A model of disease spread and containment

Benjamin Armbruster

joint work with Margaret L. Brandeau

March 14, 2006 / Stanford MS&E 292 guest lecture

Goal

- What is the cost-minimizing mix of screening and contact tracing
  - in order to find n disease cases?
  - in order to keep long-term disease prevalence below P ?
  - in order to reduce disease prevalence below P by time T ?

Intervention 1: random screening

routine/recommended screening used in high risk groups

- TB screening
  - school employees
  - prisons
  - immigrants
- syphilis screening for pregnant women
- STD screening for high risk populations

Intervention 2: contact tracing

health care provider's perspective:

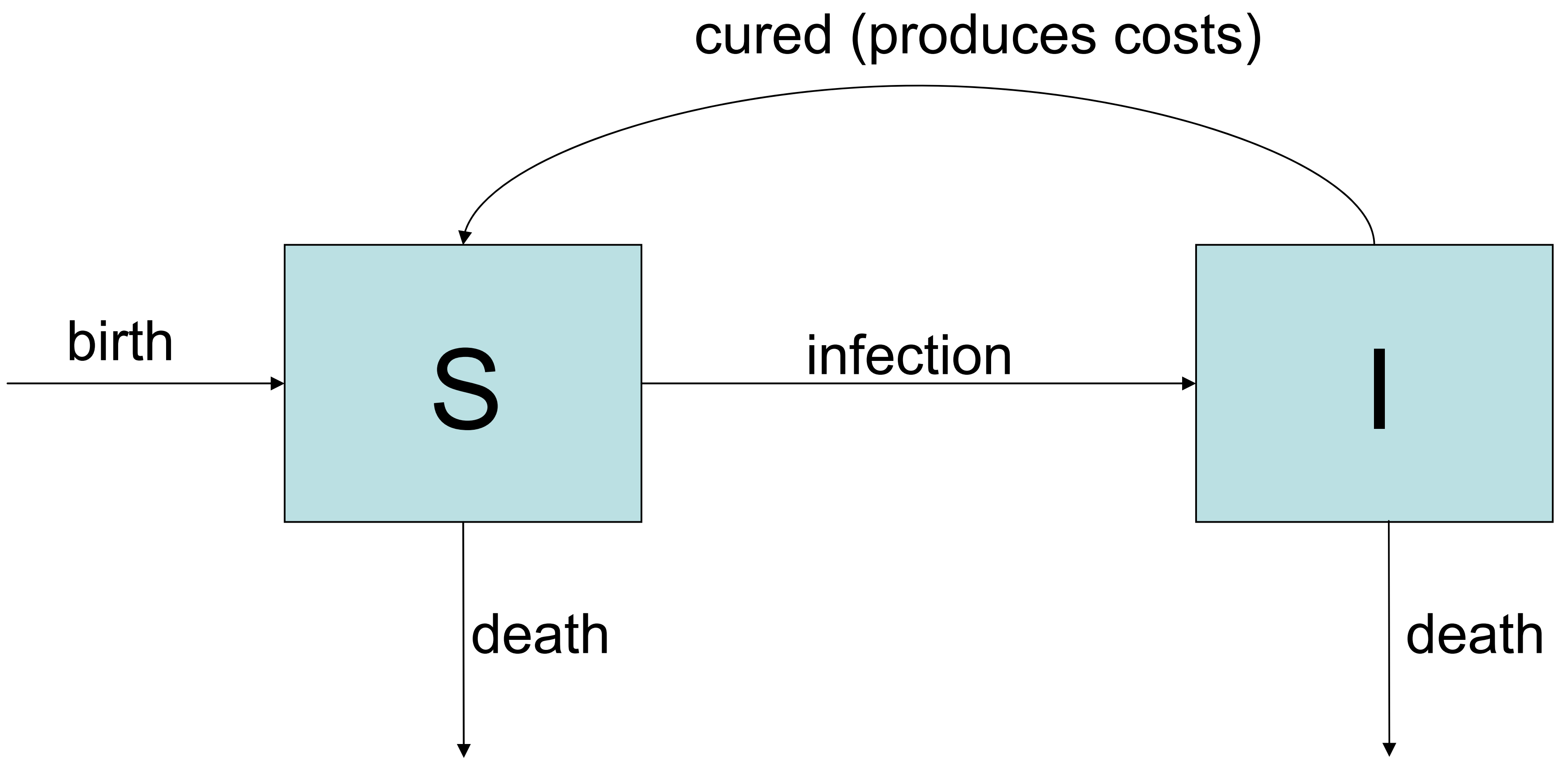
- infected person found (*index case*)
- treated
- asked for list of *contacts*
- contacts found and tested
- if contact infected go to step 1.

- standard practice for Tuberculosis (TB)
- common for HIV and other STDs
  - called *partner notification*

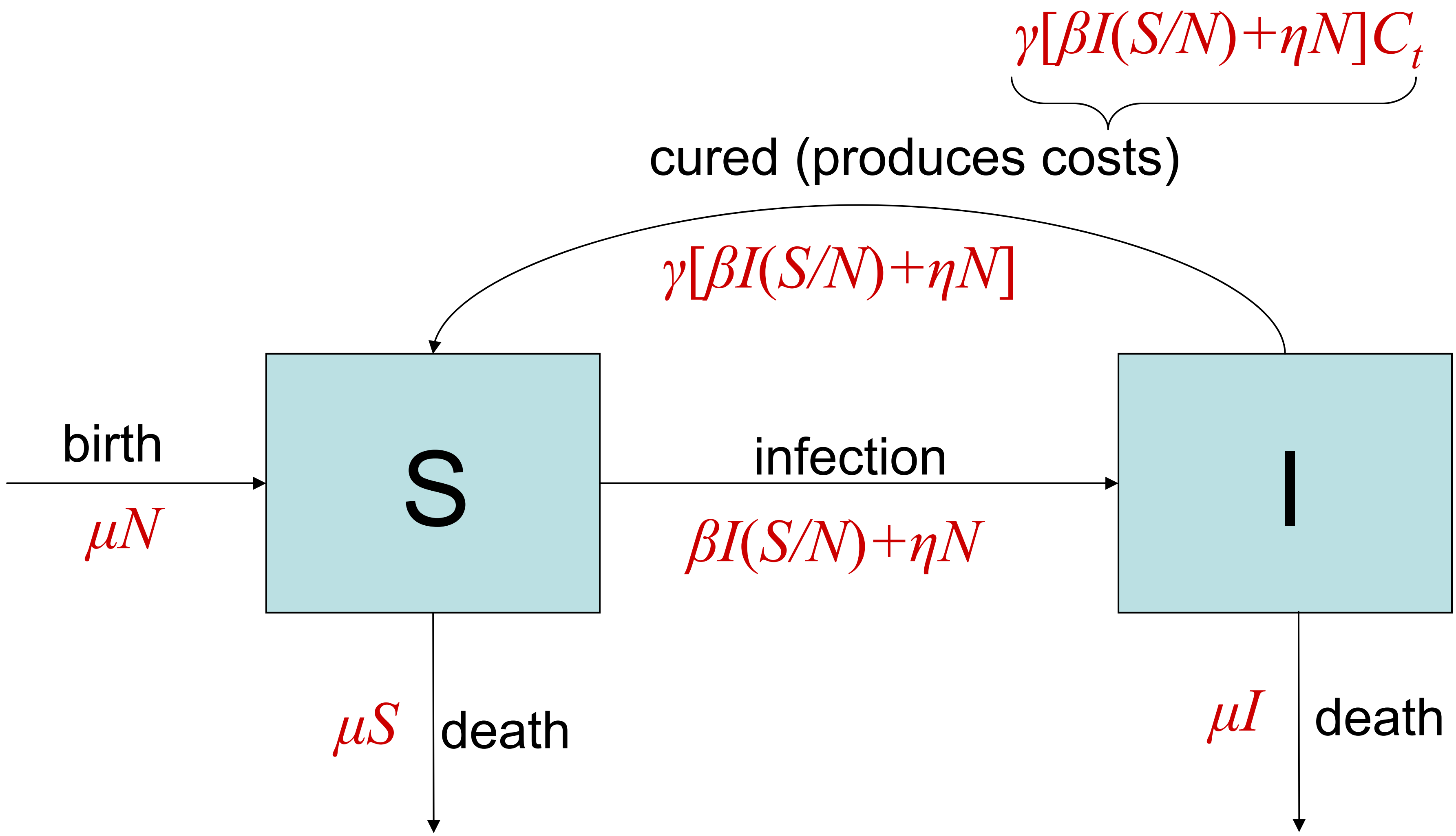
Outline

1. Model the dynamics
2. Optimal policies
3. Discussion
4. An extended model

S→I→S model

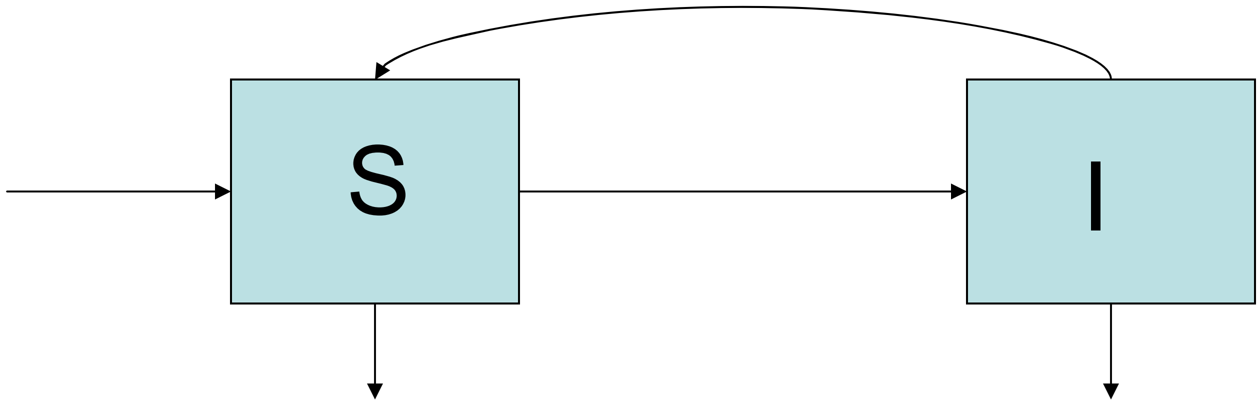


S→I→S model



$N=S+I,\ N'=0$

S→I→S model



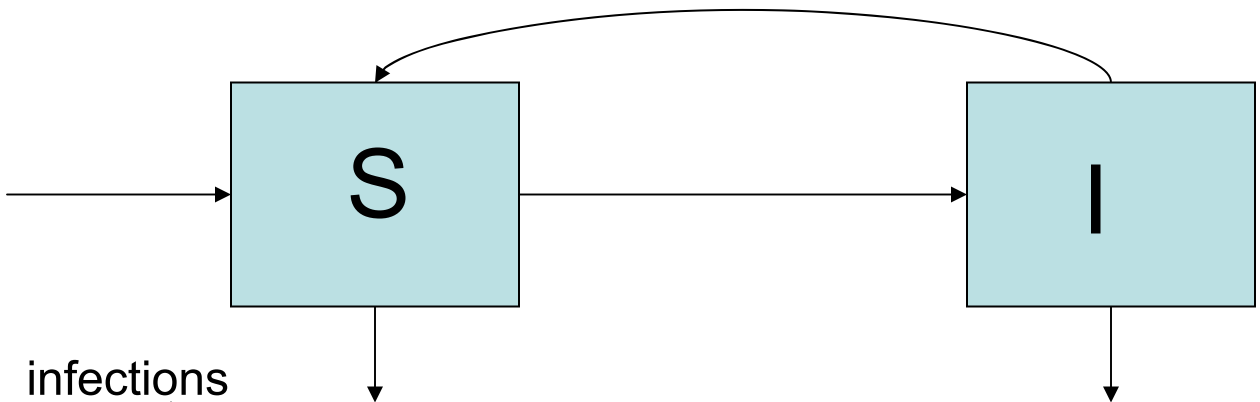
$$S' = -[\beta I(S/N)+\eta N] - \mu S + \gamma[\beta I(S/N)+\eta N]+\mu N$$

$$I' = +[\beta I(S/N)+\eta N] - \mu I - \gamma[\beta I(S/N)+\eta N]$$

$$N=S+I$$

$$\hat{C} = \gamma[\beta I(S/N)+\eta N]C_t$$

S→I→S model



$$S' = -\overbrace{[\beta I(S/N)+\eta N]}^{\text{infections}} - \mu S + \gamma[\beta I(S/N)+\eta N] + \overset{\text{births}}{\mu N}$$

$$I' = +[\beta I(S/N)+\underset{\text{infections from outside sources}}{\eta N}] - \underset{\text{deaths}}{\mu I} - \underbrace{\gamma[\beta I(S/N)+\eta N]}_{\text{get help because of symptoms; cured } = \omega(p)}$$

$$N = S + I$$

$$\hat{C} = \gamma[\beta I(S/N)+\eta N]C_t$$

Reduced model

$$S' = -[\beta I(S/N) + \eta N] - \mu S + \gamma[\beta I(S/N) + \eta N] + \mu N$$

