LP Formulations for Radiation Treatment Planning (IMRT)

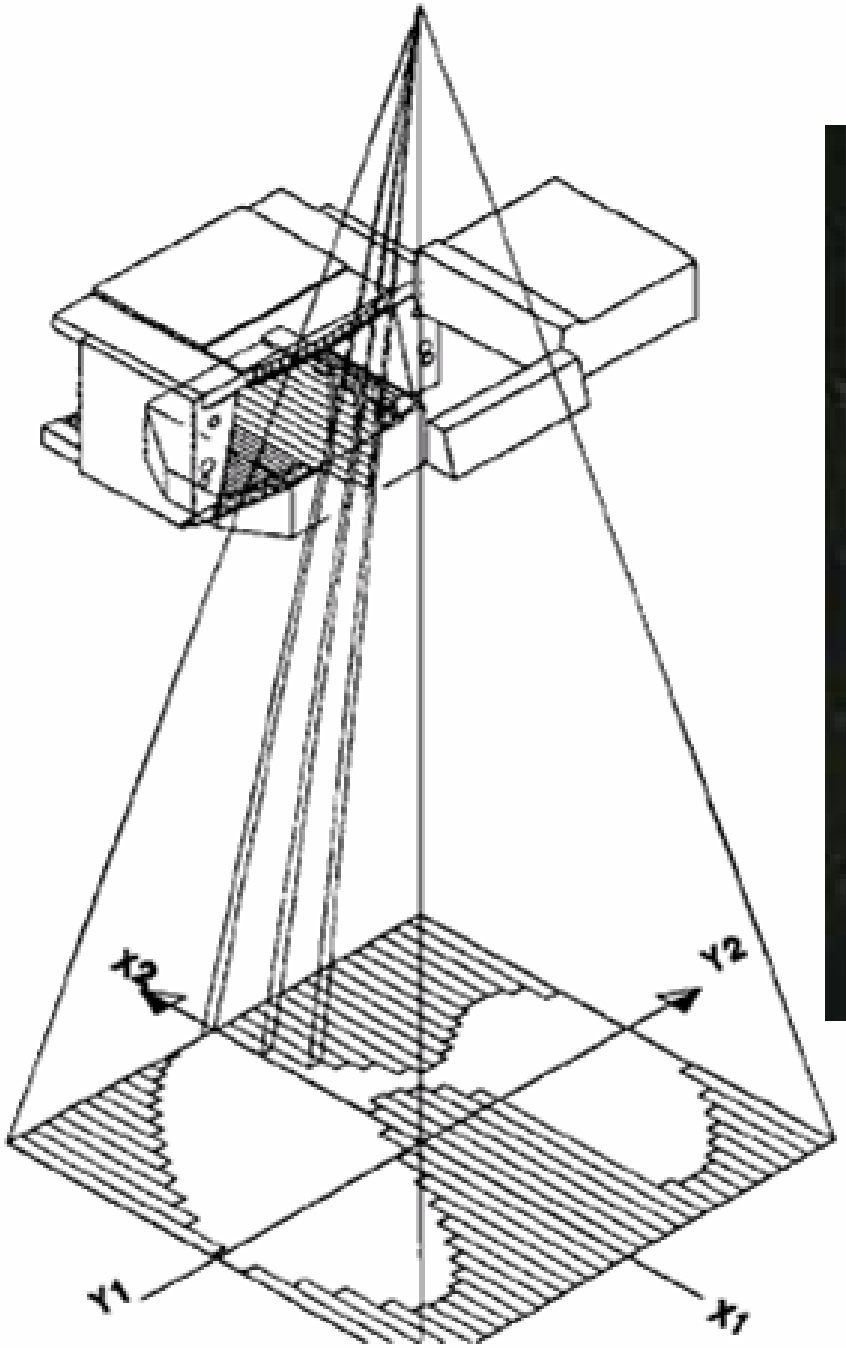
Benjamin Armbruster

joint work with Russell Hamilton, Martin Lachaine, and J. Cole Smith

Radiation Therapy

1. meet doctor
2. CT scan
3. M.D. identifies tumor, organs, writes prescription
4. dosimetry/physics creates plan
5. treatment: M-F, 5-8 wks
6. followups

Goals: tumor control and organ functionality

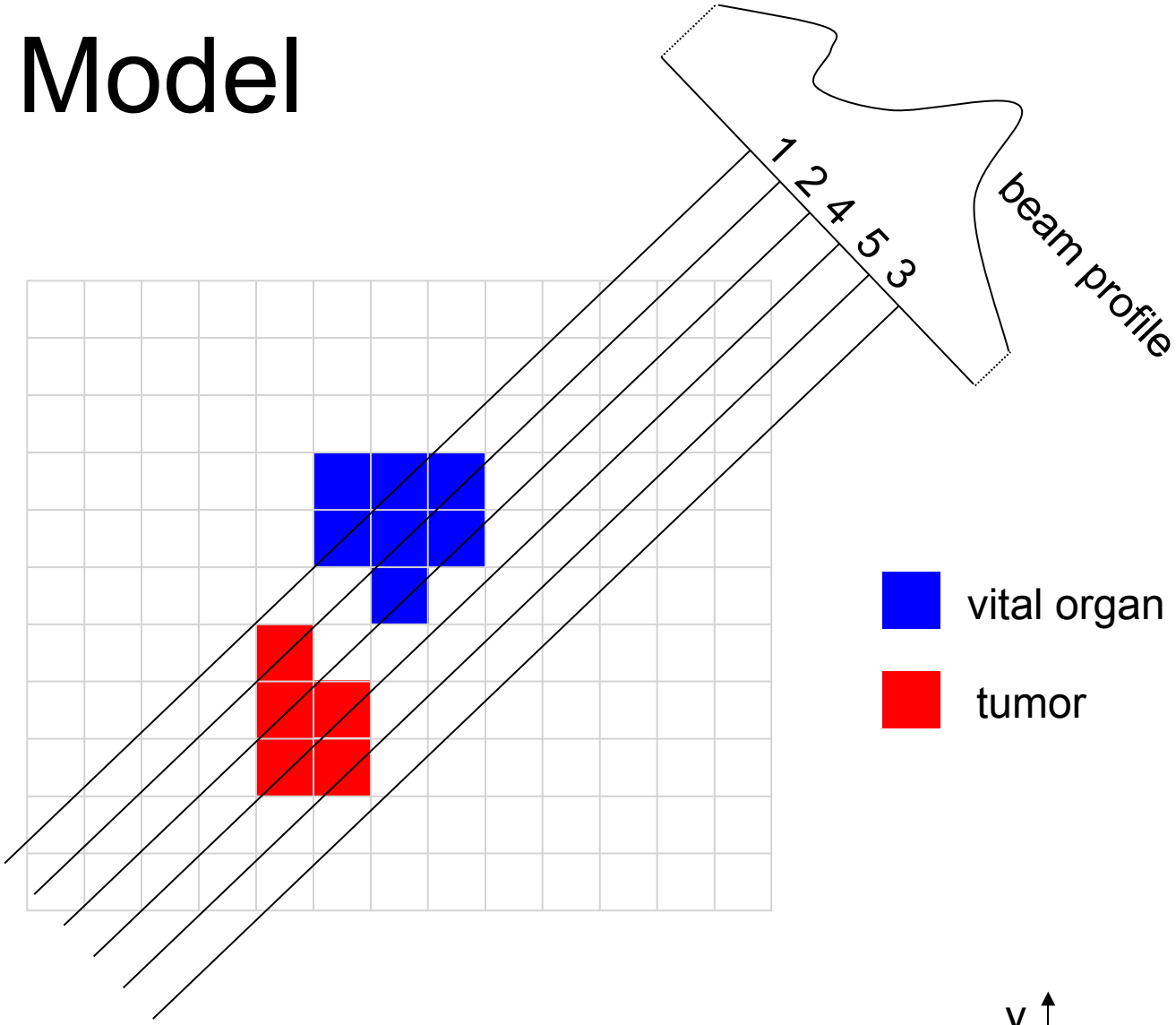


Leaves allow custom apertures

Typical Data and Problem Complexity

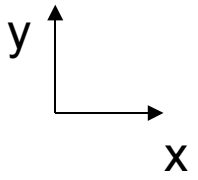
- leaves 0.5-1 cm thick
- max aperture 40 cm
- ~5-9 beam angles
- ~5-30-100 aperture shapes per beam angle
- CT 512x512 pixels (0.5-1mm), 1-5mm slices
- dose calculation 2-5mm mesh

