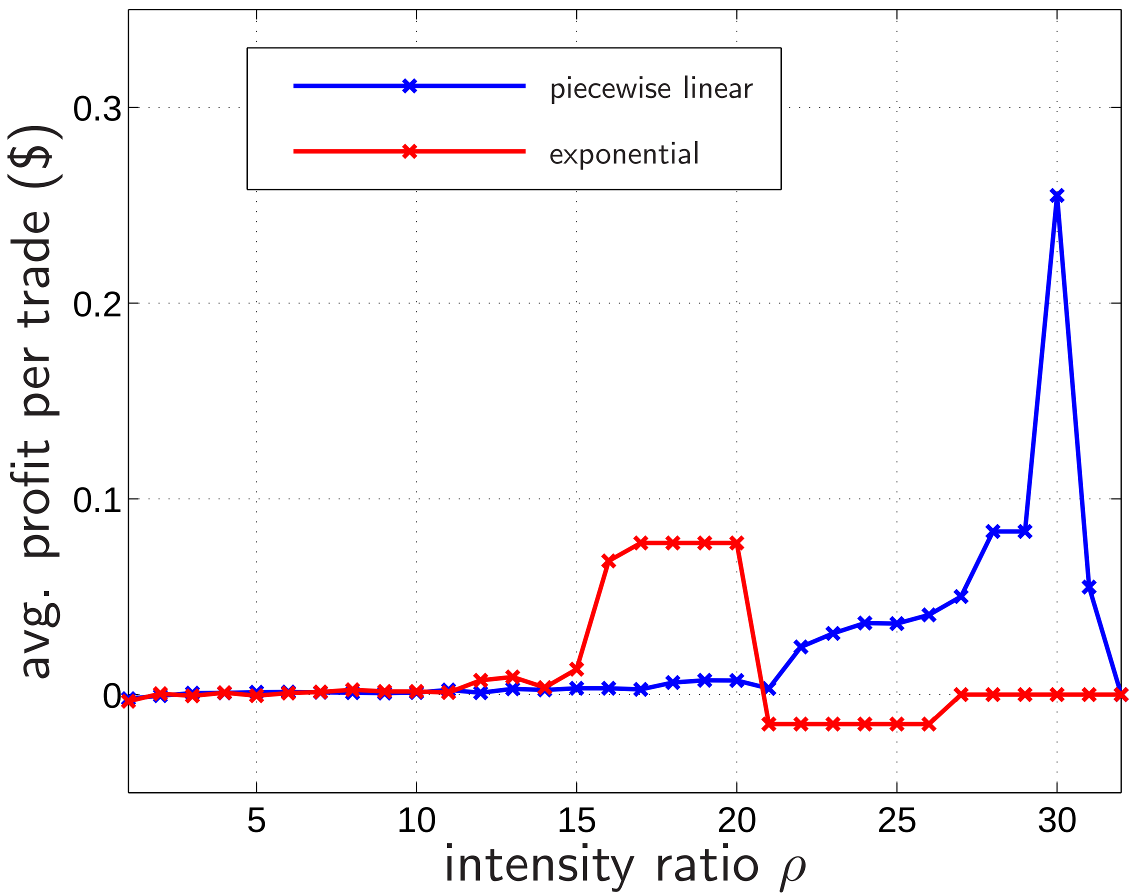
- parameter estimation using data from the previous day

- parameters applied to calculate buy and sell intensities, $\lambda_{\text{buy}}, \lambda_{\text{sell}}$ throughout trading days

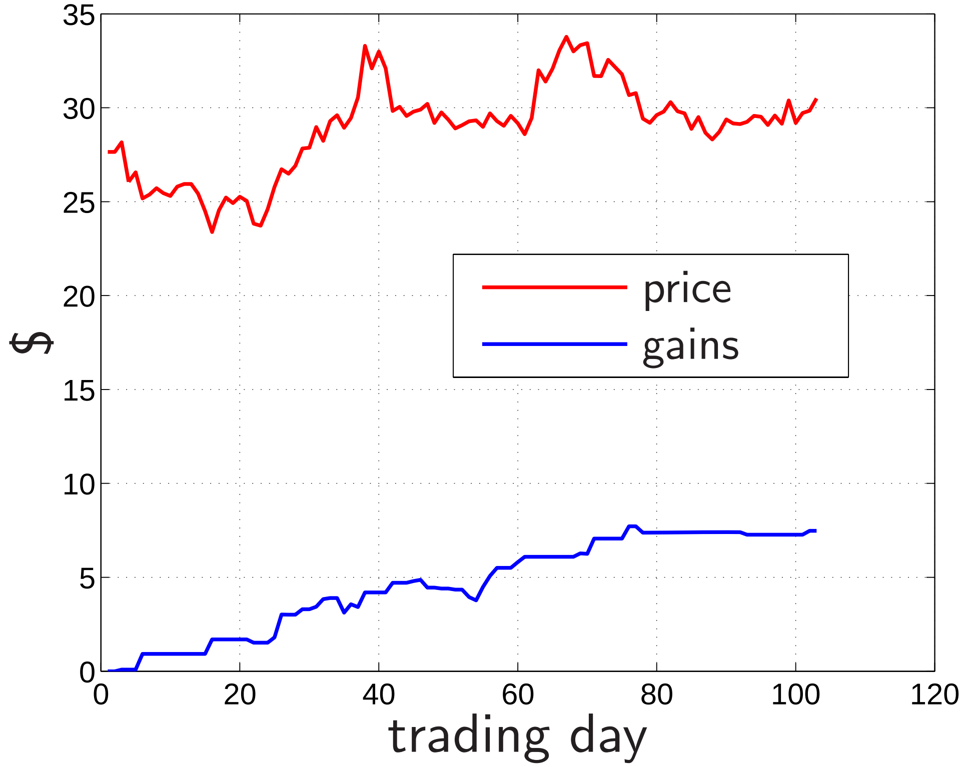
Scalp Reverse Strategy

- one share position limit
- target intensity ratio ρ that defines enter strategy:
  - buy when $\lambda_{\text{buy}}/\lambda_{\text{sell}} > \rho$
  - sell when $\lambda_{\text{sell}}/\lambda_{\text{buy}} > \rho$
- no explicit exit strategy
- can only reverse positions intraday
- liquidate position at end of day

Performance for MSFT, 01/03/07-01/11/07



Performance for TIE, 09/01/06-01/11/07



$\rho = 30$, average profit per trade $\$.087$, 86 trades (43 roundturns)

Performance for Other Stocks

stock	avg. profit/stock/day	no. of trades	ρ	period
YHOO	\$.00565	586	17	07/03/06 – 01/31/07
SNDK	\$.00755	36	15	08/01/06 – 12/29/06
VPHM	\$.01322	14	15	03/08/06 – 03/24/06
XOM	\$.08152	24	7.5	01/03/07 – 01/11/07

Good Model, Bad Strategy?

- technical indication of fundamental change?
- improvements:
  - trade size increases with intensity ratio threshold
  - exit strategy depending on price, time, and/or intensity
- incorporate price, volume or market index information in features