$$I' = +[\beta I(S/N) + \eta N] - \mu I - \gamma[\beta I(S/N) + \eta N]$$

$$\hat{C} = \gamma[\beta I(S/N) + \eta N] C_t$$

$$p = I/N \qquad C = \hat{C}/N$$

$$p' = [\beta p(1 - p) + \eta] - \mu p - \omega(p)$$

$$C = \omega(p) C_t$$

Intervention 1: random screening

random screening at rate λ

- reduces prevalence at rate λp
- cost per capita is $\lambda(C_S + C_t p)$

$$p' = [\beta p(1-p) + \eta] - \mu p - \omega(p) - \lambda p$$

$$C = \omega(p) C_t + \lambda(C_S + C_t p)$$

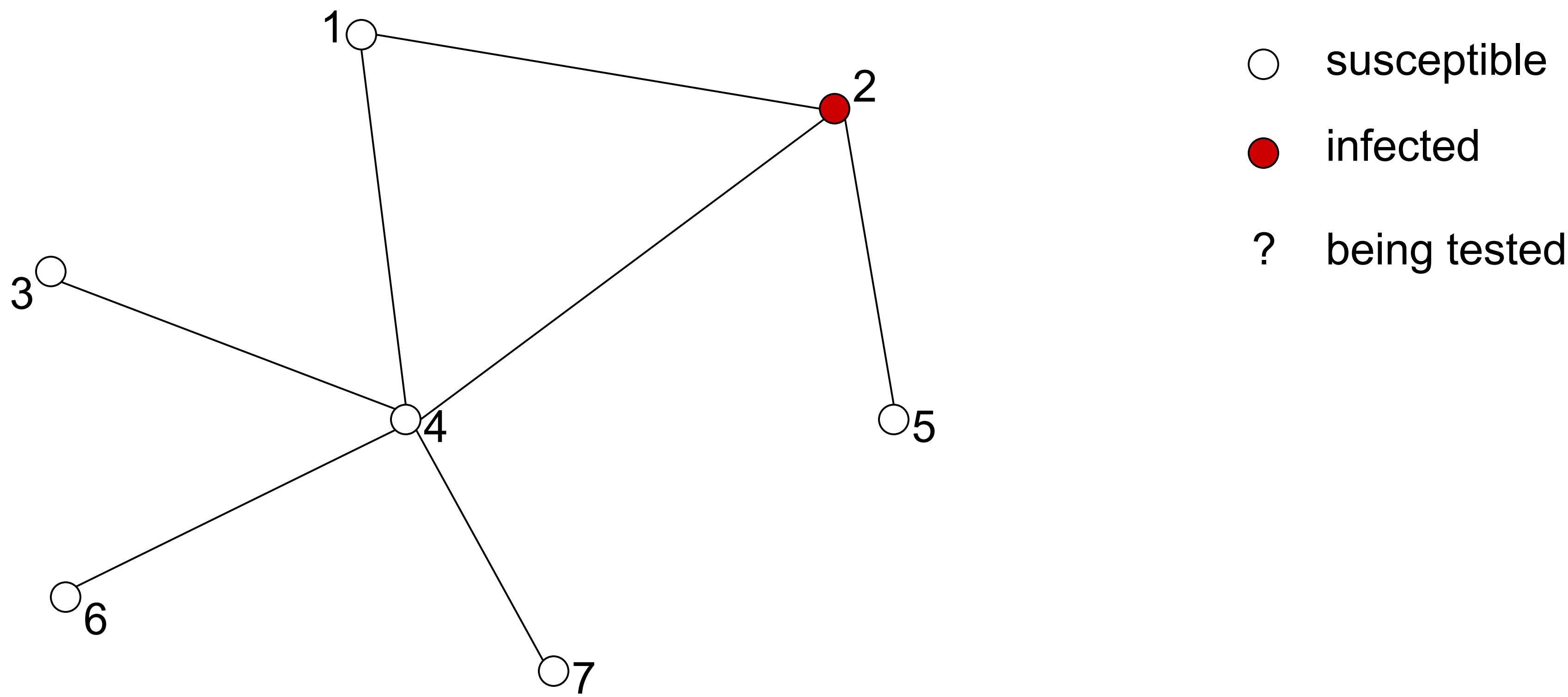
Intervention 2: contact tracing

health care provider's perspective:

- infected person found (*index case*)
- treated
- asked for list of *contacts*
- contacts found and tested
- if contact infected go to step 1.

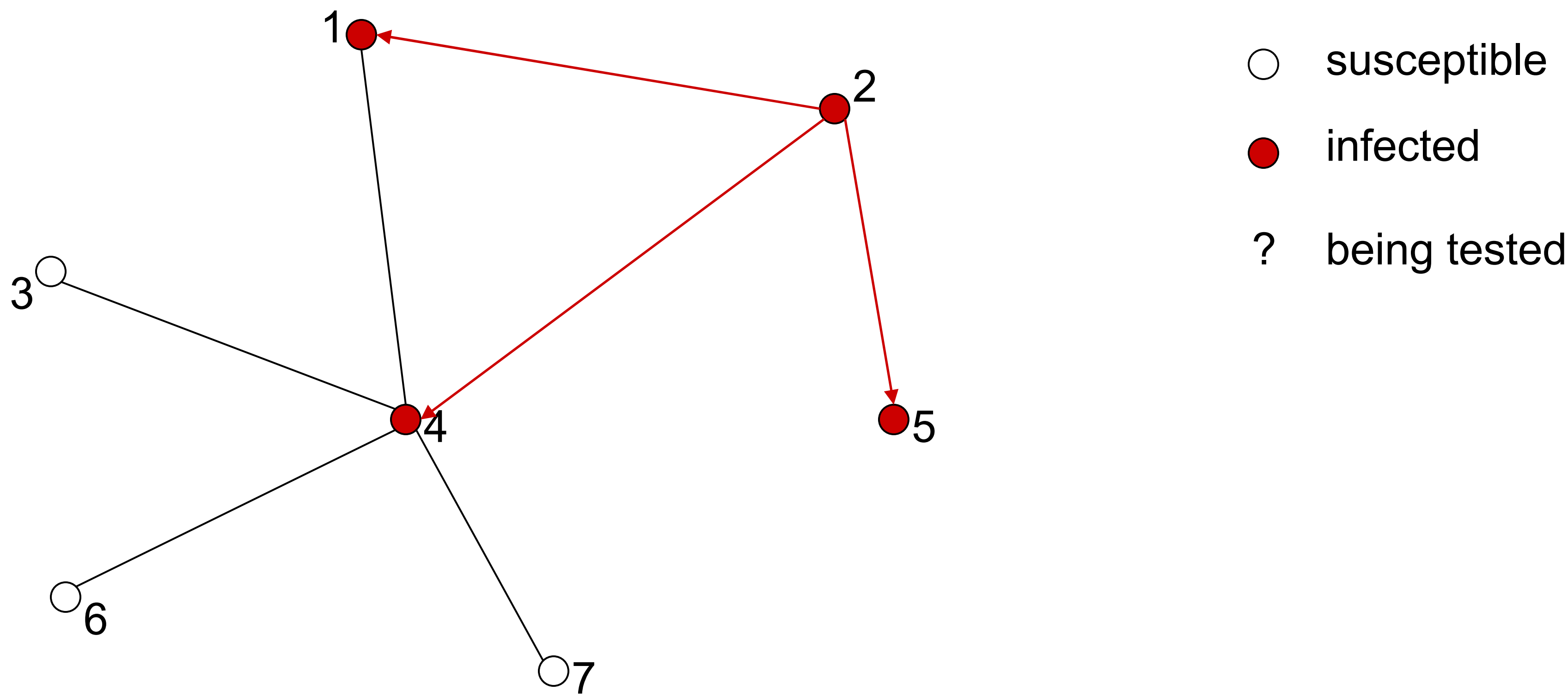
- standard practice for Tuberculosis (TB)
- common for HIV and other STDs
  - called *partner notification*

Intervention 2: contact tracing



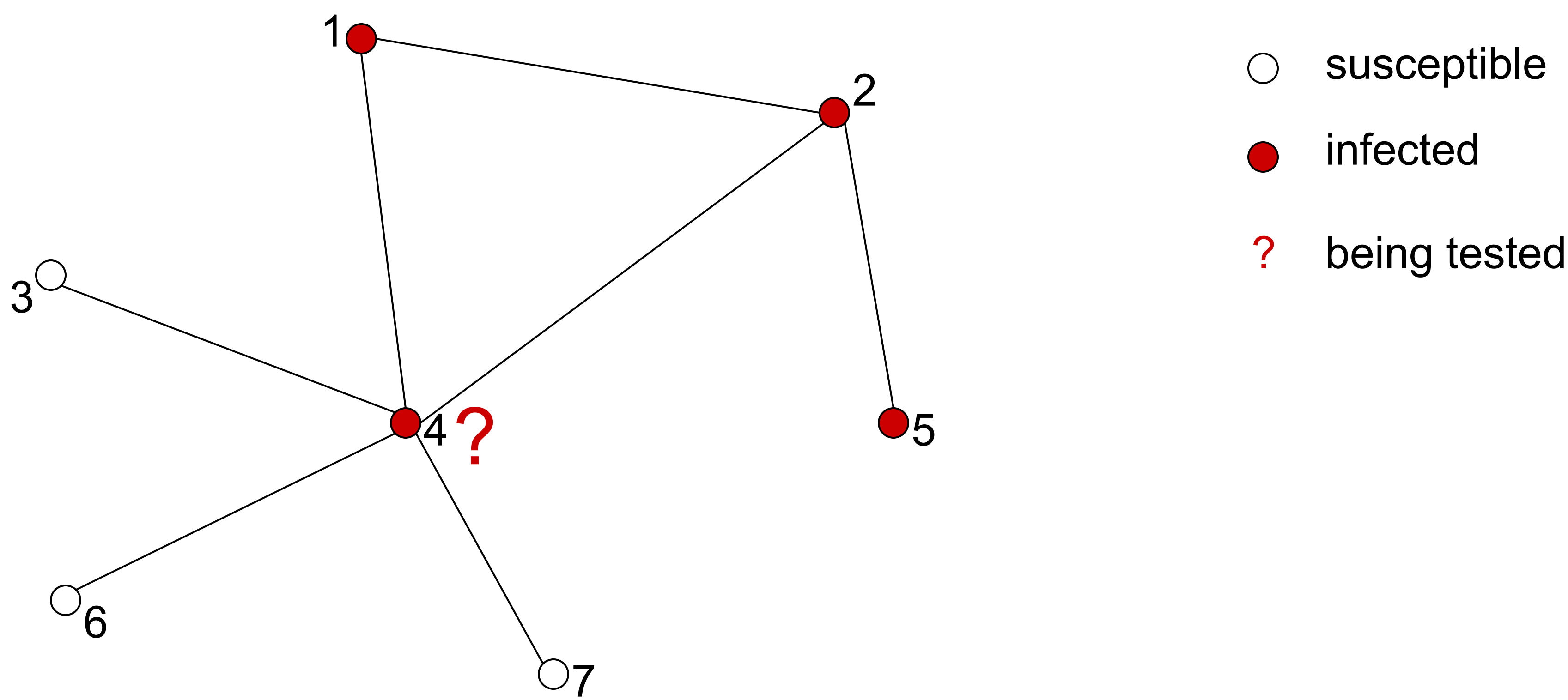
node 2 infected

Intervention 2: contact tracing



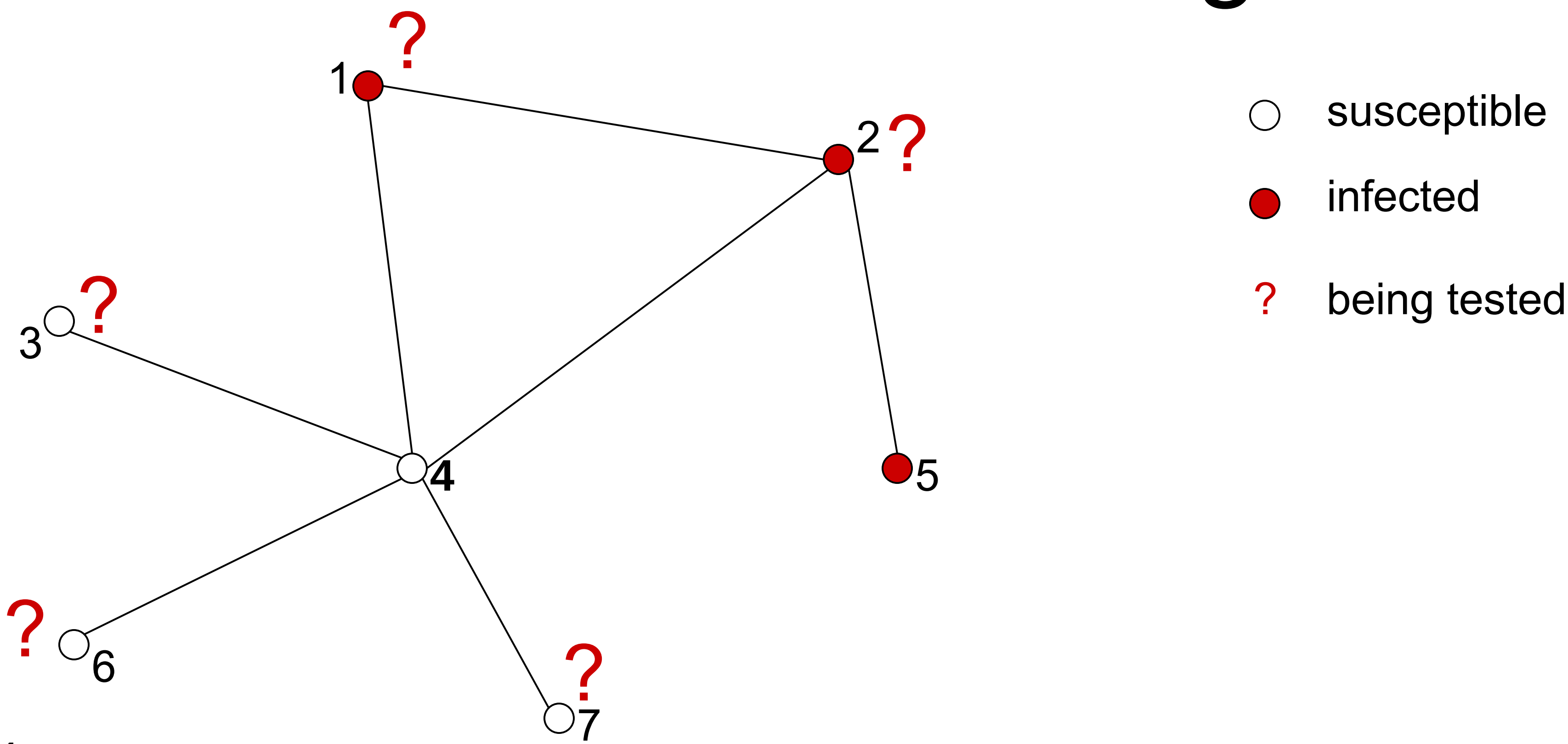
node 2 infects nodes 1,4,5

Intervention 2: contact tracing



node 4 gets tested (maybe has symptoms)