The Model

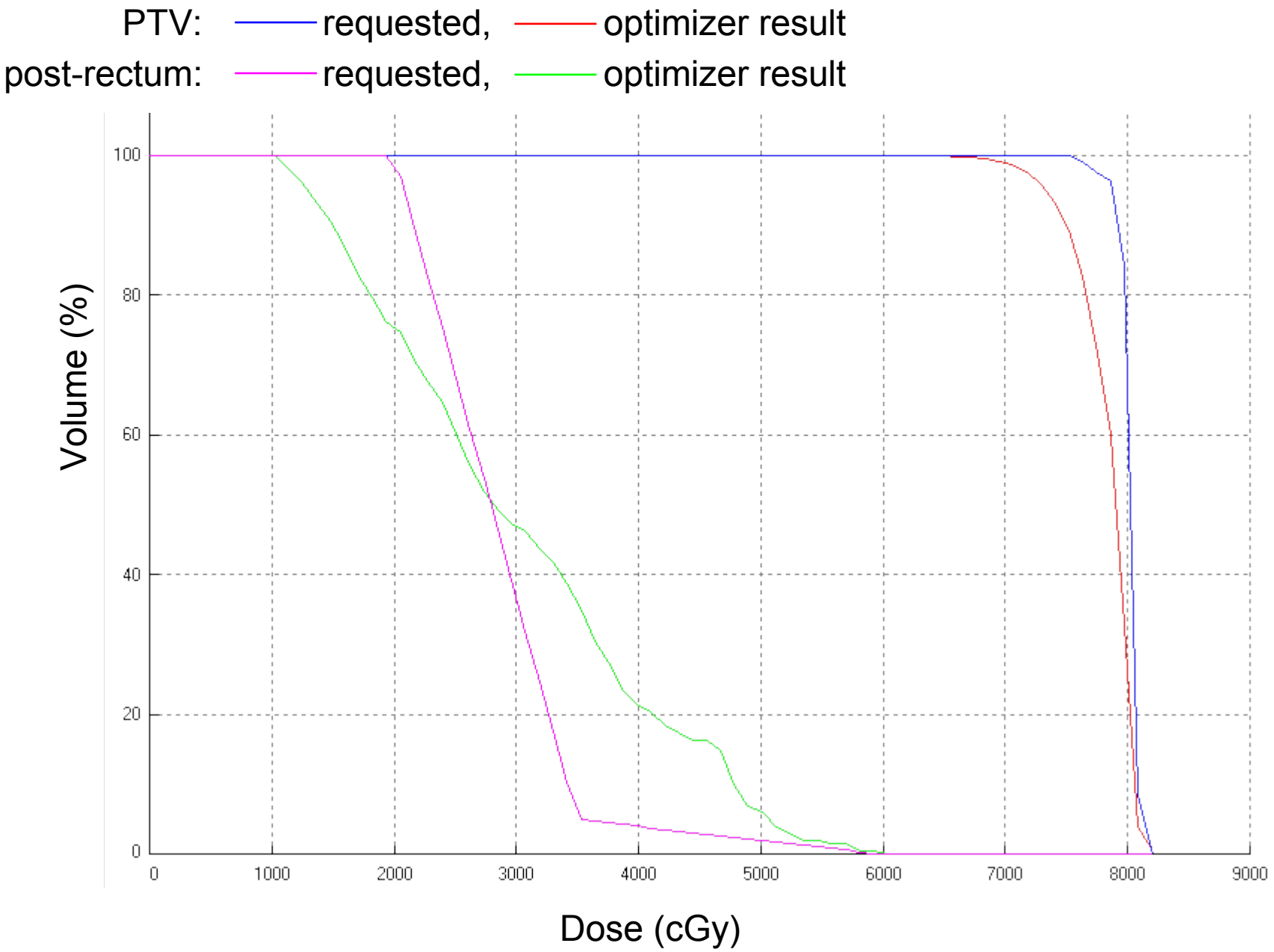


dose at location j = $\sum_i$ weight given to beamlet i
× normalized dose to location j from beamlet i