Intervention 2: contact tracing

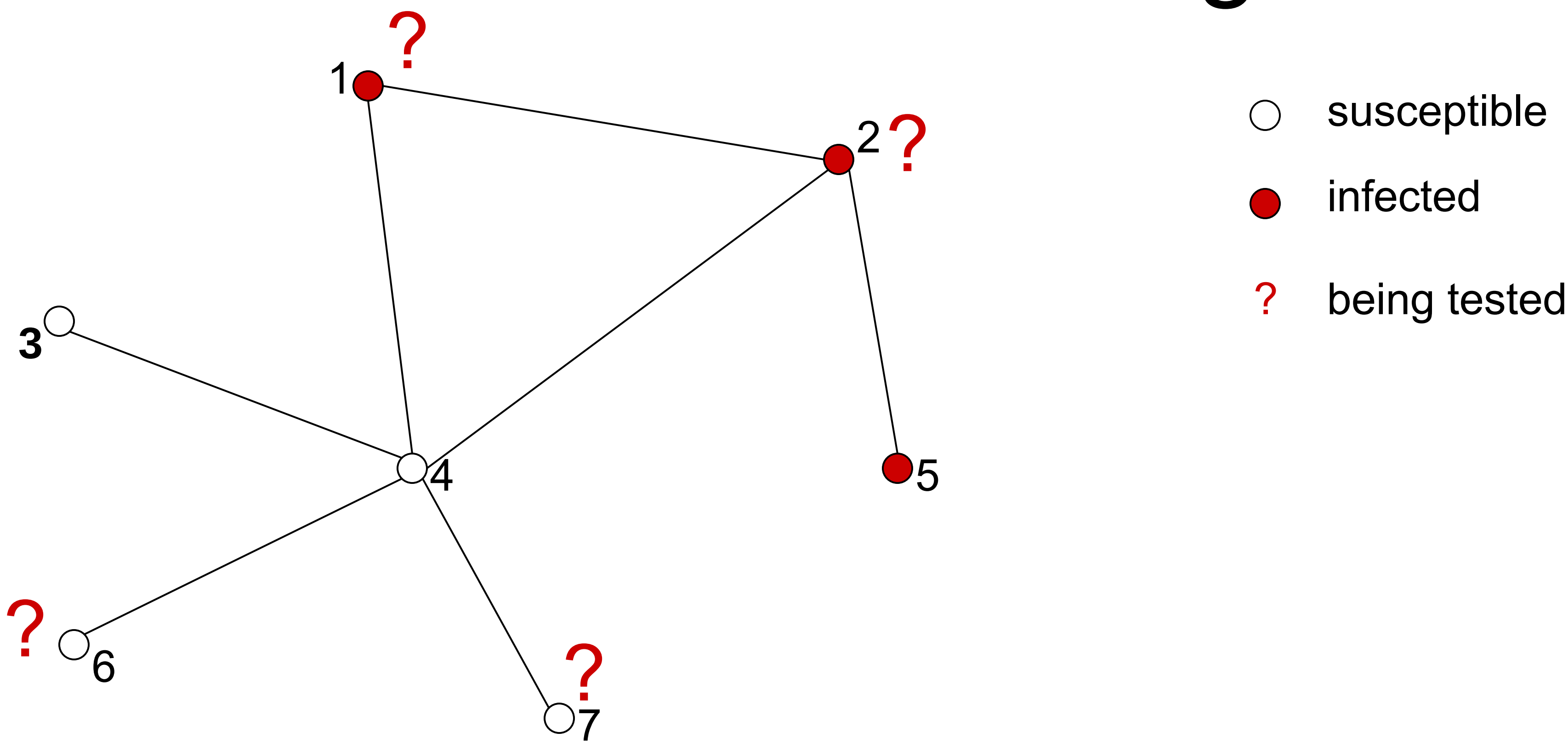


node 4

- tests positive, gets treated
- becomes a contact tracing *index case*
- names nodes 1,2,3,6,7 as contacts

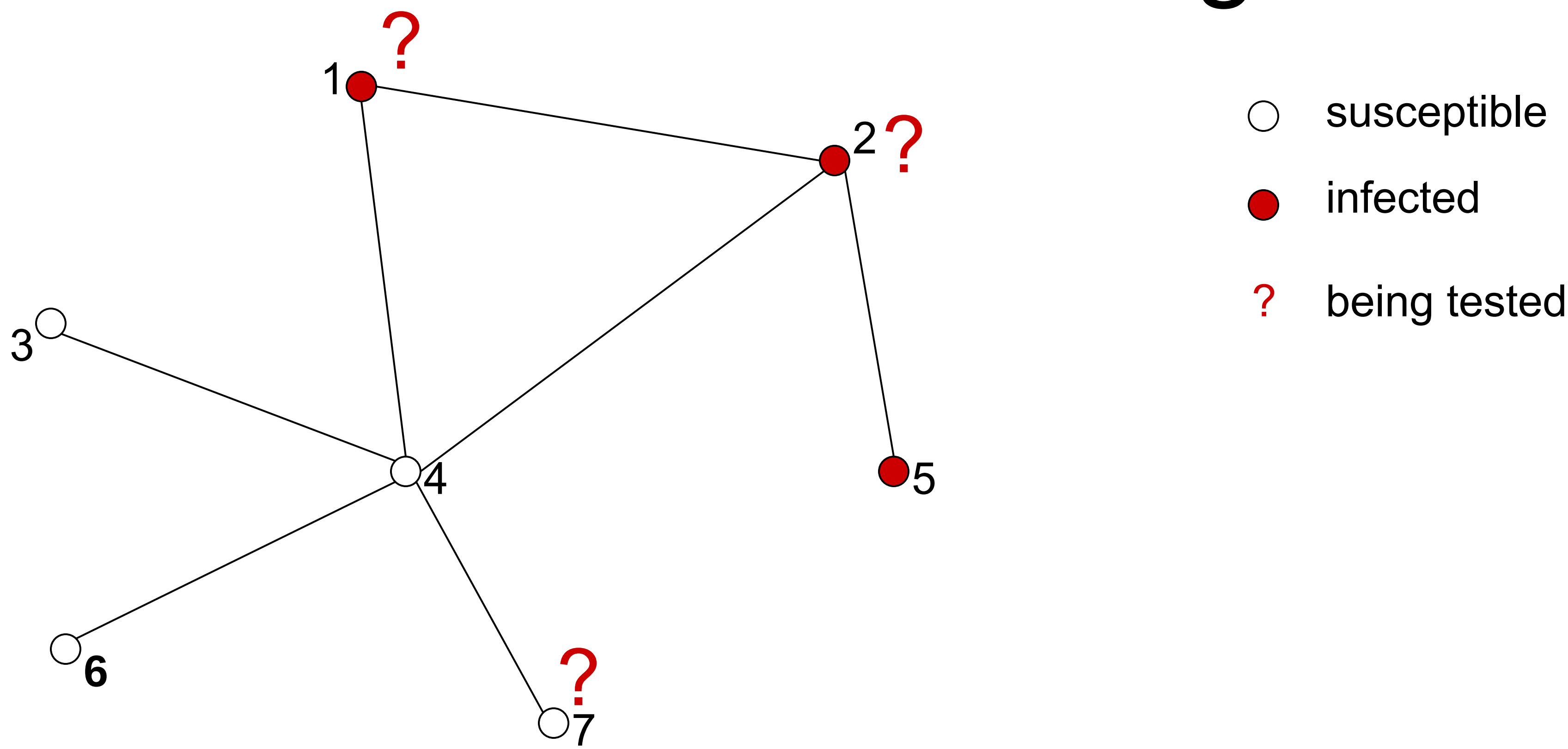
nodes 1,2,3,6,7 scheduled to be tested

Intervention 2: contact tracing



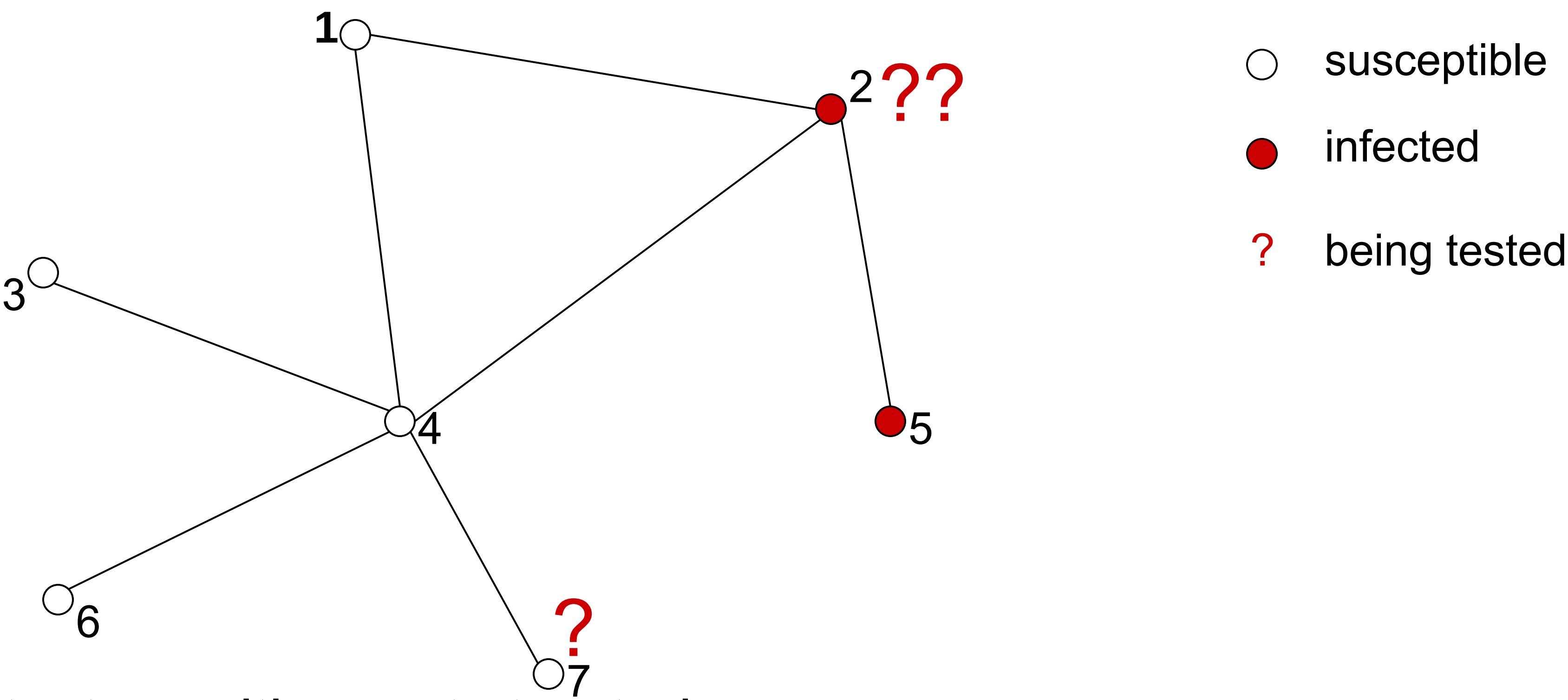
node 3 tests negative

Intervention 2: contact tracing



node 6 tests negative

Intervention 2: contact tracing



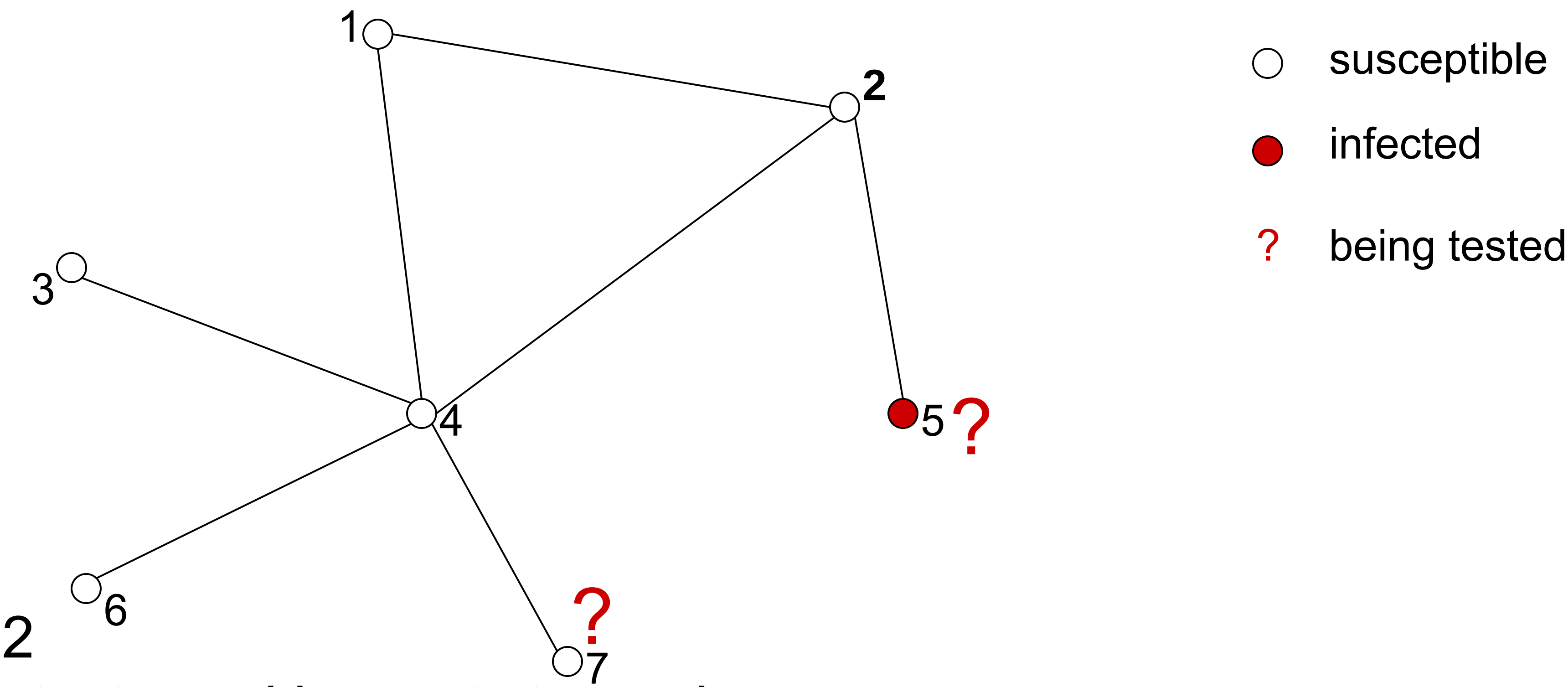
node 1

- tests positive, gets treated
- becomes a contact tracing *index case*
- names nodes 2,4 as contacts

node 4 already tested

testing node 2 gets higher priority as named by both nodes 1,4

Intervention 2: contact tracing

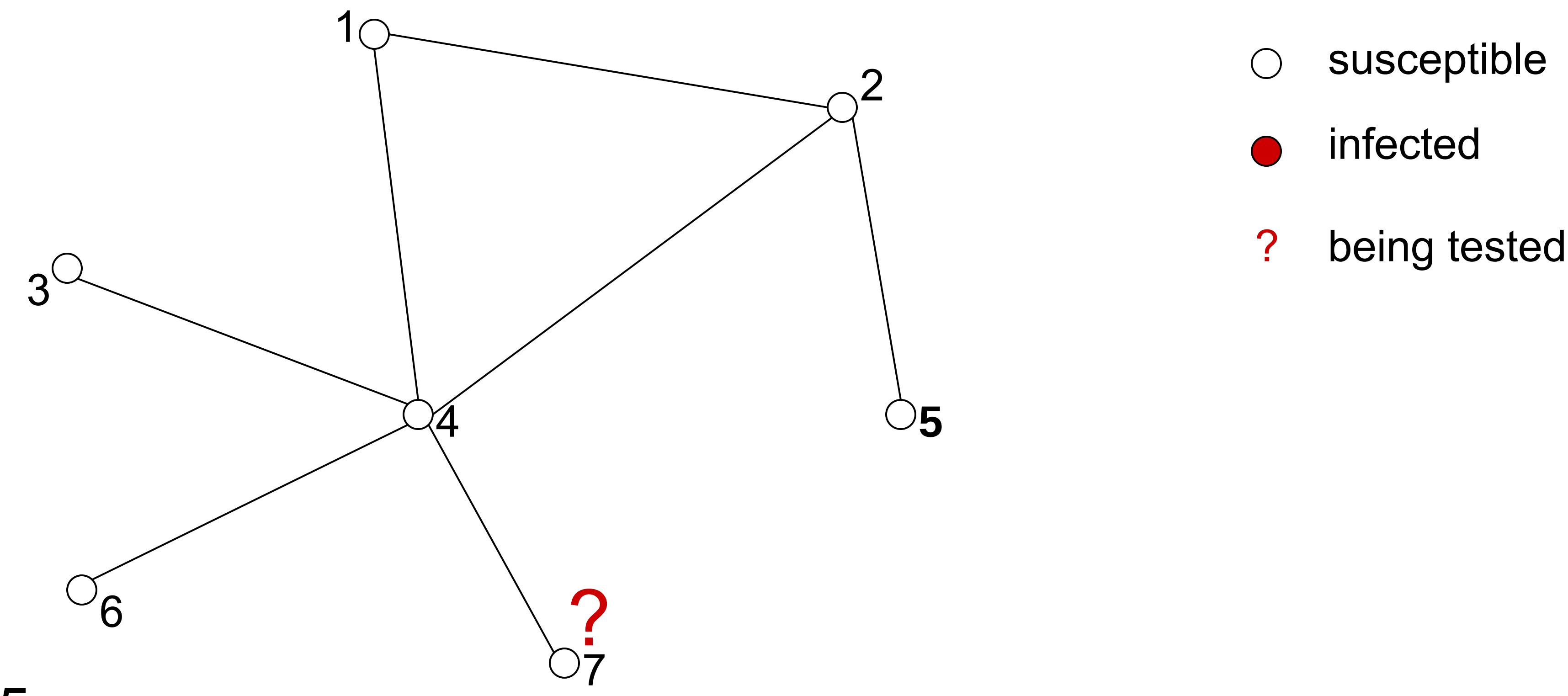


node 2

- tests positive, gets treated
- becomes a contact tracing *index case*
- names nodes 1,4,5 as contacts

nodes 1,4 already tested
node 5 scheduled to be tested

Intervention 2: contact tracing

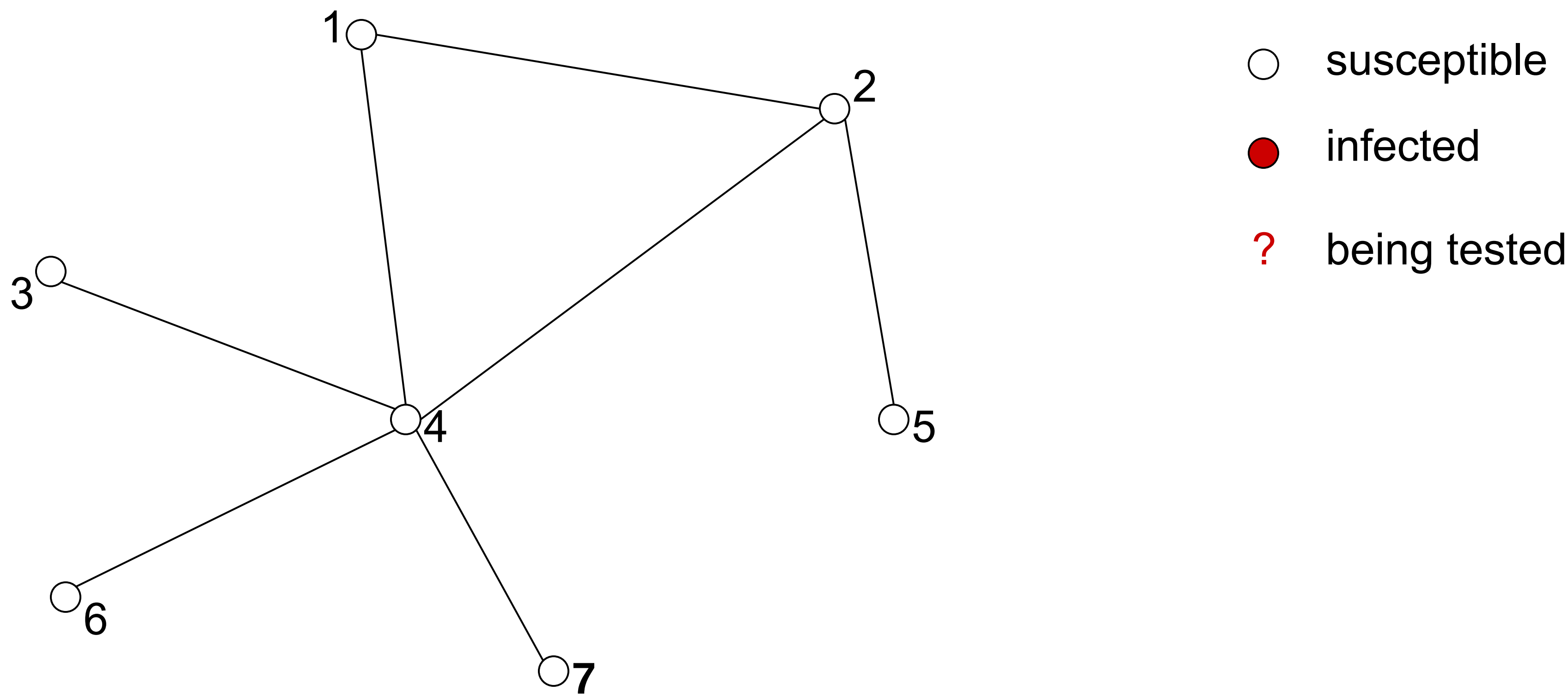


node 5

- tests positive, gets treated
- becomes a contact tracing *index case*
- names node 2 as a contact

node 2 already tested

Intervention 2: contact tracing



node 7 tests negative

Intervention 2: contact tracing

- $\delta=1$ if program exists, 0 if it does not
- K_T number of infected contacts per index case
  - $\delta K_T(\lambda p+\omega(p))$ total
- CT cost per index case
  - $\delta C_T(\lambda p+\omega(p))$

$$p' = [\beta p(1-p)+\eta] - \mu p - (1+\delta K_T)(\lambda p+\omega(p))$$

$$C(\lambda,\delta;p) = \omega(p)C_t + \lambda(C_S + C_t p) + \delta C_T(\lambda p+\omega(p))$$

Optimal intervention

tradeoff C and p

by choosing λ and δ

1. small changes (unchanged prevalence)
2. long term costs ($p' = 0$)
3. transition costs

Unchanged prevalence

$$
\begin{aligned}
Cost = \min_{\lambda,\delta} \quad & C(\lambda,\delta;p) \\
s.t. \quad & N(1+\delta K_T)\,(\lambda p+ \omega(p))=n \\
& \lambda \geq 0,\ \delta=0,1
\end{aligned}
$$

Unchanged prevalence

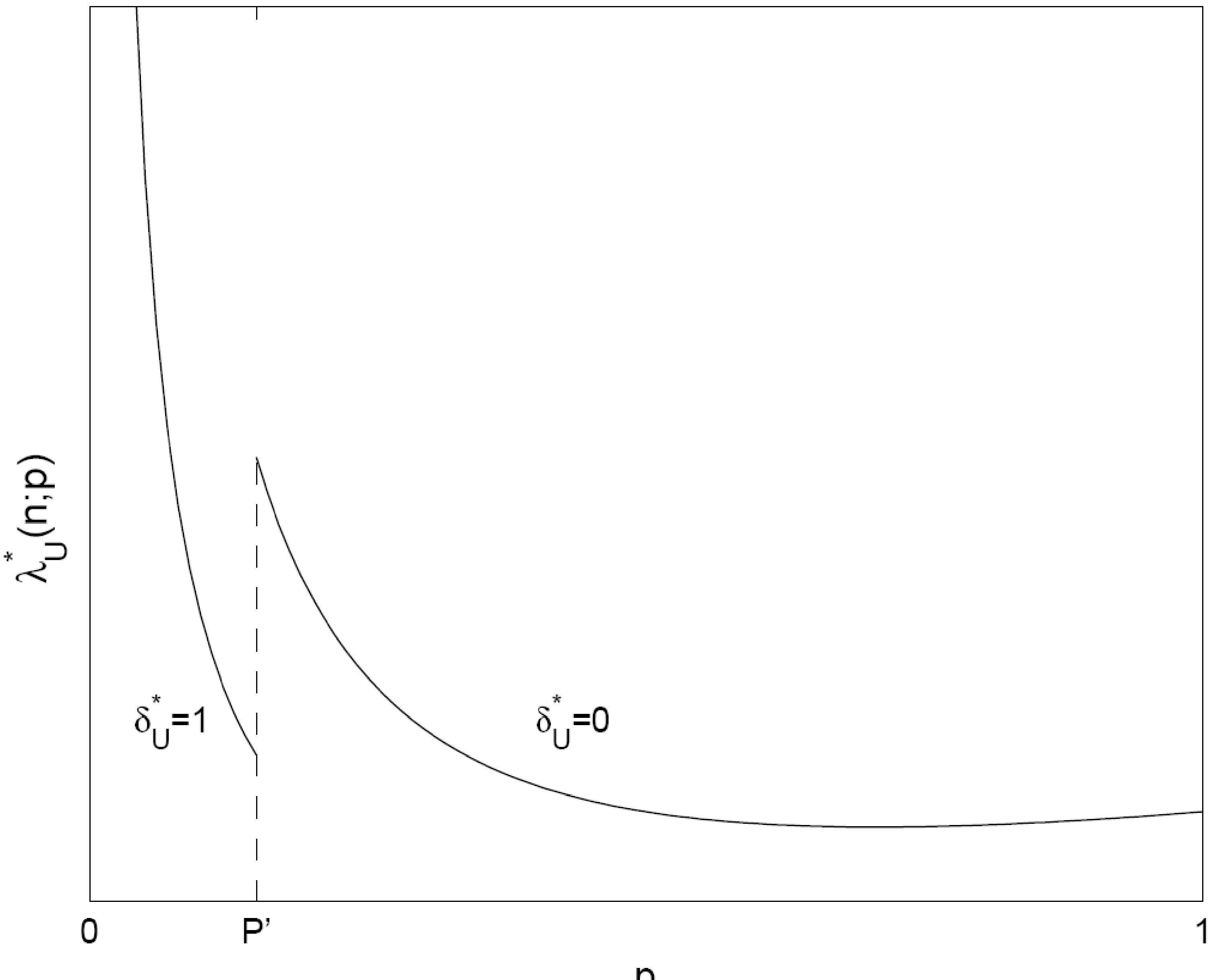
$$Cost = \min_{\lambda,\delta} \quad C(\lambda,\delta;p)$$