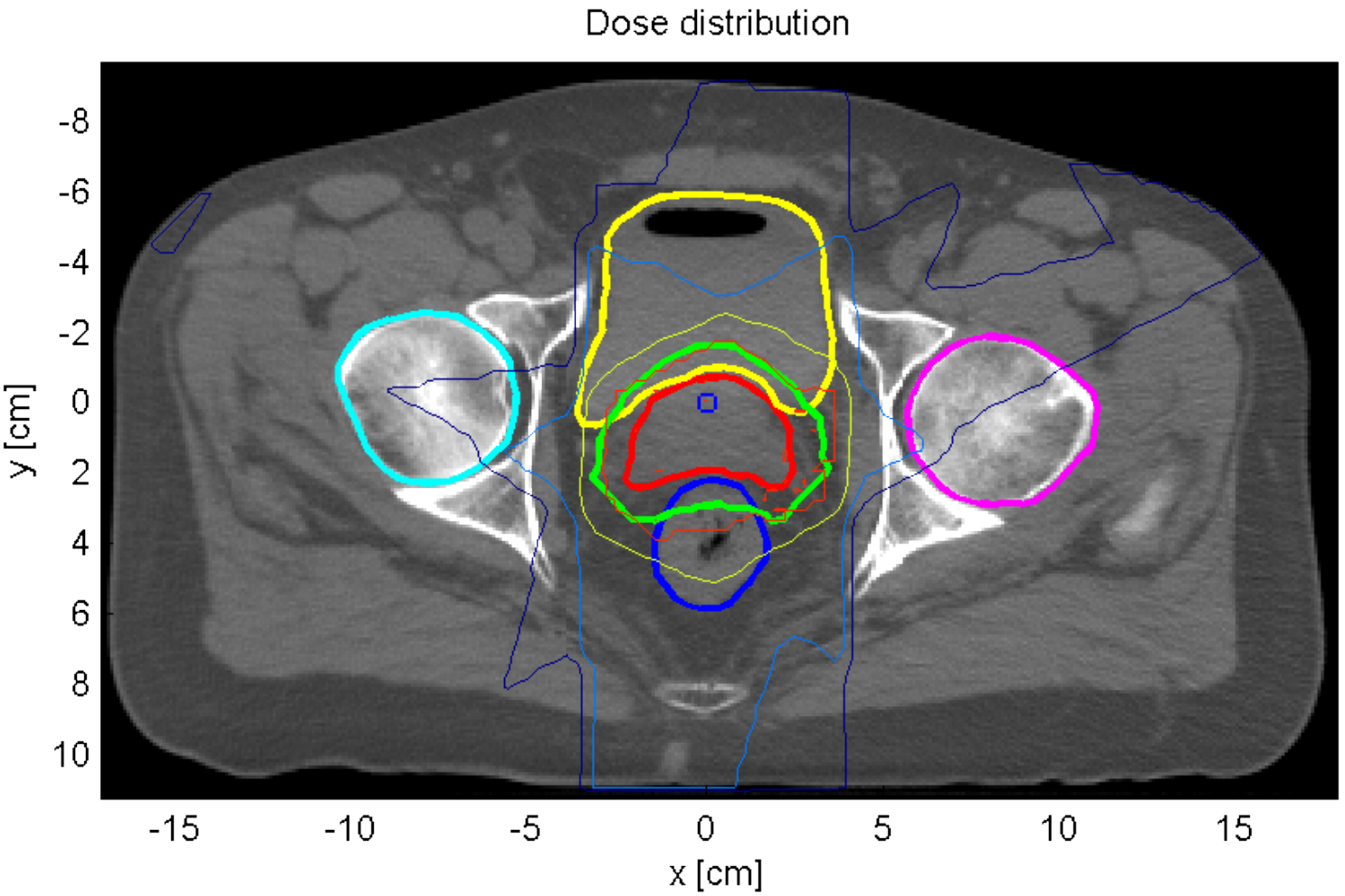
or $D_j = \sum_i w_i \times d_{ij}$



Cumulative dose-volume histogram



Plan 1



dose contour levels
- 1.15
- 1.00
- 0.80
- 0.50
- 0.30

constraints:
- PTV > 1
- PTV < x

goal: min x

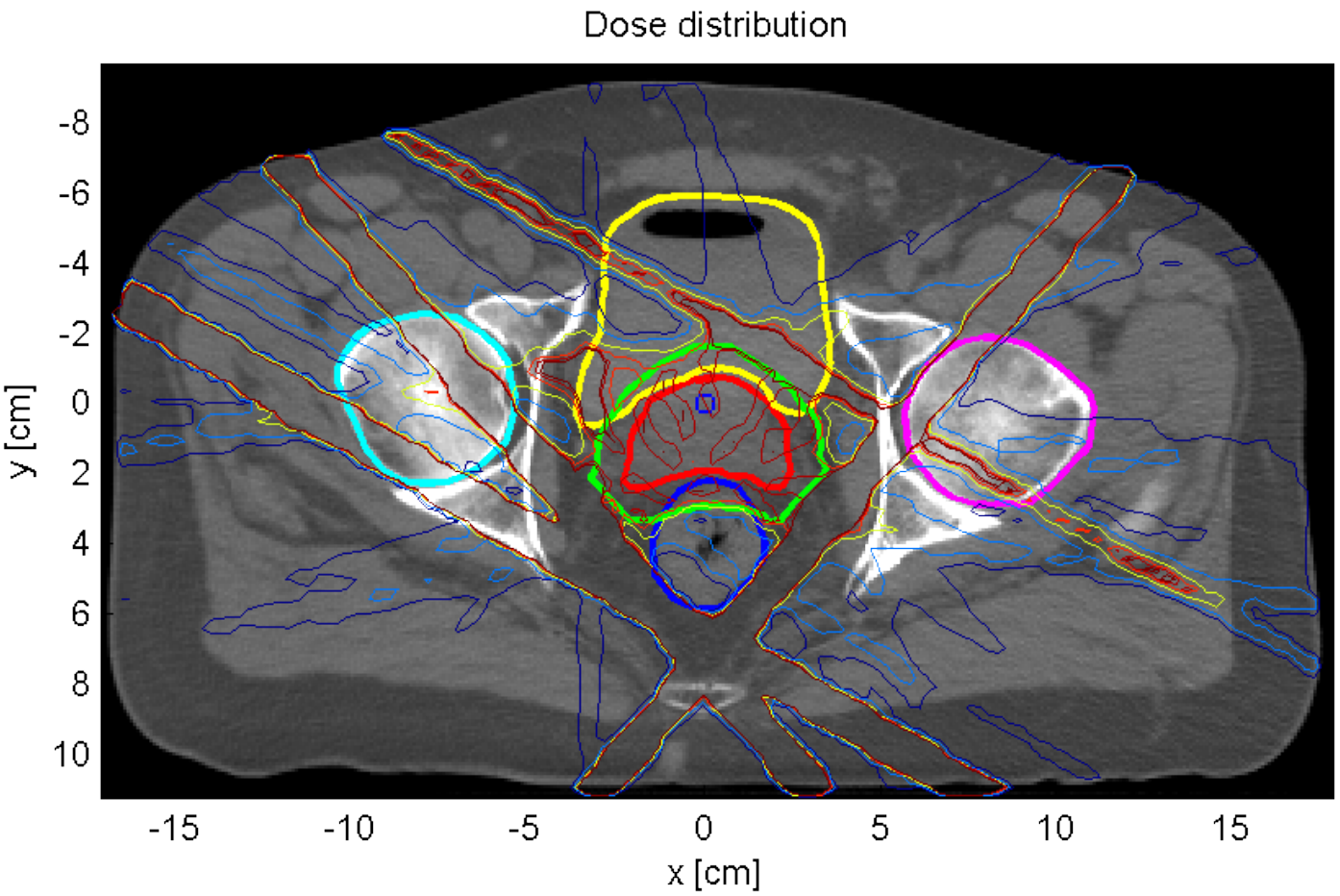
result: x=1.01

*dose normalized to prescription dose

PTV=tumor

- GTV
- bladder
- left femur
- right femur
- rectum
- PTV

Plan 2



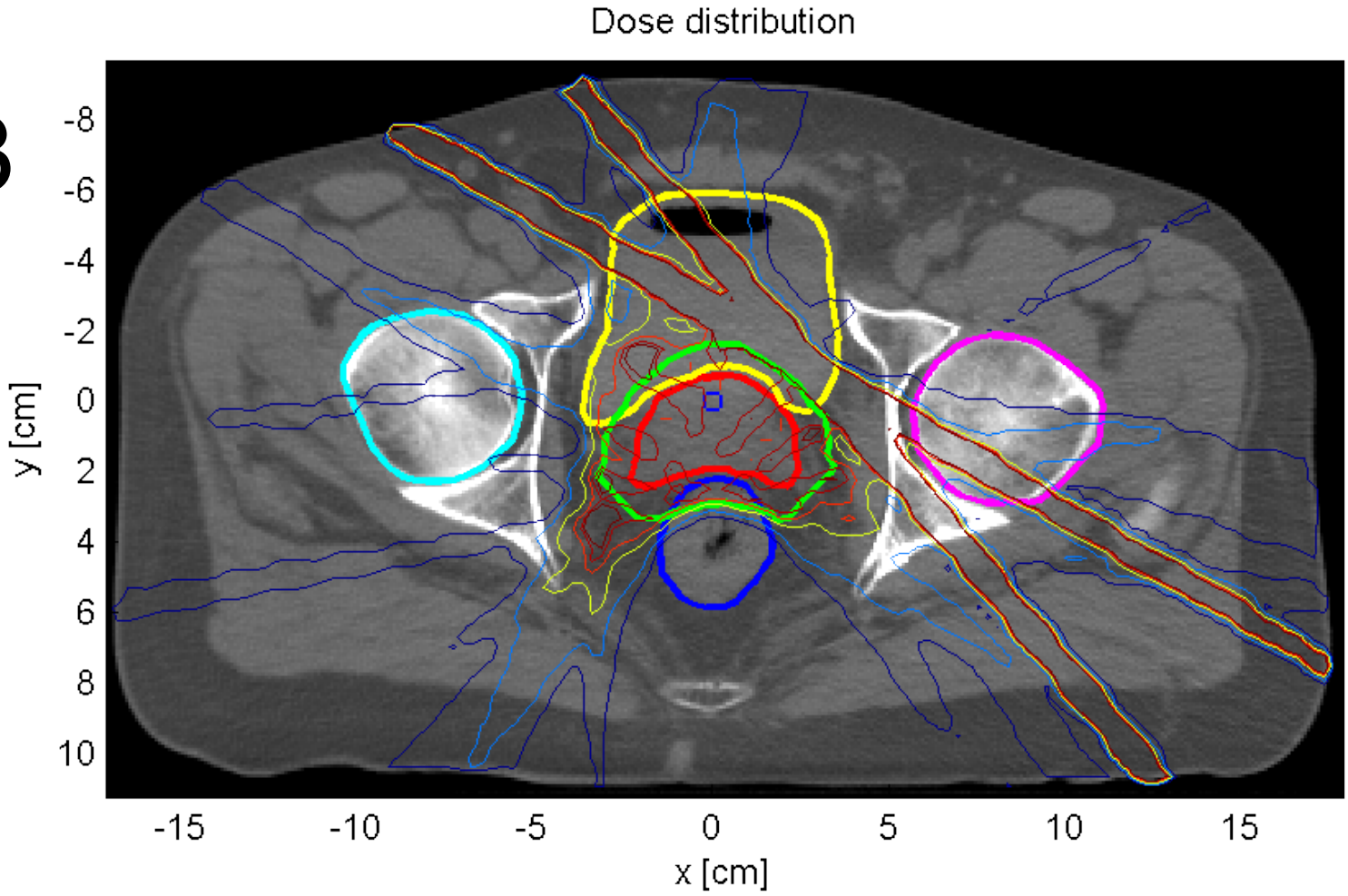