$$s.t. \qquad N(1+\delta K_T)\,(\lambda p+\omega(p))=n$$

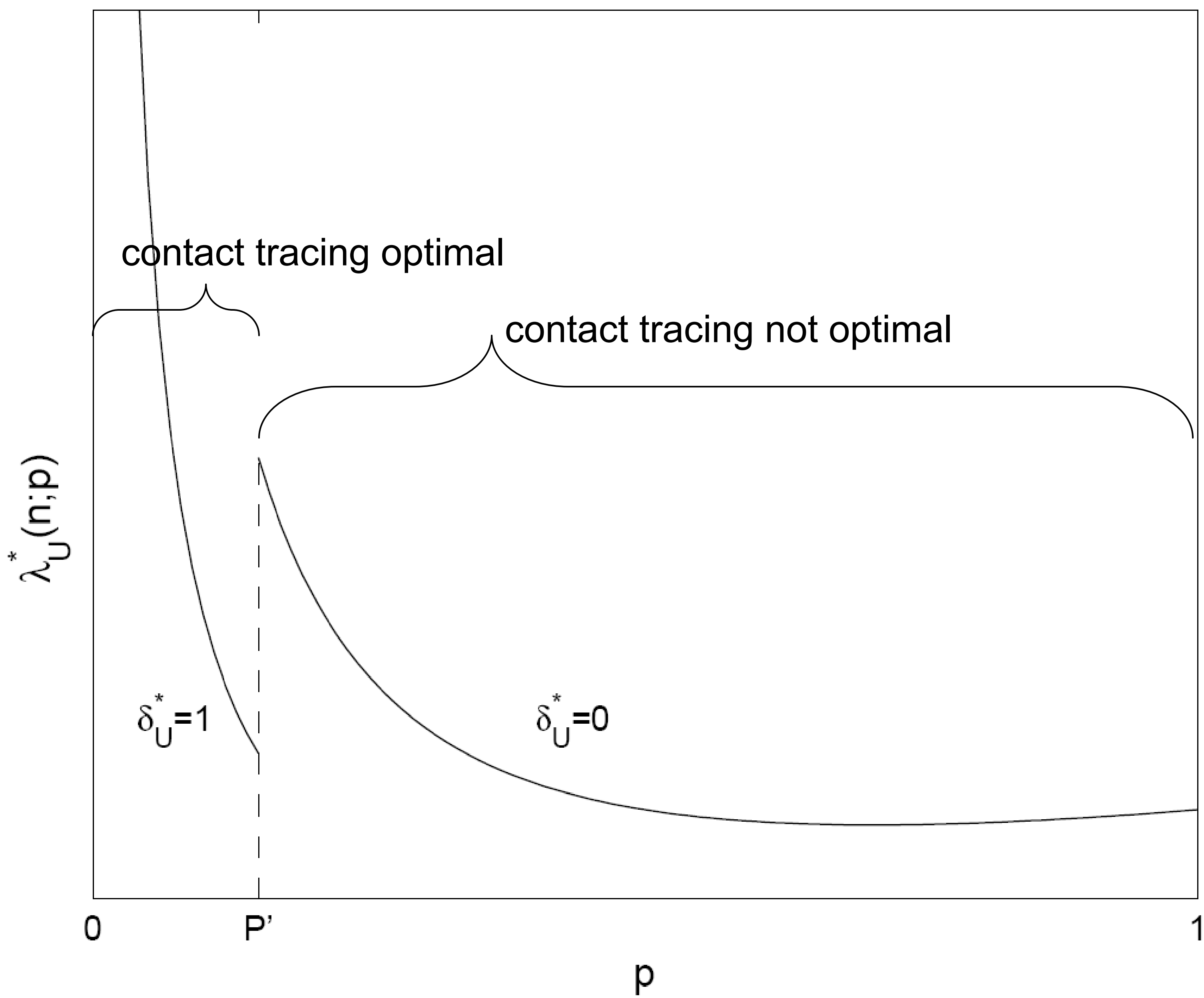
$$\lambda \geq 0,\ \delta = 0,1$$

this is the number of people we find

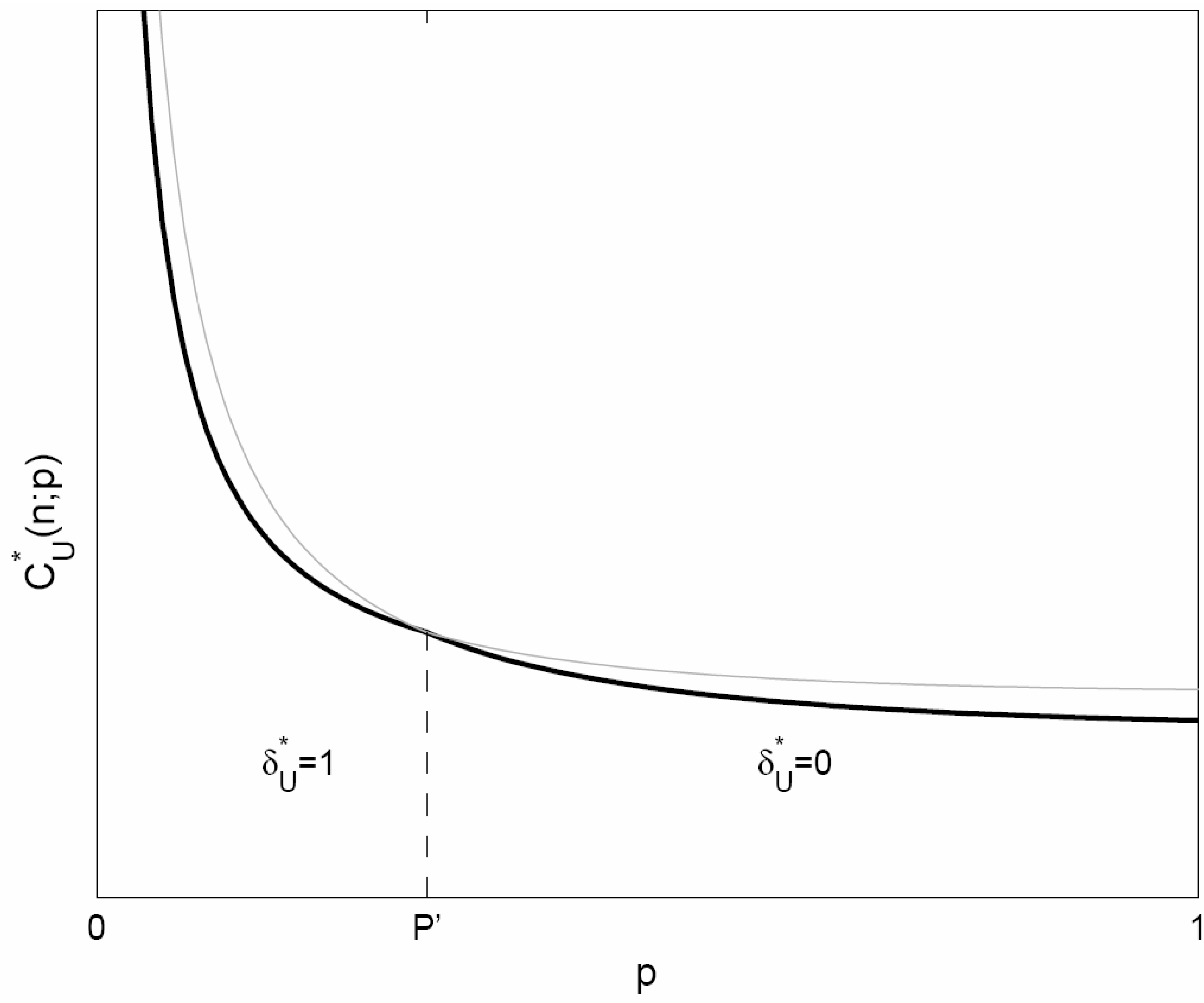
assuming p doesn't change



$\lambda^*_U(n;p)$
δ^*_U=1
δ^*_U=0
0
P'
1
p



contact tracing optimal
contact tracing not optimal
$\lambda^*_U(n;p)$
$\delta^*_U=1$
$\delta^*_U=0$
0
P'
1
p

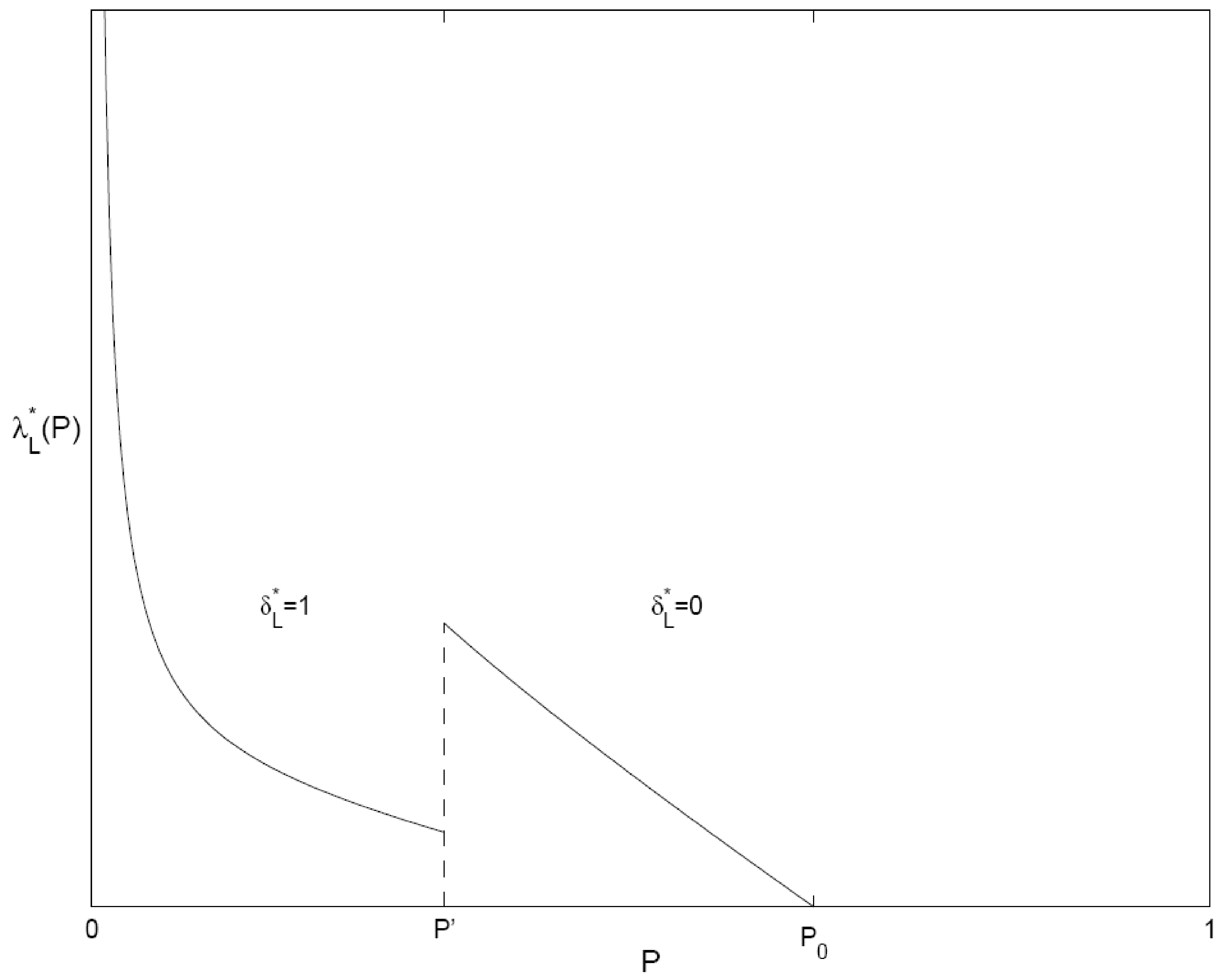


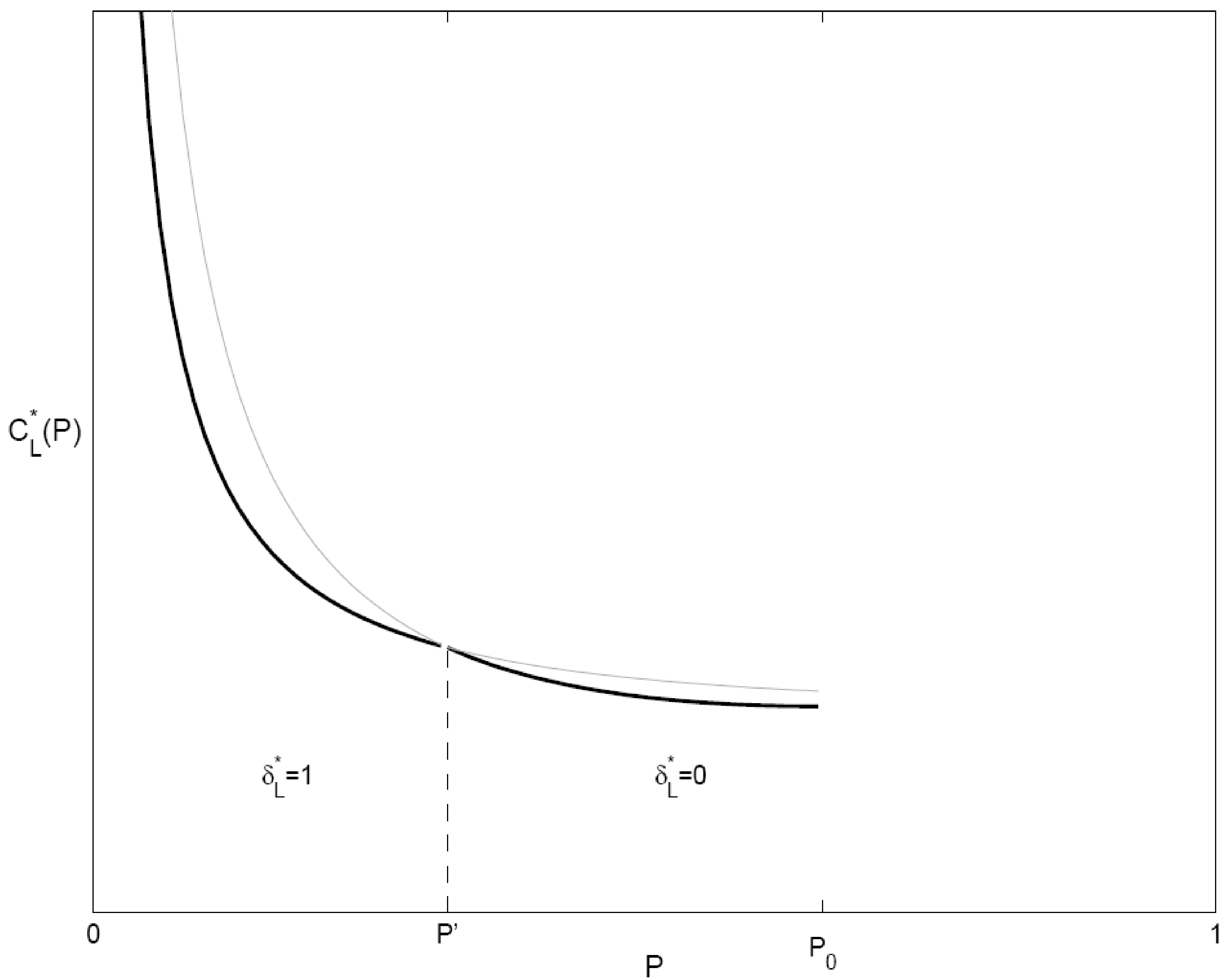
$C^*_U(n;p)$
$\delta^*_U=1$
$\delta^*_U=0$
0
P'
1
p

Long term costs

$$Cost = \min_{\lambda(t),\delta(t)} \quad C(\lambda,\delta;p)$$
$$s.t. \quad p(t)=P \text{ for all } t$$
$$\lambda(t)\geq 0,\ \delta(t)=0,1$$

$$\min_{\lambda,\delta} C(\lambda,\delta;P)$$
$$s.t.\ 0=p'= [\beta P(1-P)+\eta] - \mu P - (1+\delta K_T)\,(\lambda P+\omega(P))$$
$$\lambda\geq 0,\ \delta=0,1$$


$\lambda_L^*(P)$
$\delta_L^*=1$
$\delta_L^*=0$
0
P'
P_0
1
P



$C_L^*(P)$
$\delta_L^*=1$
$\delta_L^*=0$
0
P'
P_0
1
P

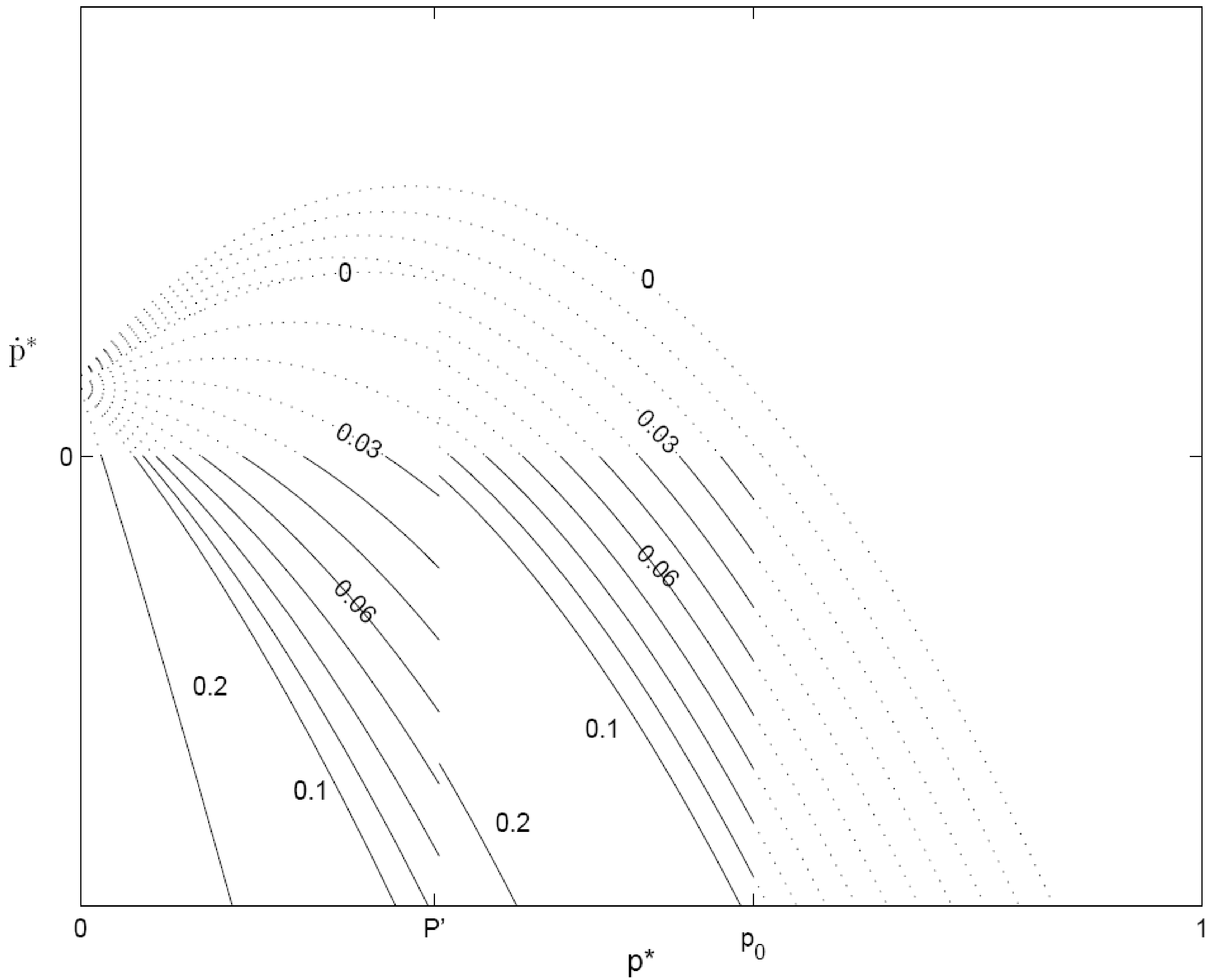
Transition costs

$$C^*_T(p_0, T, P_1) := \min_{\delta(t),\lambda(t)} \int_0^T e^{-rt} C(\lambda(t), \delta(t); p(t))\,\mathrm{d}t$$

$$s.t. \quad \dot{p}(t) = f(p(t), \lambda(t), \delta(t)) \ \forall t$$

$$p(0) = p_0, \quad p(T) \leq P_1, \quad \dot{p}(t) \leq 0 \ \forall t$$

$$\lambda(t) \geq 0, \quad \delta(t) \in \{0, 1\} \ \forall t.$$



$\dot{p}^*$
0
0
0
0.03
0.03
0.06
0.06
0.2
0.1
0.1
0.2
0
P'
p_0
1
p^*

Results

$\delta=1$ optimal if and only if $p < C_S/(C_T/K_T - C_t)$

equivalently $C_S/p < C_T/K_T - C_t$

- Unchanged prevalence
  - λ uniquely determined by feasibility
- Long term costs
  - λ uniquely determined by feasibility
- Transition Costs, $p(t)$ flat or makes jumps

Insights

- contact tracing cost-effective only when p below some threshold
- screening rate λ decreasing in prevalence p
- as p increases above the threshold λ jumps up
- model robust to different cost formulations

Model criticisms

- population in steady state: births correlated exactly balance deaths
- infections from abroad don’t depend on p (or number susceptible)
- # found by contact tracing doesn’t depend on p (or number infected)

Model criticisms

- homogenous mixing
- no delay terms for infection or contact tracing
- no model of effort or contact tracing capacity
- deterministic
- lacking realistic parameters

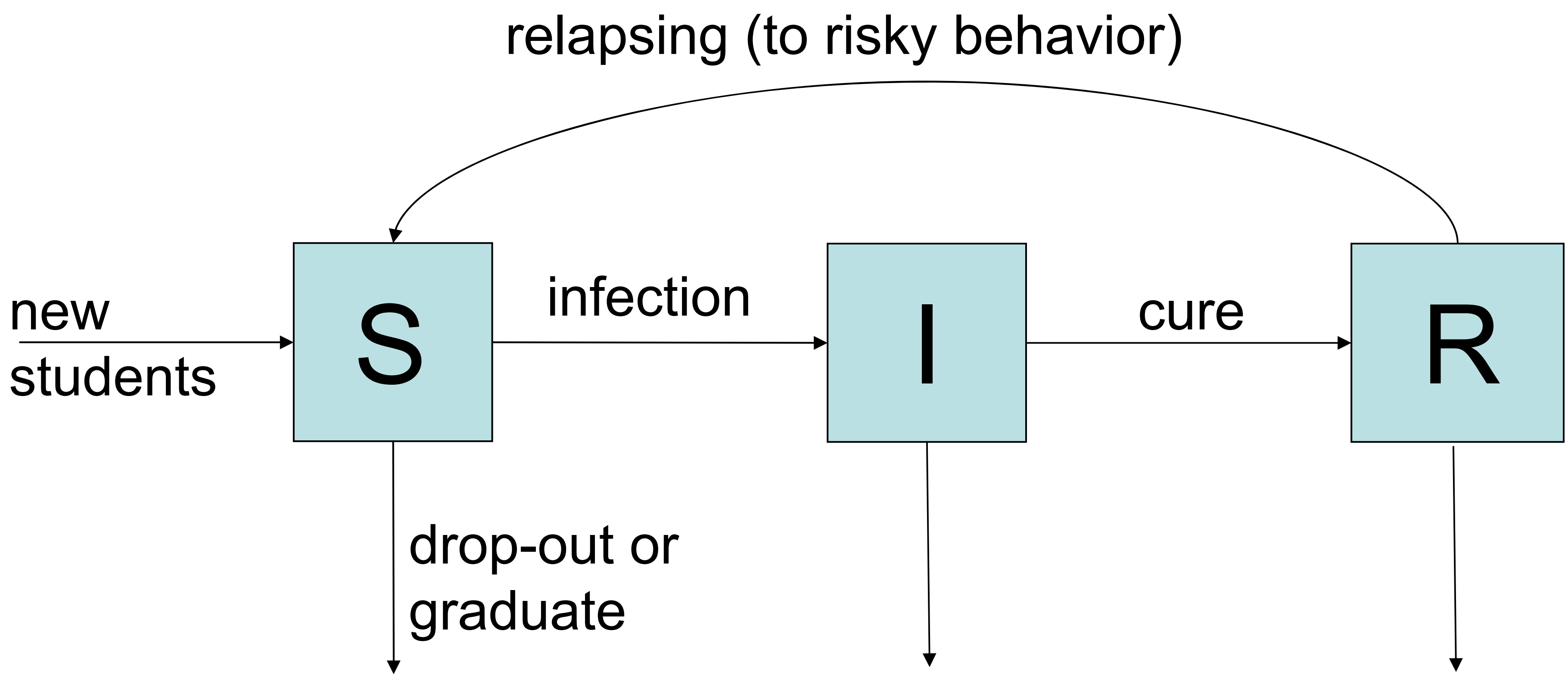
Current work

- modeling how an STD spreads in a high school

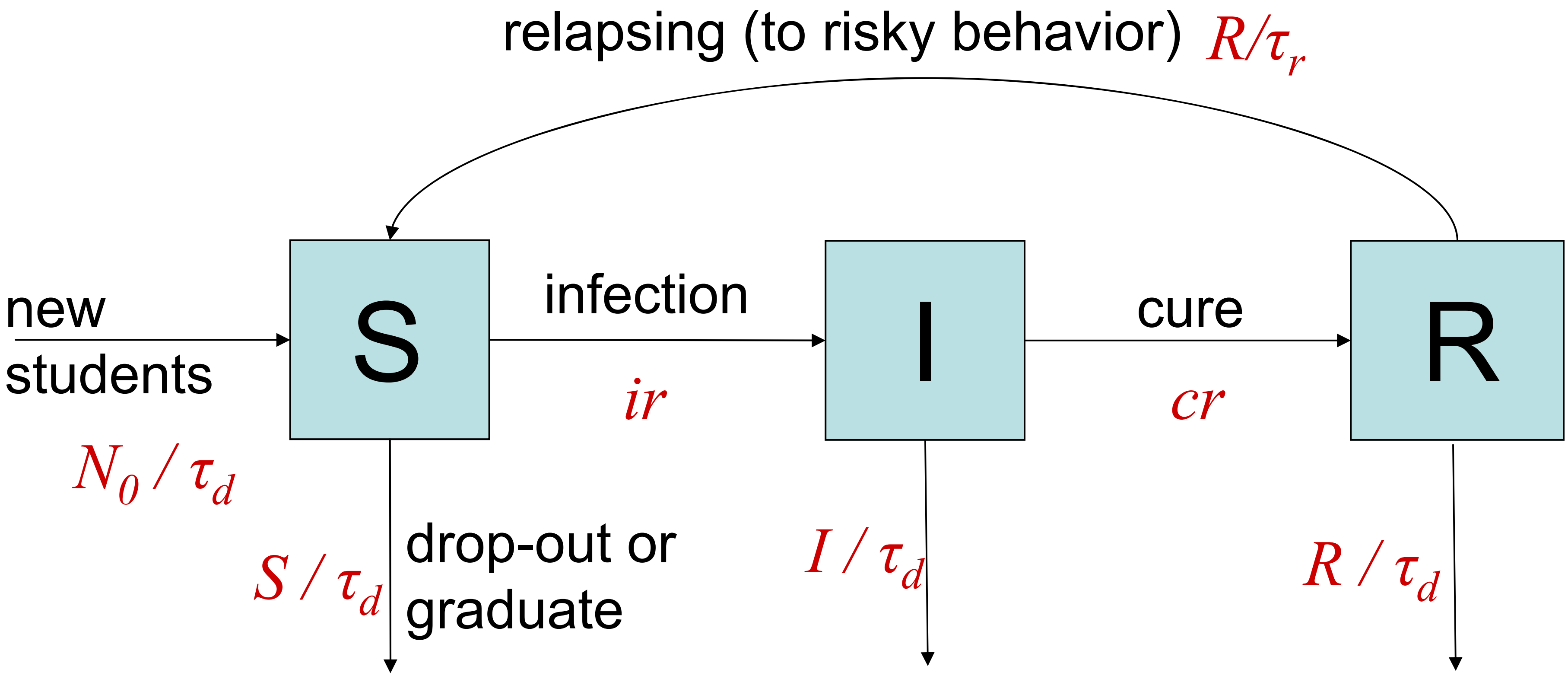
“One in 12 Philly teenage girls has chlamydia.”

Amy L. Webb. *But I didn’t know...* Philadelphia Citypaper, January 22-28, 2004.