constraints:

- PTV > 1
- PTV < 1.15
- rectum outside PTV < x

goal: min x

result: x=0.59

Plan 3



constraints:
- PTV > 1
- PTV < 1.15
- avg. dose in rectum outside PTV < x

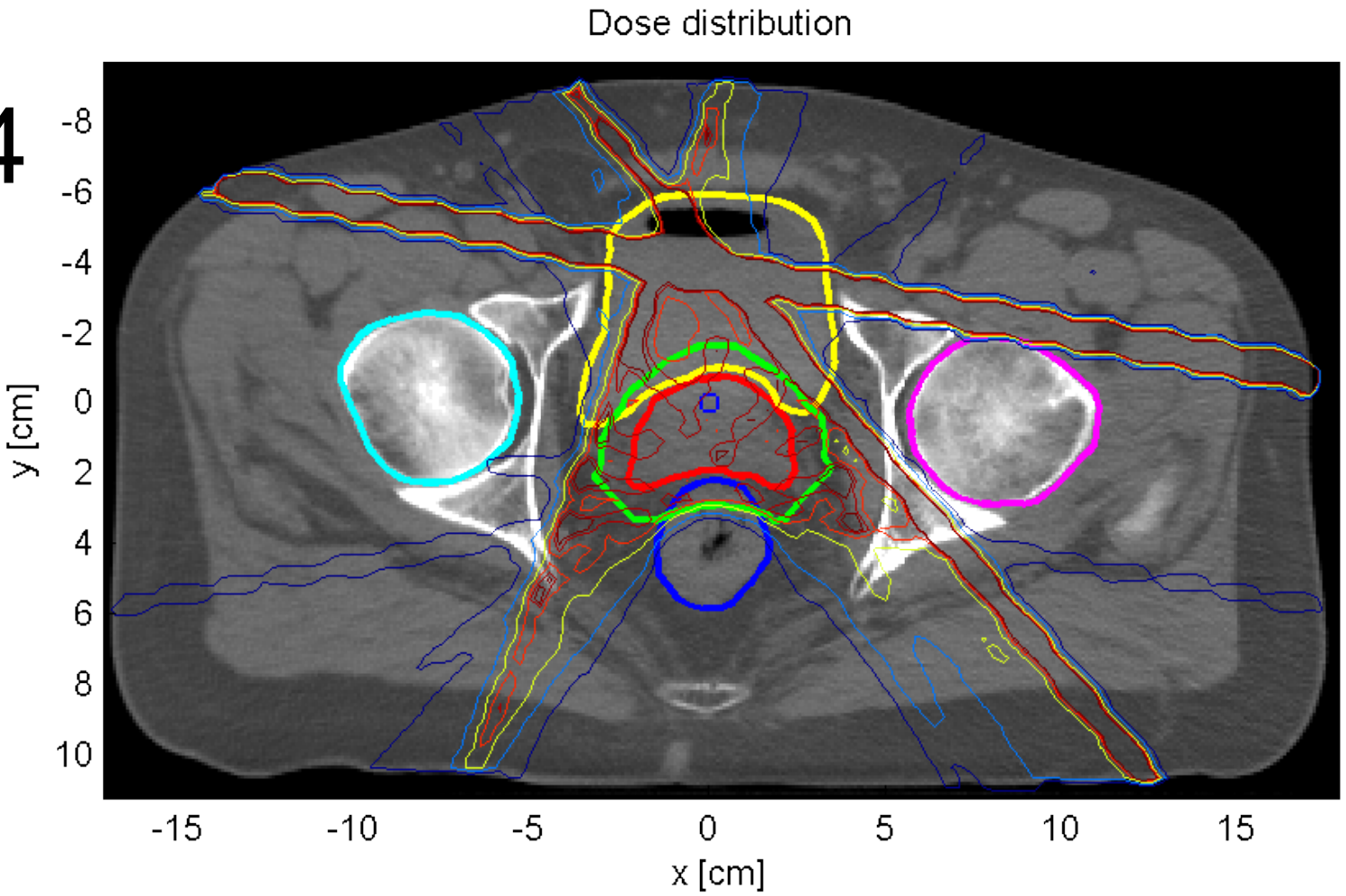
goal: min x

result: x=0.15

dose contour levels
1.15
1.00
0.80
0.50
0.30

GTV
bladder
left femur
right femur
rectum
PTV

Plan 4



constraints:
- PTV > 1
- PTV < 1.15
- femurs < 0.25
- avg. dose in rectum outside PTV < x

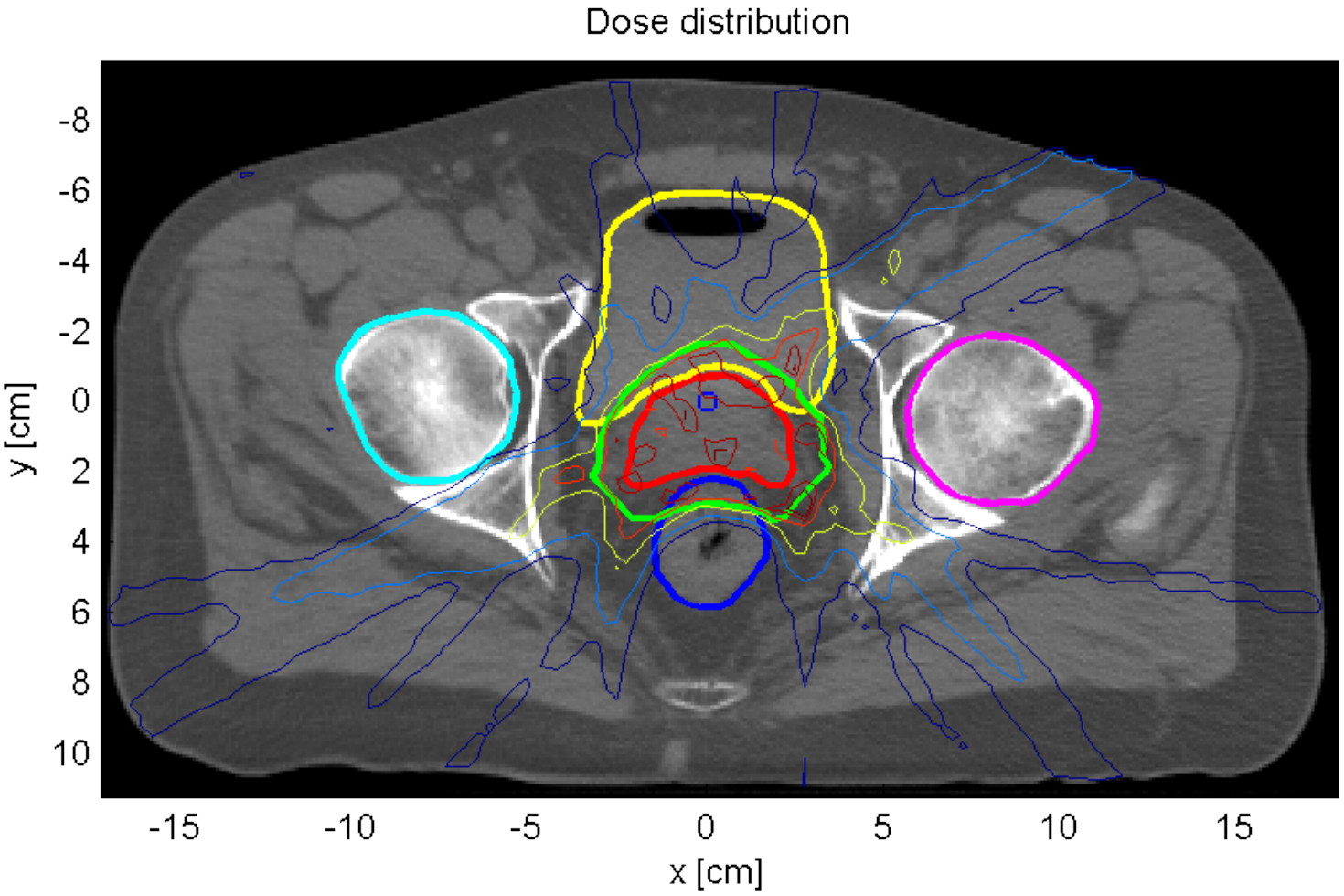
goal: min x

result: x=0.30

dose contour levels
1.15
1.00
0.80
0.50
0.30

GTV
bladder
left femur
right femur
rectum
PTV

Plan 5



constraints:

- PTV > 1
- PTV < 1.15
- femurs < 0.25
- avg. dose in rectum outside PTV < 0.33
- avg. dose in bladder < x

goal: min x

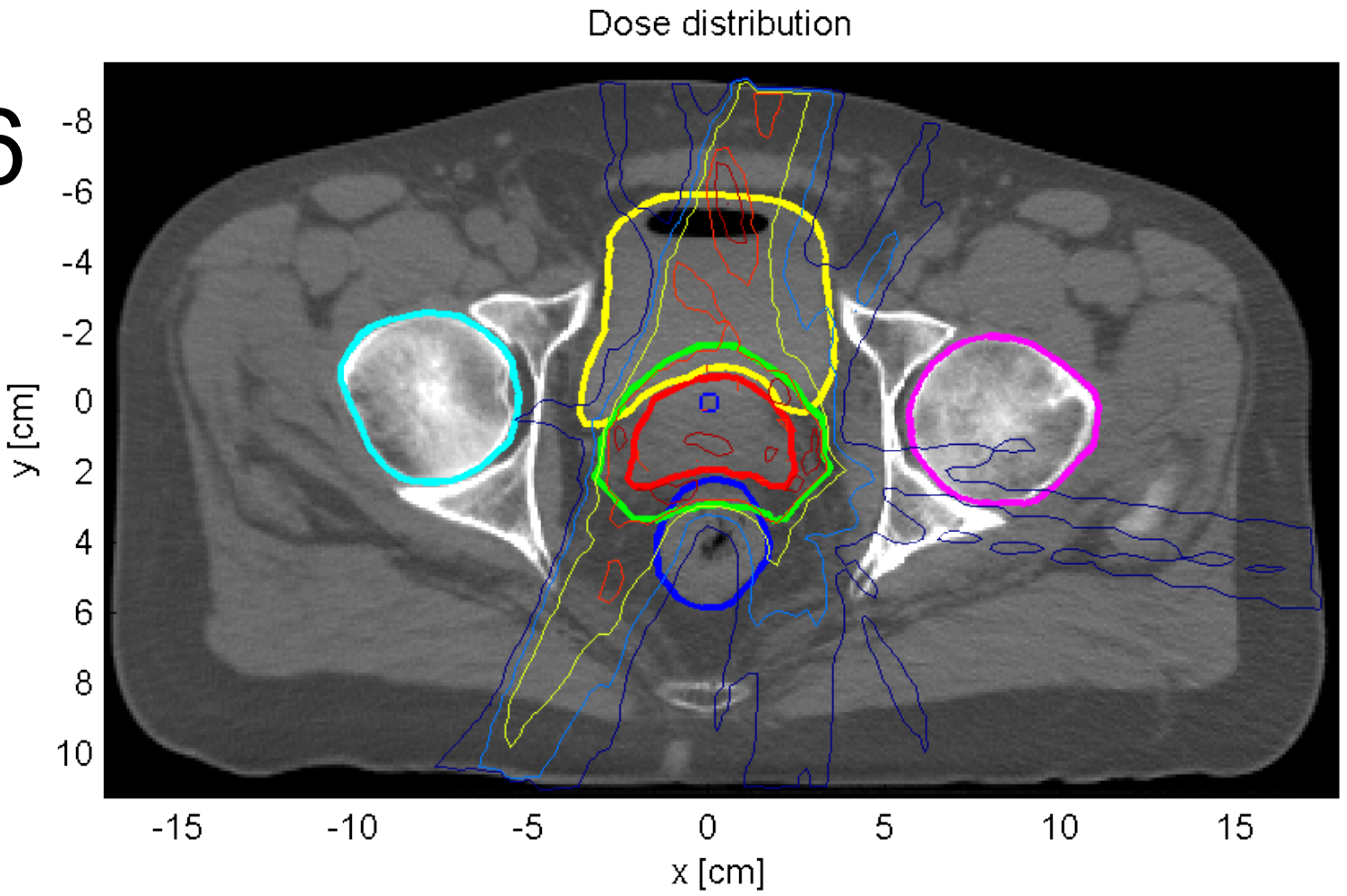
result: x=0.54

dose contour levels

1.15
1.00
0.80
0.50
0.30

GTV
bladder
left femur
right femur
rectum
PTV

Plan 6



constraints:
- PTV > 1
- PTV < 1.15
- femurs < 0.375
- avg. dose in rectum outside PTV < 0.50
- avg. dose in bladder < 0.71
- avg. external dose < x