Stochastic S→I→R model



Stochastic S→I→R model

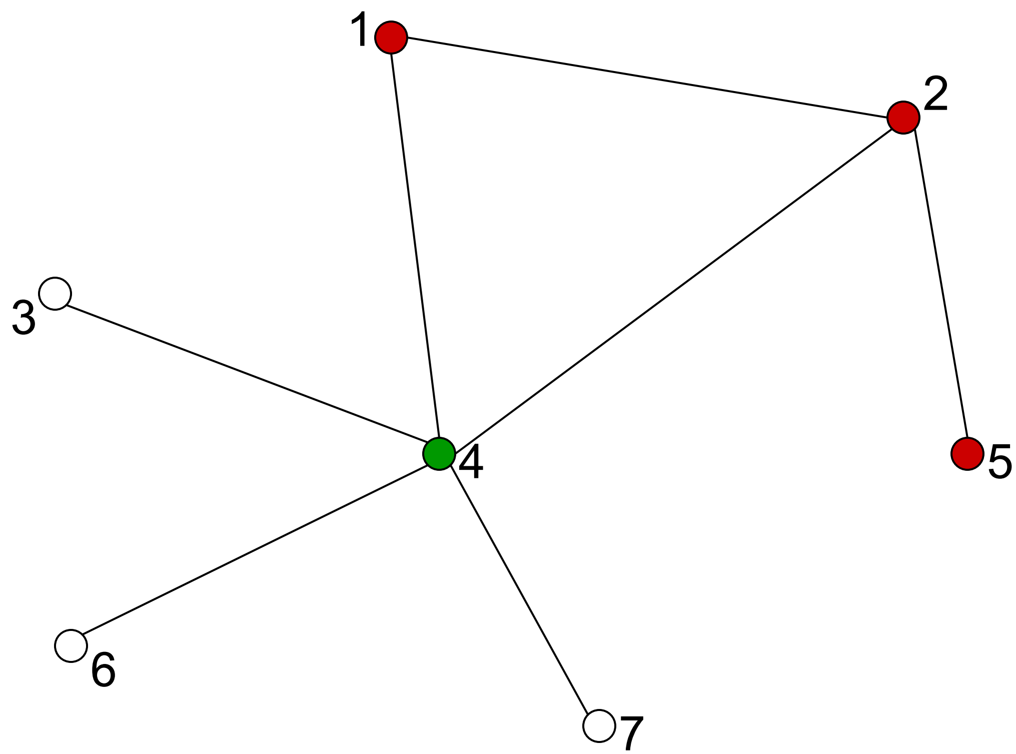


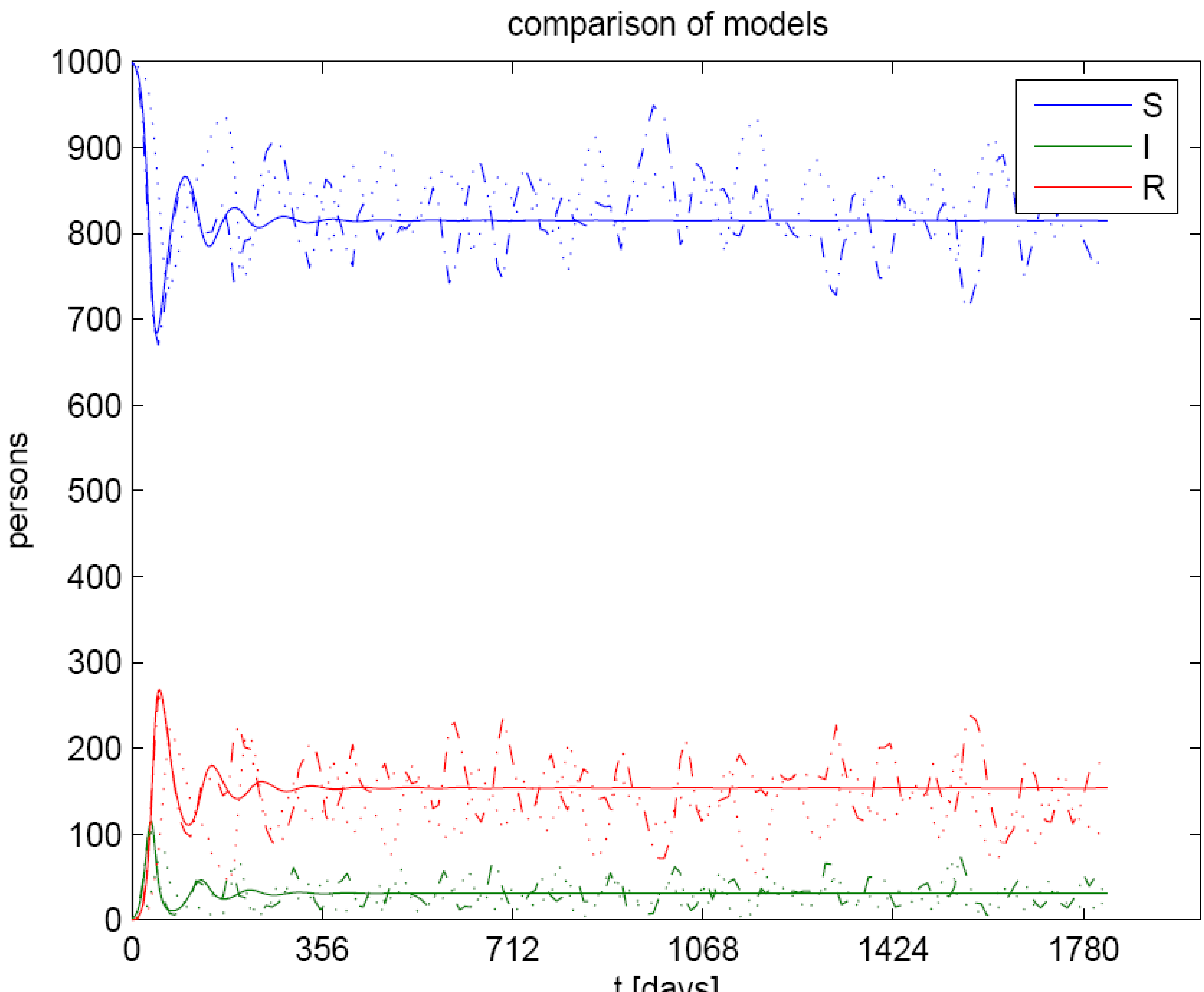
$N=S+I+R$

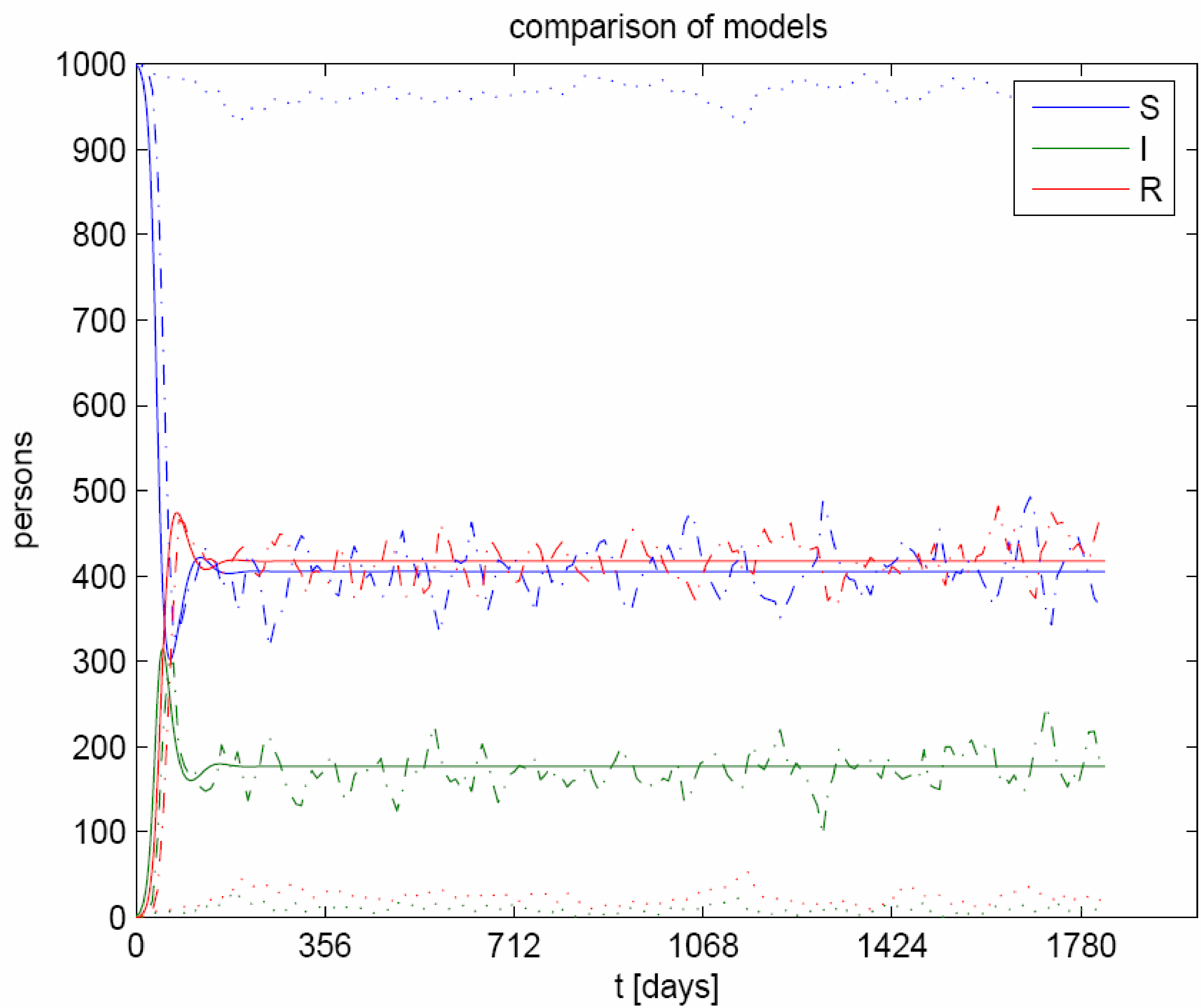
$ir=+S/\tau_{oi} \quad + I(S/N) / \tau_{ii}$

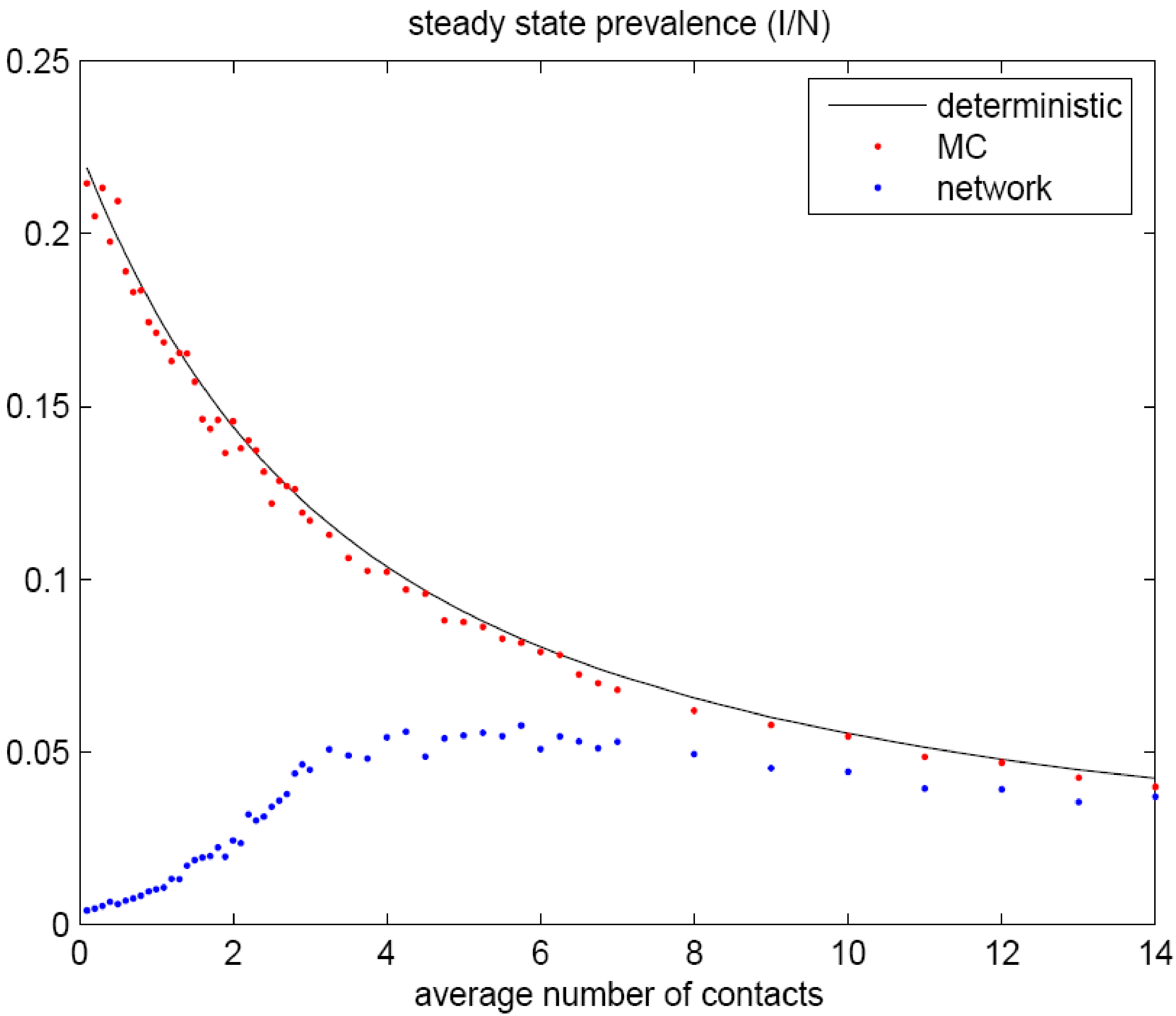
$cr=+I/\tau_h \quad +(I/N) / \tau_s \quad +\delta\, R\, d\, (I/N) / \tau_c$