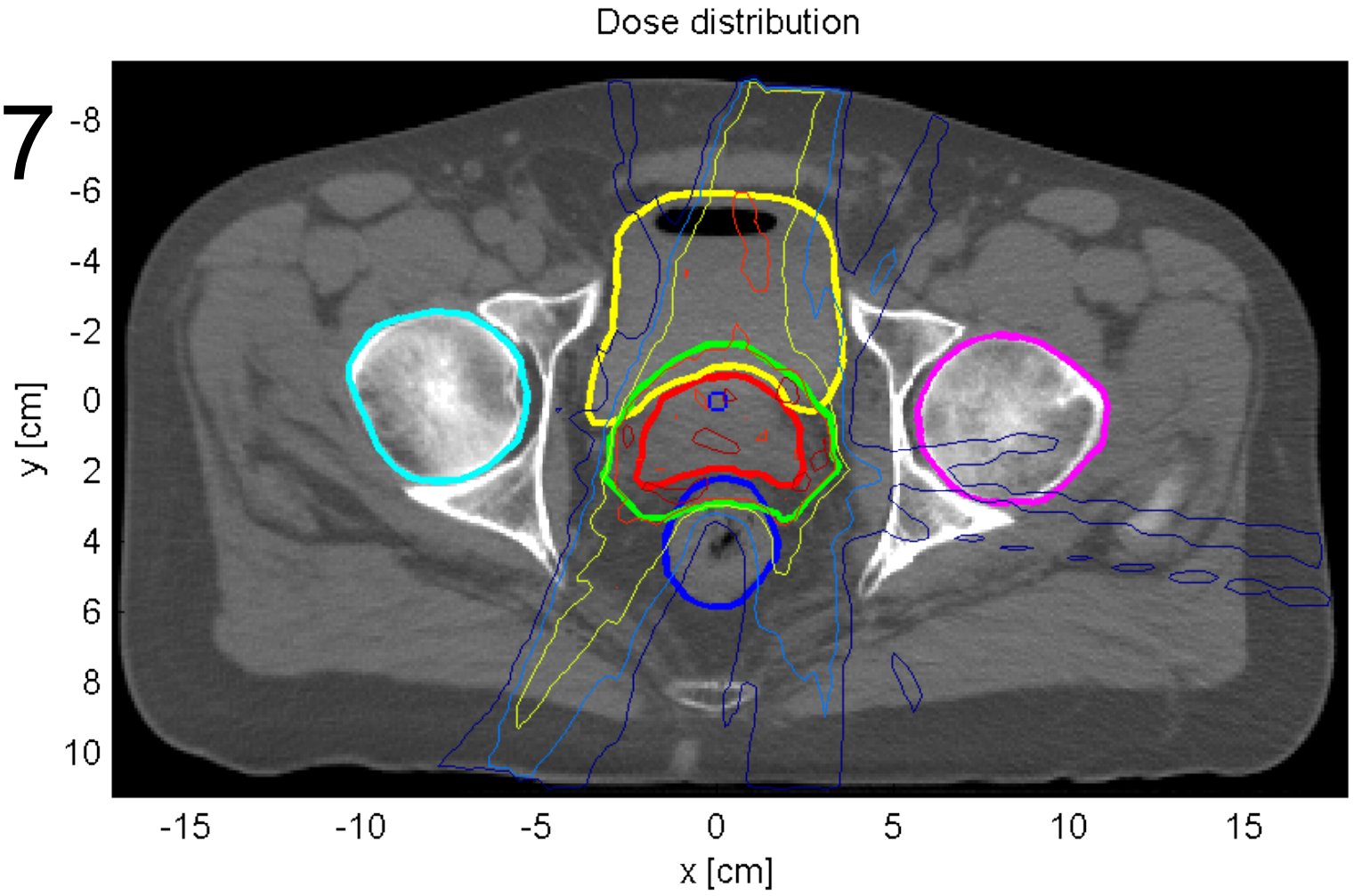
goal: min x

result: x=0.17

dose contour levels
1.15
1.00
0.80
0.50
0.30

GTV
bladder
left femur
right femur
rectum
PTV

Plan 7



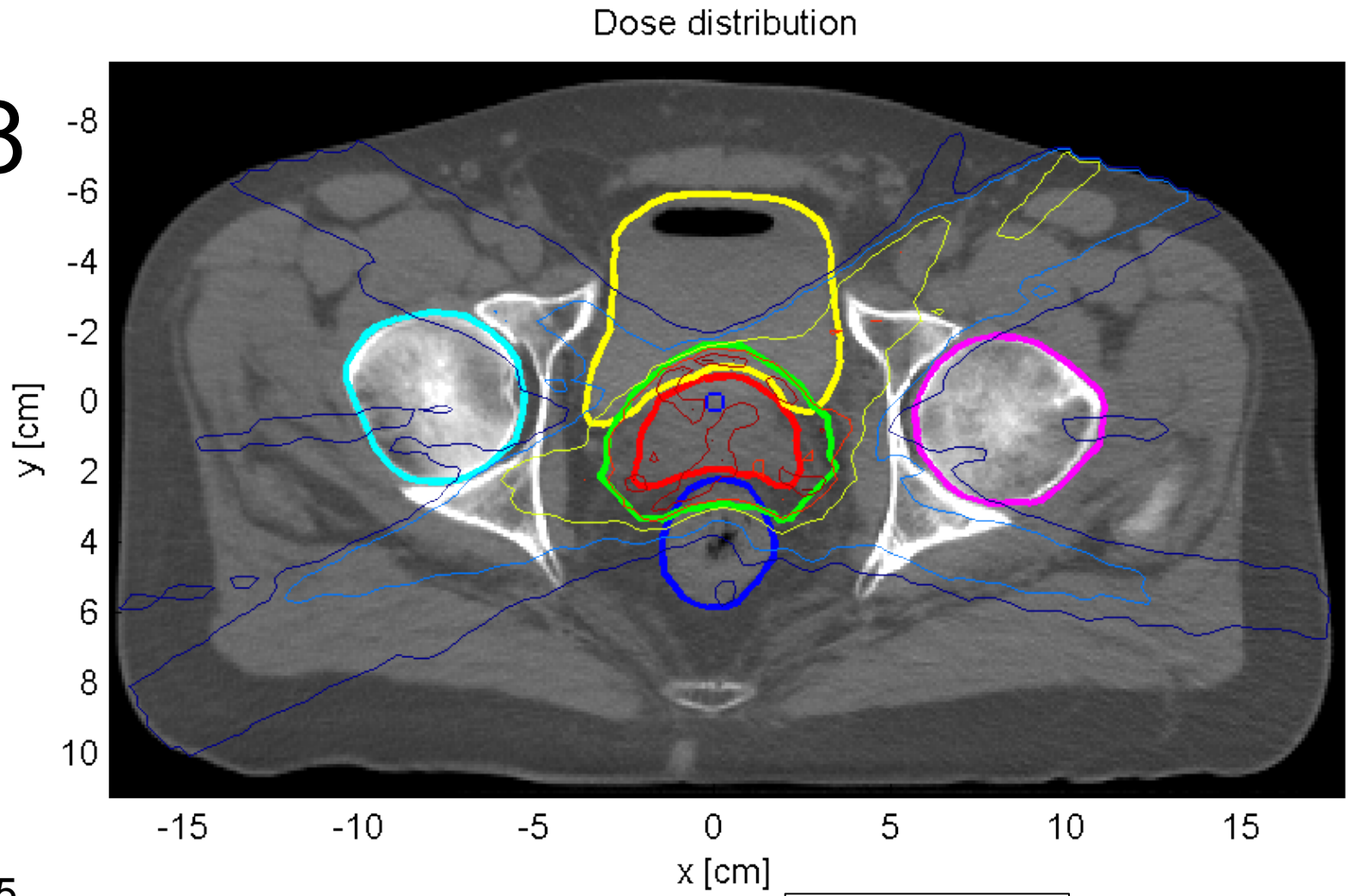
constraints:

- PTV > 1
- PTV < 1.15
- femurs < 0.375
- avg. dose in rectum outside PTV < 0.50
- avg. dose in bladder < 0.71
- external < 1
- avg. external dose < x

goal: min x

result: x=0.17

Plan 8



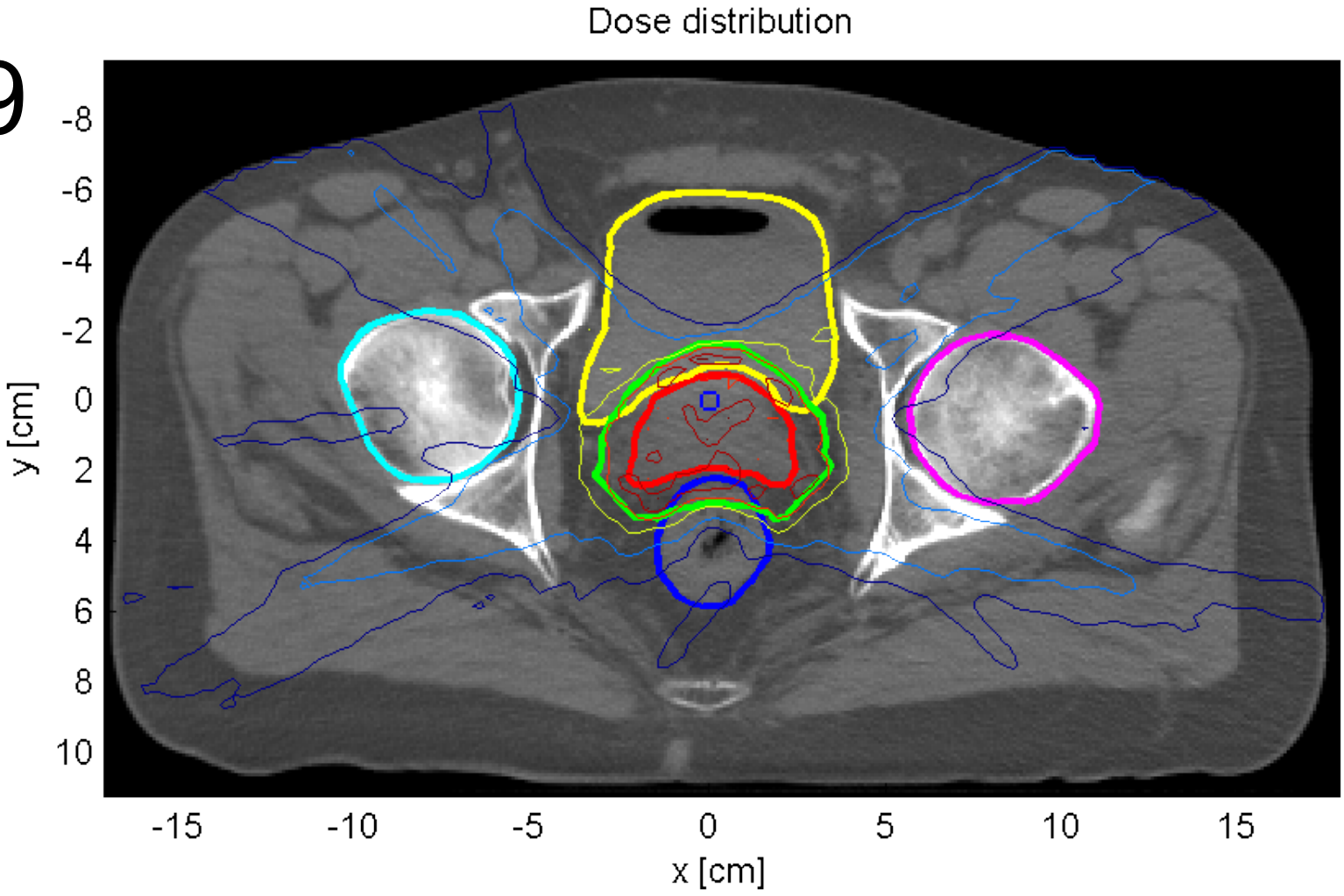
constraints:

- PTV > 1
- PTV < 1.15
- femurs < 0.375
- avg. dose in rectum outside PTV < 0.50
- avg. dose in bladder < x
- external < 1
- avg. external dose < 0.26

goal: min x

result: x=0.37

Plan 9



constraints:

- PTV > 1
- PTV < 1.15
- femurs < 0.375
- avg. dose in rectum outside PTV < 0.50
- avg. dose in bladder < x
- external < 0.80
- avg. external dose < 0.26

goal: min x

result: x=0.39

Hard constraints fit tumor control

- probability cell survives: $s_j = \exp[-D_j \alpha]$
  - $s_j << 1$
- probability all clonagenic tumor cells die:

$p = \prod 1 - s_j \approx 1 - \sum s_j$

$\approx 1 - \max s_j = 1 - \exp[-\alpha \min D_j]$

hence $p = f(\min dose)$

Hard constraints fit serial organs?

- serial organs: need all cells to functions
  - example: spinal cord
- probability all cells survive, p:

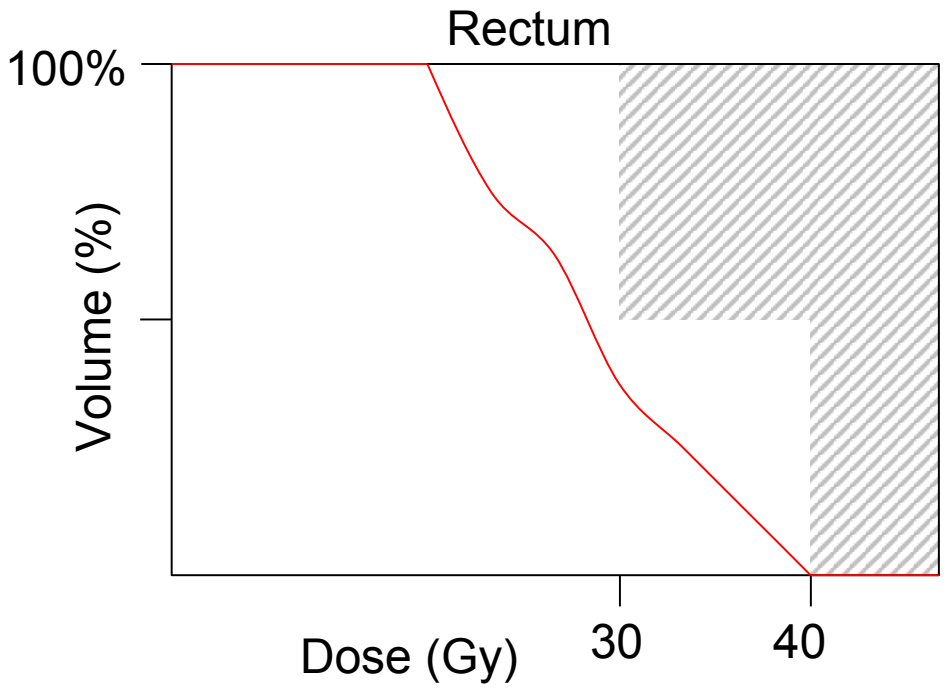
  recall $s_j = \exp[-D_j \alpha]$

  $p = \prod s_j = \exp[-\alpha \sum D_j]$

  hence $p = f$(mean dose)