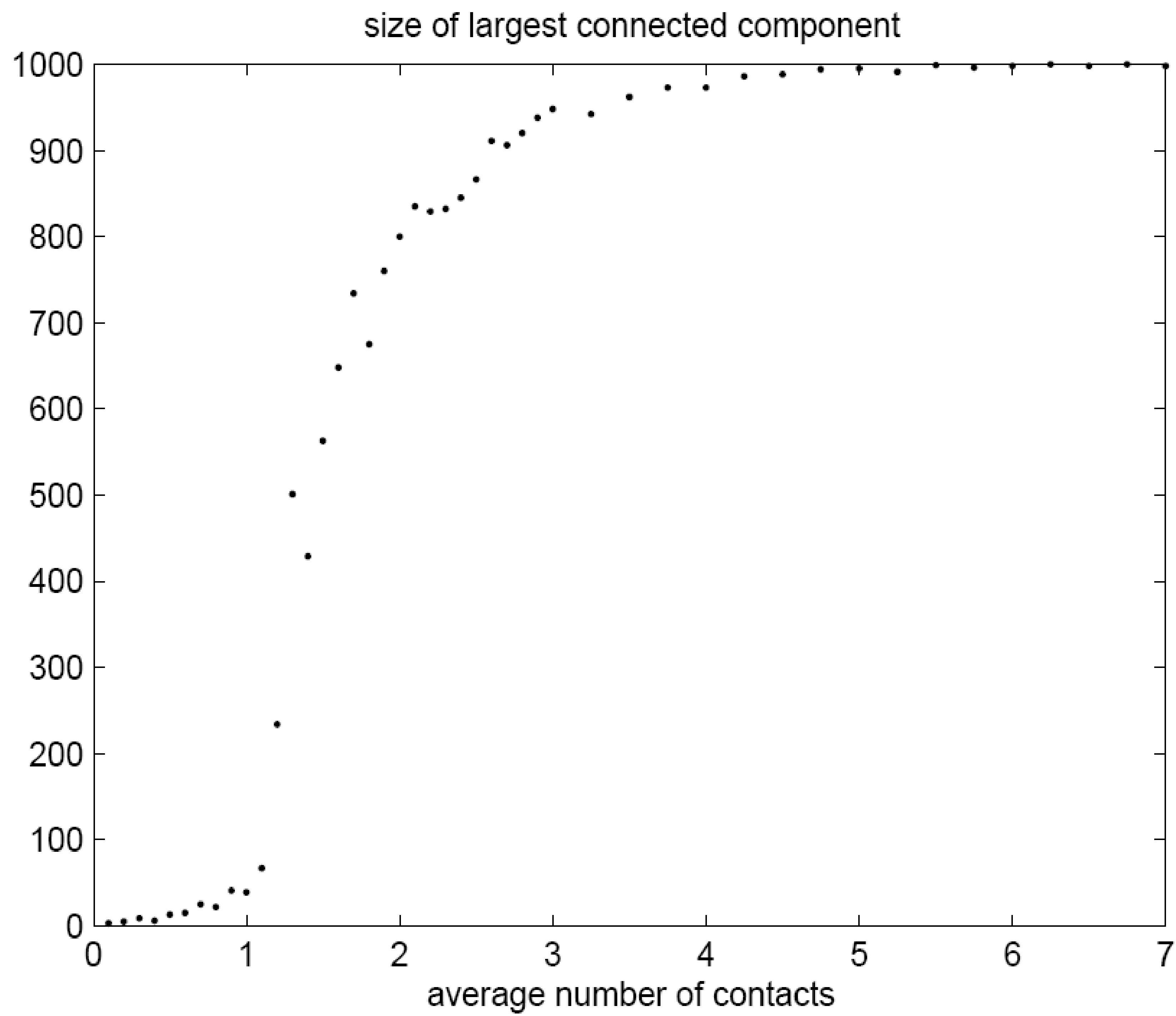
A network model



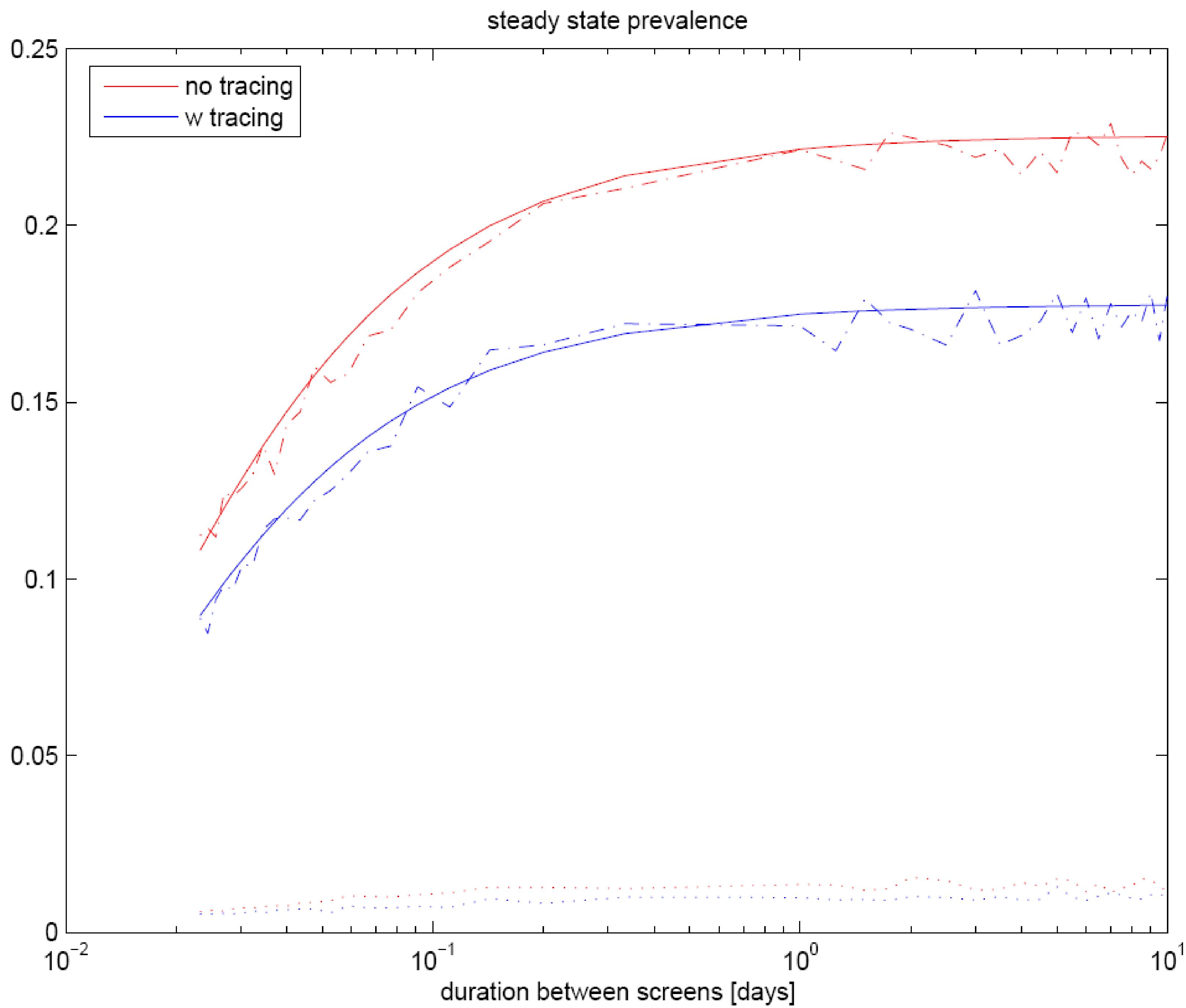

comparison of models
persons
t [days]
S
I
R
1000
900
800
700
600
500
400
300
200
100
0
0
356
712
1068
1424
1780

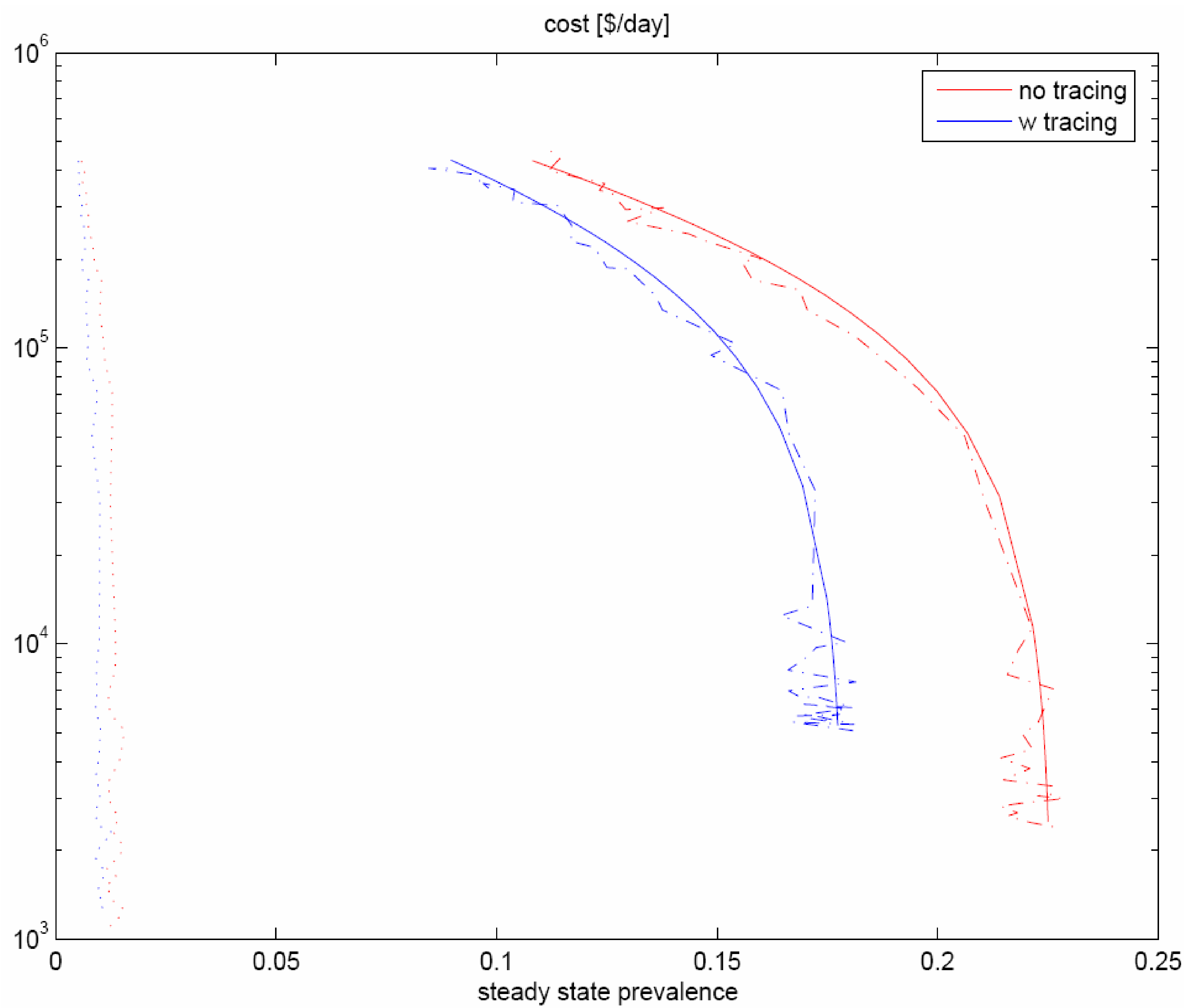

comparison of models
persons
t [days]
S
I
R
1000
900
800
700
600
500
400
300
200
100
0
0
356
712
1068
1424
1780

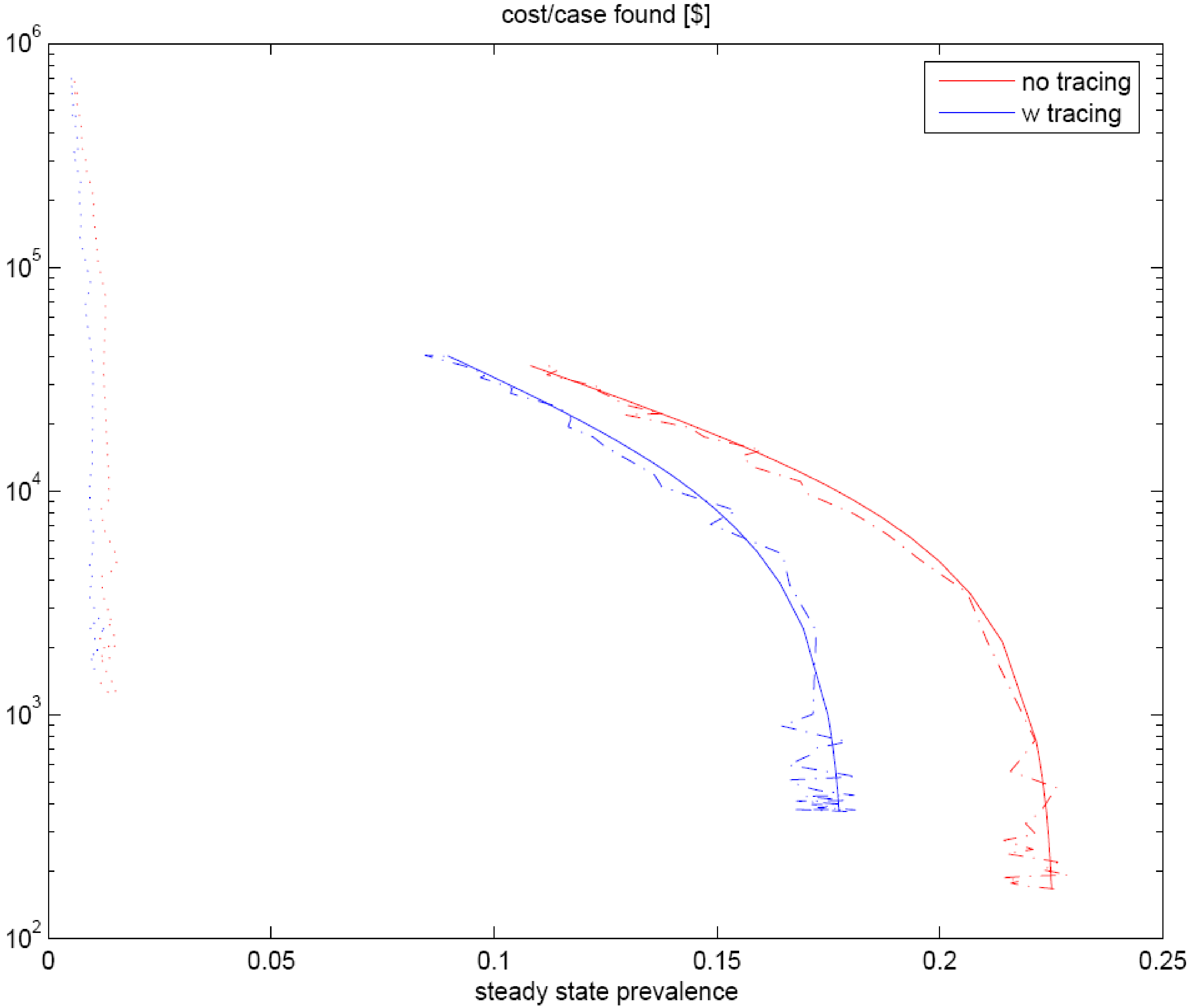

steady state prevalence (I/N)
deterministic
MC
network
0.25
0.2
0.15
0.1
0.05
0
0
2
4
6
8
10
12
14
average number of contacts



size of largest connected component
1000
900
800
700
600
500
400
300
200
100
0
0
1
2
3
4
5
6
7
average number of contacts


steady state prevalence
no tracing
w tracing
0.25
0.2
0.15
0.1
0.05
0
10^{-2}
10^{-1}
10^{0}
10^{1}
duration between screens [days]


cost [$/day]
no tracing
w tracing
10^6
10^5
10^4
10^3
0
0.05
0.1
0.15
0.2
0.25
steady state prevalence



cost/case found [$]
no tracing
w tracing
10^6
10^5
10^4
10^3
10^2
0
0.05
0.1
0.15
0.2
0.25
steady state prevalence

Questions?

Optimal mix of screening and contact tracing

for endemic diseases

www.stanford.edu/~barmbrus/policystatics.pdf