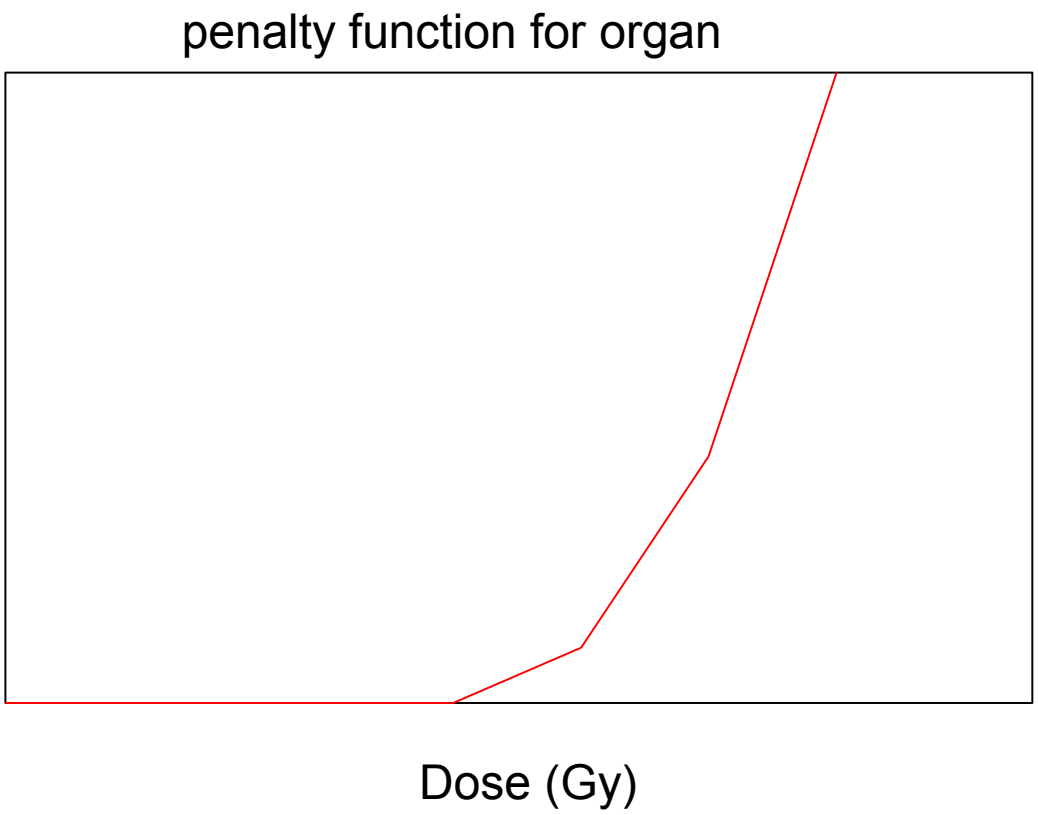
- caveat: $s_j = \exp[-D_j \alpha - D_j^2 \beta]$ and data ambiguous on importance of β

Cumulative dose-volume histogram



- $D_j \leq 40$ for all j in Rectum
- $D_j \leq 30 + BigNum * x_j$
  - x_j binary
  - $\sum x_j \leq 0.5 *$ num mesh points in Rectum

penalty functions

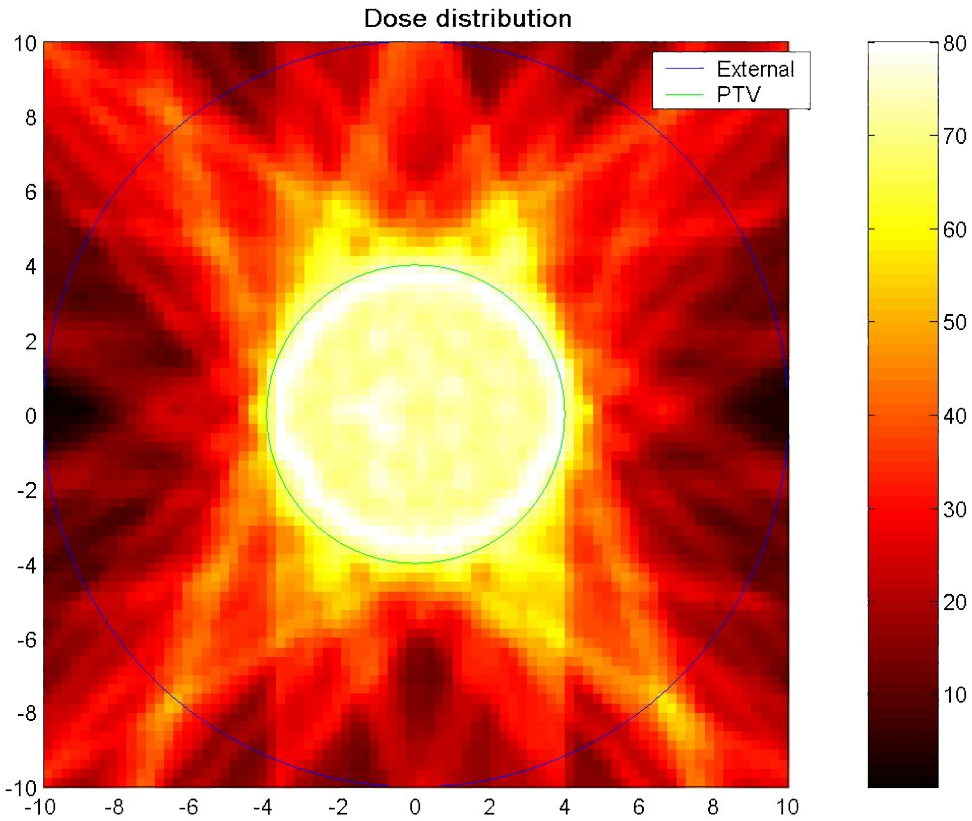


Tail averages

- $\text{avg}_{\text{over X\% of volume with highest dose}}\ D_j \le U$
- $\text{avg}_{\text{over X\% of volume with lowest dose}}\ D_j \ge L$
- each such constraint needs an artificial variable for every mesh point
- introduced by Romeijn et al.
  - (Phys. Med. Biol. 48)

- formulation for upper tail average:
  - $w_j \ge D_j - z, \qquad w_j \ge 0$
  - $z + 1/(\text{X\% vol}) \sum w_j \le U$
  - we want $w_j = \max(0, D_j - z)$
  - we want $z = \min_{\text{over X\% of volume with highest dose}} D_j$

gradient objective



$$\min \sum_{v} \mathbf{u}_v \bullet \nabla dose_v$$

Questions